Exam III

Solutions

Part A: Choose the best answer; 4 points each.

1. You are sitting in a boat floating on still water in a small pond. In the boat is a large iron ball. You drop it into the water and it sinks.

   A. The boat floats higher in the water and the level of water in the pond rises.

   ☛B. The boat floats higher in the water and the level of water in the pond falls. [While the ball is in the boat, enough water is displaced to support its weight; when it is in the water, only its volume is displaced.]

   C. The boat floats higher in the water but the level of water in the pond does not change.

   D. The level of the boat in the water and the level of water in the pond do not change.

2. A file card is resting on top of two paper clips on a table as shown. A narrow jet of air can be directed into the page, either over or under the card.

   A. In either case the card will experience an upward force.

   B. In either case the card will experience a downward force.

   C. If the jet is directed over it the card experiences a downward force, but if the jet is directed under it the card experiences an upward force.

   ☛D. If the jet is directed over it the card experiences an upward force, but if the jet is directed under it the card experiences a downward force. [Pressure is lower in the jet.]

3. Two equal masses are executing SHM with the same frequency, but the amplitude of #1 is twice that of #2.

   A. The total energy of #1 is twice that of #2.

   ☛B. The maximum acceleration of #1 is twice that of #2.

   ☛C. The maximum speed of #1 is twice that of #2.

   D. None of the above is true.

   [$a_{\text{max}} = \omega^2 A$ and $v_{\text{max}} = \omega A$.]

4. A pendulum has string length R and mass m. It is started from rest with the string making angle θ with the vertical. Which of these is wrong?

   A. Its total energy is $mgR(1-\cos\theta)$.

   B. If $\theta << 1$ (radians), the motion is approximately SHM.

   ☛C. Increasing θ decreases the period of the oscillation. [Increases.]

   D. One of the above is wrong.

Part B: Choose T or F, depending on whether the statement is true or false; 3 points each.

1. A bubble in a glass of beer forms near the bottom and rises, its volume increasing as the water pressure decreases; its motion upward is also with increasing speed. **T** [Buoyant force increases with volume.]

2. When a damped oscillator has lost half its initial energy, the amplitude of its motion is half its initial value. **F** [$E \sim A^2$.]

3. When sound waves interfere destructively their energy is converted to heat. **F** [It goes elsewhere.]

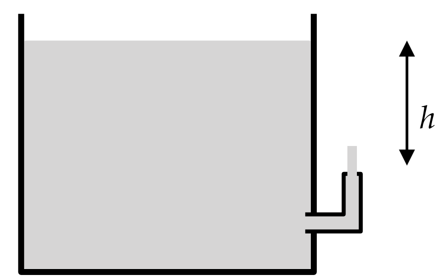
Part C: Problems. Indicate your method clearly. An answer supported by no argument, or a fallacious one, may receive little or no credit. Each part of a problem carries the value of 5 points.

1-1. Questions about fluids. Call the density of water ρ.

a. A block of wood of floats in a bucket of water, with half of its volume below the surface. If the system were in an elevator moving upward with acceleration a, what fraction of the block's volume would be submerged? Justify your answer.

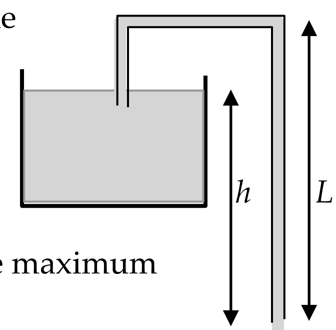
a. Half. Both the weight and the buoyant force are proportional to g, so they change together.

b. A large water tank has a pipe fitted near its bottom as shown, which directs the emerging stream upward. Show that the water in the stream rises to the same height h as the level of water in the tank.



b. Compare the top the tank water to the top of the stream. In both cases the speed is zero, and in both cases the pressure is air pressure, so by Bernoulli's theorem the heights must be the same.

1-2. The siphon shown is used to drain water from the tank. The bottom of the pipe is at distance h below the level of water in the tank, and the vertical length of the pipe is L.



