Exam II

Solutions

Part A: Choose the best answer; 4 points each.

1. A raft is floating at rest in still water. A person sitting on the raft gets up and walks from the end of the raft nearest the shore to the end farthest from the shore, stops walking and sits down. Neglecting resistance from the water:

   A. The raft will continue to move slowly away from the shore.

   B. The raft will continue to move slowly toward the shore.

   C. The raft will stop, but will have moved away from the shore.

   ☛D. The raft will stop, but will have moved toward the shore. [CM does not move.]

2. A wheel rolls without slipping along a horizontal surface, then moves onto an incline. If the incline has friction so that the wheel continues to roll without slipping, its CM rises a height *h* before rolling back down. If the incline were frictionless instead:

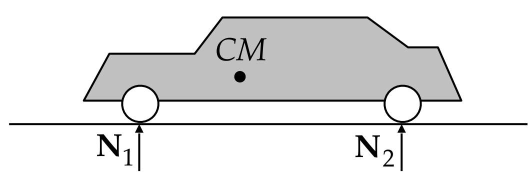
   A. The wheel's CM would rise a height greater than *h*.

   B. The wheel's CM would rise height *h*.

   ☛C. The wheel's CM would rise a height less than *h*. [The wheel still has rotational kinetic energy, so its potential energy would be smaller.]

   D. Whether the CM rises higher or lower than *h* depends on the detailed properties of the wheel.

3. Shown is an auto with the normal forces on the front and rear wheels indicated. The auto is slowing down by braking. [Consider torques about the CM.]



   A. Braking makes no change in the normal forces.

   B. Both normal forces increase.

   ☛C. N_1 increases and N_2 decreases. [Friction, to the right, gives a counter-clockwise torque, so the normal forces change to give a clockwise torque.]

   D. N_2 increases and N_1 decreases.

4. Satellite A is in a circular orbit of radius R around the earth. Satellite B is in an elliptical orbit with semi-major axis $a = R$. The satellites have the same mass. Which of the following is wrong?

   A. The satellites have the same total energy.

   B. The time for a complete orbit is the same for both.

   ☛C. When satellite B is at its point of closest approach to the earth, its speed is less than that of satellite A. [Greater. Potential energy is smaller.]

   D. When satellite B is at distance R from the earth, its speed is the same as satellite A.

Part B: Choose T or F, depending on whether the statement is true or false; 3 points each.

1. If all external forces on a system are conservative, total mechanical energy is conserved. **F** [Must include internal forces.]

2. A bike rider rounding a curve should lean the bike so that her effective weight acts along the line between her CM and the point of contact of the tires with the road. **T**

3. Melting of the glaciers after the ice age, which spread water from the northern lands more widely over the earth, resulted in a slight lengthening of the day. **T** [Increased the moment of inertia, so reduced the angular speed.]

Part C: Problems. Indicate your method clearly. An answer supported by no argument, or a fallacious one, may receive little or no credit. Each part of a problem carries the value of 5 points.

1-1. Two questions about the CM.

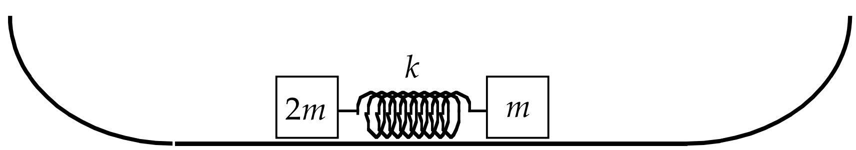
a. Take earth's mass to be $M = 6\times10^{24}$ kg, moon's mass to be $m = 7\times10^{22}$ kg, and the earth-moon distance (center to center) to be $d = 4\times10^{5}$ km. Find the distance of the CM of the system from earth's center. [Do it algebraically first, then put in the numbers.]

a. Take the center of the earth as origin. Then the CM is at distance $r_{CM} = \dfrac{md}{M+m} = \dfrac{(7\times10^{22})(4\times10^{5})}{6.07\times10^{24}} = 4.61\times10^{3}$ km. [Earth's radius is 6.37×10^{3} km, so the CM is inside the earth.]

b. For a system of N particles with masses m_i, located relative to the CM by position vectors $\mathbf{r}_i$, we have by definition of the CM that $\sum_i m_i\mathbf{r}_i = 0$. The gravitational force on a particle is $\mathbf{F}_i = m_i\mathbf{g}$, where $\mathbf{g}$ is a constant. Show that the total gravitational torque about the CM, $\sum_i \mathbf{r}_i \times \mathbf{F}_i$, is zero.

b. The total torque is $\sum_i \mathbf{r}_i \times m_i\mathbf{g} = \sum_i m_i\mathbf{r}_i \times \mathbf{g} = \left(\sum_i m_i\mathbf{r}_i\right)\times\mathbf{g} = 0$. [This is why we say gravity acts at the CM: it never produces a torque about the CM.]