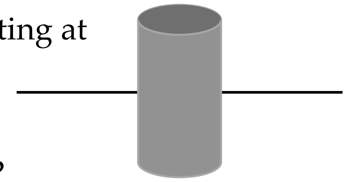
c. Find the speed of the water flowing from the bottom end of the pipe.

d. The pressure cannot be negative anywhere. Find the maximum length L.

c. Compare the top of the water in the tank to the stream coming out of the pipe. At both points the pressure is air pressure, so Bernoulli's theorem says $P_0 + \rho g h + 0 = P_0 + 0 + \frac{1}{2}\rho v^2$. This gives $v^2 = 2gh$.

d. Compare the stream at the top of the pipe to the stream at the bottom. We find $P + \rho g L + \frac{1}{2}\rho v^2 = P_0 + 0 + \frac{1}{2}\rho v^2$. This gives for the pressure at the top of the pipe $P = P_0 - \rho g L$. Since P cannot be negative, $L \le P_0 / \rho g$. [For water, this is about 32 ft.]

2-1. A wooden cylinder of mass M and cross section area A is floating at rest in water. It is pushed vertically deeper into the water a small distance x.



a. When it is released, what is the net upward force on it?

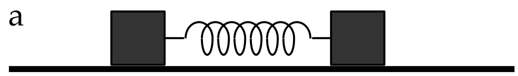
b. Show that it will execute SHM and find ω.

[Answers in terms of M, A, the density of water ρ, g and x.]

a. Pushing it down a distance x displaces a volume of water equal to Ax, so the buoyant force is the weight of this amount of water, or $F_B = (\rho A g) \cdot x$.

b. The force is proportional to the displacement and opposite to it, so the motion is SHM. The standard form is $F = M\omega^2 \cdot x$, so $\omega^2 = \rho A g / M$.

2-2. Two blocks of mass m are connected as shown by a spring of stiffness k on a frictionless table. They are pushed together, compressing the spring by distance d, and released from rest. [Answers in terms of m, k, and d.]



c. What is the amplitude A of the oscillation of each block about its equilibrium position?

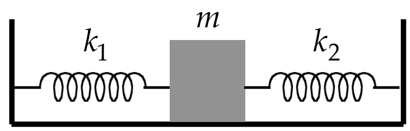
d. Find the maximum speed $v_{\max}$ of each block and use $v_{\max} = \omega A$ to find the angular frequency ω of the oscillations.

c. The CM remains at rest halfway between the blocks, so their amplitudes are $d/2$.

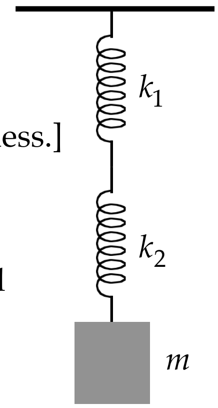
d. The total energy is $E = \frac{1}{2}kd^2$. When it is all kinetic energy we have $E = 2 \cdot \frac{1}{2} m v_{\mathrm{m}}{}^2 = m v_m{}^2$, so $v_m{}^2 = kd^2 / 2m$. Since also $v_m = \omega \cdot (d/2)$ we find $kd^2 / 2m = \omega^2 d^2 / 4$, or $\omega^2 = 2k/m$.

3. Questions about a block and two springs.

a. A block is attached to two springs which are attached as shown to two walls. The block slides on a frictionless surface. It is pushed a bit from equilibrium and released. Find the angular frequency ω of the subsequent oscillation. [If the block is moved a distance x from equilibrium, what is the force on it?]



b. The block shown is hanging at rest, supported by two springs connected together. What its the total amount x by which the two springs are stretched? [The springs are massless.]