1-2. The two blocks shown on a frictionless surface are pushed together to compress the spring a distance x. They are released at the same time, and the spring, which is not attached to them, expands. [Answers in terms of m, k and x.]



c. What are the speeds of the two blocks after they leave the spring and before they reach the curved parts of the track? [What is conserved?]

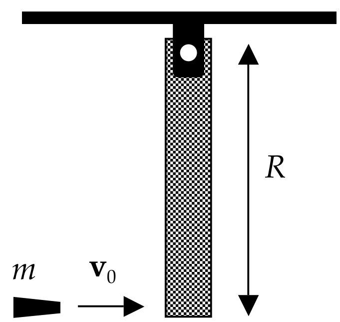
d. While they are on the curved part, the spring is removed, so when they come back they collide and stick together. What is the velocity of the combined block after the collision?

c. Momentum and energy are conserved. Call v the speed of the heavier block. Then the speed of the lighter one is $2v$. Conservation of energy gives

$$\tfrac{1}{2}kx^2 = \tfrac{1}{2}(2m)v^2 + \tfrac{1}{2}m(2v)^2 = 3mv^2, \text{ or } v^2 = \frac{kx^2}{6m}.$$

d. When they collide they have the same speeds but opposite directions. The total momentum is again zero, so when they stick together they come to rest.

2. A pellet of mass m moves as shown and strikes the bottom of the hanging object in an elastic collision. The hanging object has mass $3m$ and moment of inertia $I = mR^2$ about the pivot.



a. What quantities are conserved in the collision?

b. Find the velocity **v** of the pellet after the collision. [Use $v_0{}^2 - v^2 = (v_0 + v)(v_0 - v)$.]

c. Find the height h (above its original position) to which the CM of the object rises in its swing after the collision.

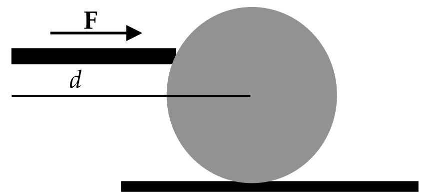
a. Energy and angular momentum about the pivot. (Normal forces from the pivot prevent conservation of linear momentum.)

b. Conservation of angular momentum: $mRv_0 = mRv + I\omega$. Conservation of energy: $\tfrac{1}{2}mv_0{}^2 = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2$. Using the given value of I we have $v_0 - v = R\omega$ and $v_0{}^2 - v^2 = (R\omega)^2$. Using the hint, we find $v_0 + v = R\omega$, which together with $v_0 - v = R\omega$ shows that $v = 0$.

c. The kinetic energy of the object after the collision is the initial kinetic energy of the pellet. By conservation of energy we have $\tfrac{1}{2}mv_0{}^2 = (3m)gh$, or $h = v_0{}^2 / 6g$.

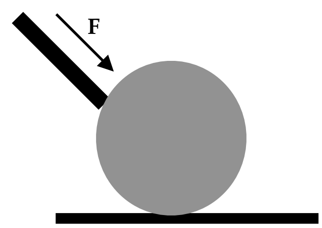
3-1. A billiard ball has mass *m*, radius *R*, and moment of inertia $\frac{2}{5}mR^2$. The ball is struck a sharp blow by a cue stick. The force, acting for a short time, imparts momentum $m\mathbf{v}_0$ to the CM of the ball. If it does not act through the CM, it also produces a torque about the CM, giving the ball angular momentum $\mathbf{r}\times m\mathbf{v}_0$, where **r** is the vector from the CM to the point where the ball is struck.

a. Suppose the ball is struck as shown, horizontally but at distance *d* above the center. The surface has friction, so the ball can roll. Find the value of *d* for which it will roll from the start without slipping first. [Relate the linear momentum to the angular momentum about the CM.]