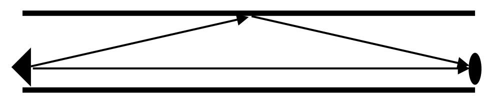
c. Suppose the hanging block is now pulled down a bit and released. What is the angular frequency ω of the resulting oscillation? [What is the effective stiffness k of the combined springs?]

a. If the block moves distance x to the right, it compresses the spring on the right and stretches the one on the left. These both result in a force to the left, with total force $F = k_1x + k_2x = (k_1 + k_2)x$. This is like a single spring of stiffness $k = k_1 + k_2$, so the angular frequency is $\omega^2 = (k_1 + k_2)/m$.

b. Both springs exert an upward force equal to the weight of the block, so we have $k_1y_1 = k_2y_2 = mg$. The total displacement is

$$y = y_1 + y_2 = mg/k_1 + mg/k_2 = mg\left(\frac{1}{k_1} + \frac{1}{k2}\right).$$

c. If it were a single spring of stiffness k we wold have $y = mg/k$, so the equivalent single spring has stiffness given by $\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$, or $k = \frac{k_1k_2}{k_1 + k_2}$. The angular frequency is $\omega^2 = \frac{1}{m} \cdot \frac{k_1k_2}{k_1 + k_2}$



4-1. A loudspeaker is at one end of a narrow hallway and a listener stands against the wall at the other end, as shown from above. A sound wave from the speaker directly to the listener travels distance 10 m. A wave reflected off the opposite wall as shown travels 11 m to reach the listener. These waves have equal amplitude as they reach the listener. The speed of sound is 340 m/s, and there is no phase change by reflection at the wall. The speaker emits sound of a single frequency, which is being slowly increased from 20 Hz.

a. What is the first frequency for which the listener hears no sound?

b. What is the first frequency for which the listener hears maximum intensity?

a. The path difference is 1 m. For destructive interference $k\Delta x = \pi$, or $\frac{2\pi}{\lambda}\cdot 1 = \pi$, so $\lambda = 2$ m. Using $f = v/\lambda$ we find $f = 340/2 = 170$ Hz.

b. Now $k\Delta x = 2\pi$, so $\lambda = 1$ m and $f = 340/1 = 340$ Hz.

4-2. Two waves are described by these equations:

$$y_1(x,t) = A\cos(kx_1 - \omega t)$$
$$y_2(x,t) = 2A\cos(kx_2 - \omega t).$$

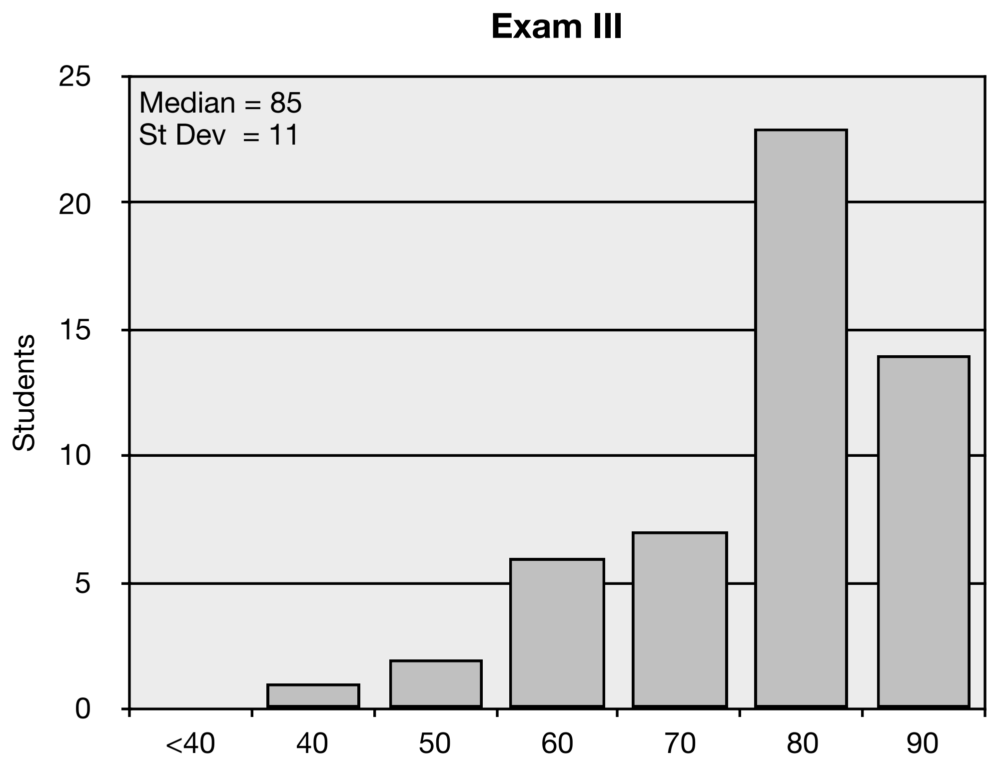
The intensity of wave 1 alone is I_0. Both waves have wavelength $\lambda = 0.5$ m.

c. Find the smallest value of $|x_2 - x_1|$ for which there is constructive interference, and find the intensity (in terms of I_0) in that case..

d. Do the same for destructive interference.

c. Using $\delta = k\Delta x$ we see that $\Delta x = 0$ gives $\delta = 0$, which is constructive interference, so that is the smallest value. The total amplitude is $A + 2A = 3A$, so the intensity is 9 times that of y_1 alone, or $9I_0$.

b. Set $\delta = \pi$. We find $\pi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{0.5}\Delta x = 4\pi\Delta x$, so $\Delta x = 0.25$ m. The total amplitude is $2A - A = A$, so the intensity is I_0.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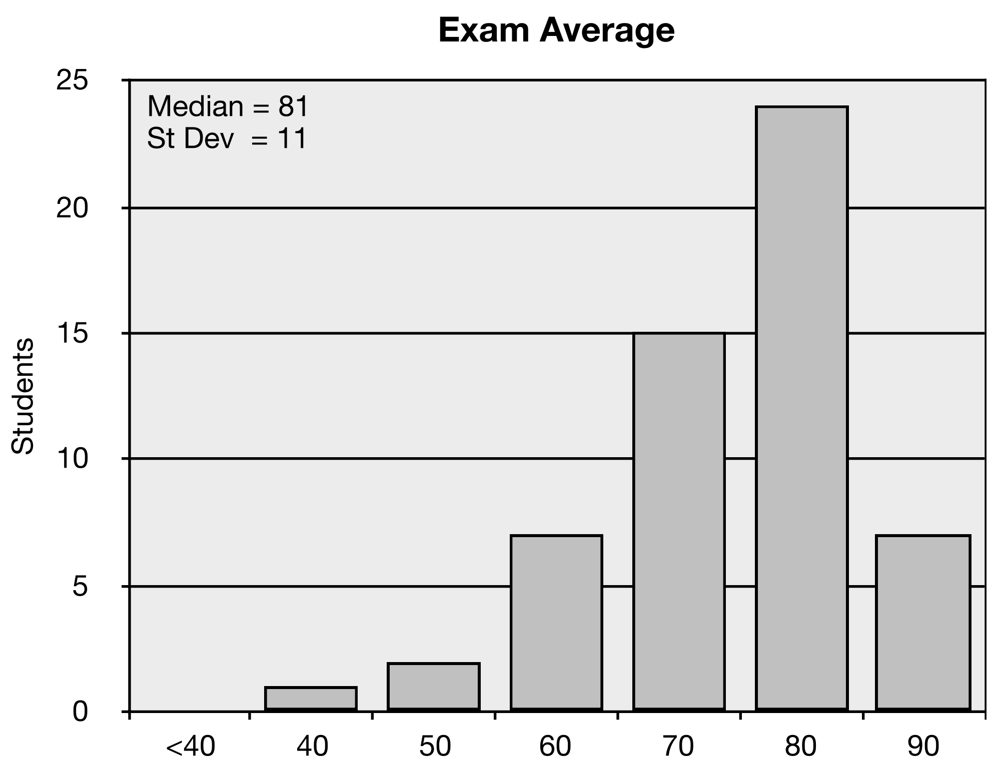
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