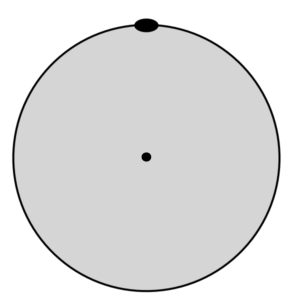
a. The momentum of the CM is mv_0 and the angular momentum about the CM is $dmv_0 = I\omega_0$. To roll, it must have $\omega_0 = v_0 / R$, so $dmv_0 = I(v_0 / R) = \frac{2}{5}mRv_0$. This gives $d = \frac{2}{5}R$.

b. In a trick shot, the player strikes the ball at a downward angle as shown. The ball jumps into the air as it moves forward. Explain why this happens. [Consider forces exerted by the table.]



b. The momentum given to the ball is transferred to the table in a large force for a short time. The reaction force is equally large (by Newton's 3rd law) and propels the ball off the table. It is as though the ball had been thrown at the table and bounced off. Because the table is very massive, it hardly moves.

3-2. A vertical disk mounted on a frictionless axle, as shown from the side, has attached to its top a small lead weight. The weight has mass *m* and the disk has radius *R* and moment of inertia *I* about the axle. Initially the system is at rest, but it is given a negligible nudge and rotates clockwise.



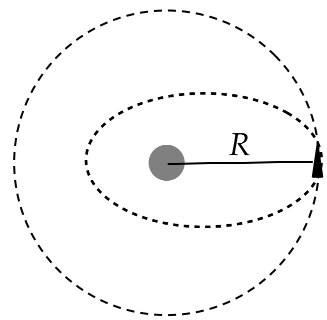
c. What is conserved in the motion? [Think carefully.]

d. What is the angular speed of rotation when the weight is at the bottom of the disk?

c. Only energy. Because of the lead weight, the CM of the system is not at the pivot, so gravity and the normal forces from the pivot do not cancel, and gravity gives an external torque about the pivot.

d. Take the bottom of the disk to be where gravitational potential energy is zero. The total moment of inertia about the pivot is $I_{tot} = I + mR^2$, so by conservation of energy $mg(2R) = \frac{1}{2} I_{tot}\omega^2 = \frac{1}{2}(I + mR^2)\omega^2$, or $\omega^2 = \dfrac{4mgR}{I + mR^2}$.

4. A spaceship is in the circular orbit shown orbit around the earth. Its kinetic energy is 3×10^{14} J. [Answers are numerical.]



a. Find its potential energy U and its total energy, E. [How are K ,U and E related for a circular orbit?]

b. At the point shown, the engines are fired to reduce its kinetic energy to 2×10^{14} J. What is its new total energy, E_1?

c. Express its distance of closest approach to earth in terms of R. [What is the ratio E / E_1?]

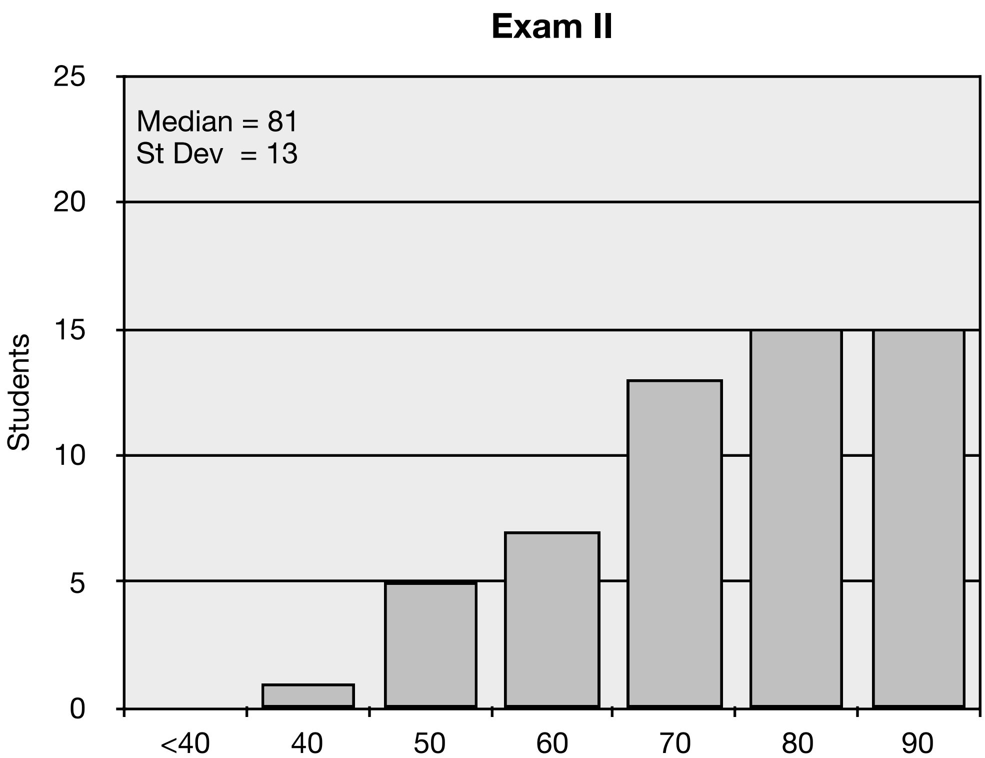
d. Copy the drawing and sketch the new orbit on it carefully.

a. For a circular orbit, $G\dfrac{Mm}{r^2} = m\dfrac{v^2}{r}$, so $K = \frac{1}{2}mv^2 = \frac{1}{2}\dfrac{GMm}{r} = -\frac{1}{2}U$. So $U = -2K = -6\times 10^{14}$ J, and $E = -K = -3\times 10^{14}$ J.

b. The potential energy does not change because the position is the same. The new total energy is $E_1 = K + U = 2\times 10^{14} - 6\times 10^{14} = -4\times 10^{14}$ J.

c. Since $E = -\dfrac{Gmm}{2R}$ and $E_1 = -\dfrac{GMm}{2a}$, we have $E / E_1 = a / R$. From the numbers we find $a = \frac{3}{4}R$. The total length of the new orbit is $2a = \frac{3}{2}R$. The present location is the point of greatest distance, so the point of closest approach is at distance $R/2$.

d. Drawing shows a pozsslibe orbit. (Minor axis depends on how the engines change the angular momentum.)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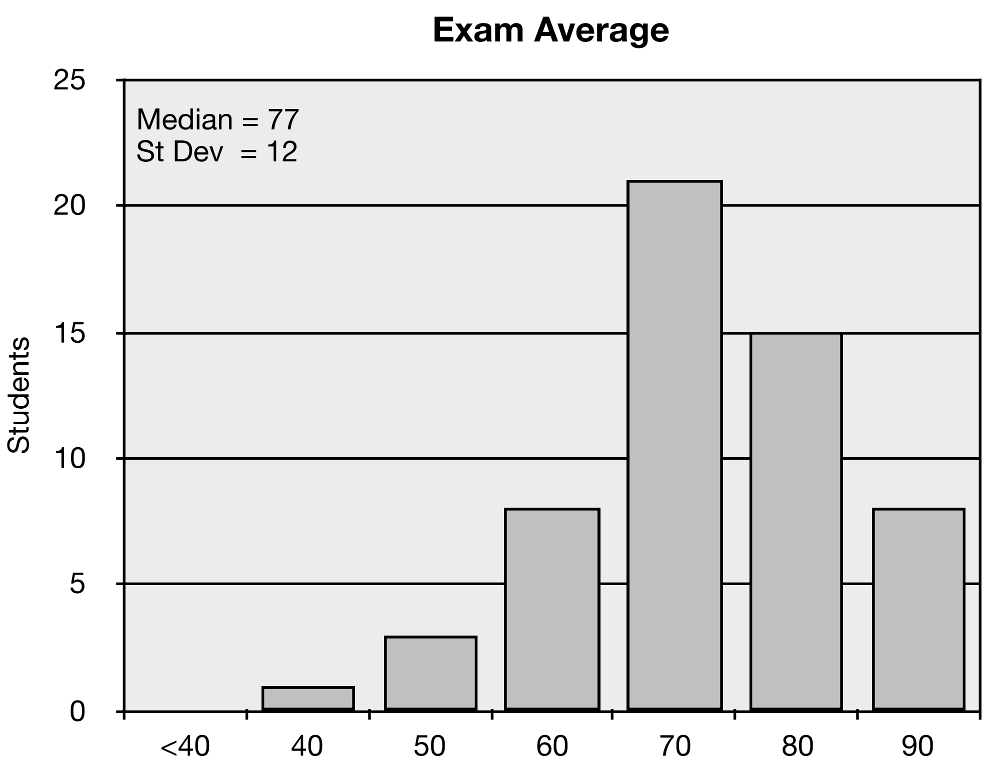
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