Final Exam

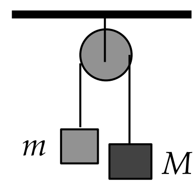
Solutions

Part A. Multiple choice questions. Check the best answer. Each question carries a value of 4 points.

1. A system is subject to zero net external force.

- [ ] Its total linear momentum and angular momentum are certainly conserved, but its total mechanical energy might not be.
- [ ] Its total linear momentum, angular momentum and mechanical energy are all certainly conserved.
- [√] Its total linear momentum is certainly conserved, but its total angular momentum and mechanical energy might not be.
- [ ] None of these three quantities are certainly conserved.

2. In the situation shown, $M > m$ and the pulley has mass. Which of the following is NOT true when the system is released from rest?

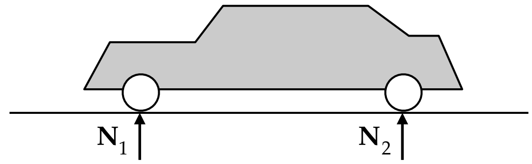


- [ ] The tension in the right hand string is less than Mg.
- [ ] The tensions in the two strings are different.
- [√] The tension in the left hand string is less than mg. [Greater.]
- [ ] Mechanical energy is conserved if the pulley has no friction.

3. A wheel rolls without slipping on a surface. Which of the following is *wrong*?

- [ ] If it is accelerating, there must be a torque about its rotation axis.
- [ ] If the surface is not horizontal there must be friction.
- [ ] The CM speed of the wheel is equal to its radius times its angular speed.
- [√] One of the above is not true. [They are all true.]

4. Shown is an auto with normal forces on the front and rear wheels indicated. When the auto accelerates to the left:

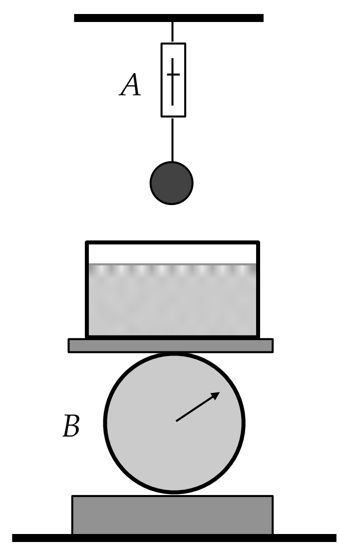


- [ ] The normal forces remain the same.
- [ ] N_1 increases while N_2 decreases.
- [√] N_2 increases while N_1 decreases. [To provide a counter-clockwise torque about CM to cancel the clockwise torque from friction forces to the left.]
- [ ] How the normal forces change depends on which are the drive wheels.

5. Concerning Kepler's three laws, as explained by Newtonian gravity, which of the following is NOT true?

- [ ] One can be used to determine the mass of a planet if it has a small moon.
- [√] One says the quantity T^2 / a^3 (T = period, a = semi-major axis) is the same for the earth's orbit around the sun and moon's orbit around the earth. [Must be orbits around the same object.]
- [ ] One says that planetary orbits are ellipses with the sun at a focus.
- [ ] One is equivalent to conservation of angular momentum of the orbit.

6. A metal ball, hanging from scale *A* as shown, is lowered into a beaker of water resting on scale *B*. As the ball is lowered into the water until it is hanging submerged, what happens to the scale readings?

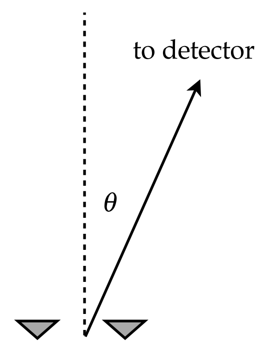


- [ ] Nothing.
- [ ] Both readings decrease.
- [ ] Scale *A* reading decreases and scale *B* reading does not change.
- [√] Scale *A* reading decreases and scale *B* reading increases. [Reaction to buoyant force.]

7. A violin string is played simultaneously with a tuning fork of frequency 440 Hz, and 4 beats per second are heard. The tension in the string is increased and the beats disappear. The initial frequency of the violin string was:

- ☐ 444 Hz
- ☐ 442 Hz
- ☐ 438 Hz
- ☑ 436 Hz [Increasing tension increases speed and thus frequency.]

8. Shown are two speakers emitting sound waves of wavelength λ, in phase with each other. The speakers are 3λ apart. As the detector moves from $\theta = 0$ to $\theta = \pi/2$, the number of angles at which maximum intensity will be received is:



- ☐ 2.
- ☐ 3.
- ☑ 4. [Phase differences are 0, 2π, 4π, and 6π.]
- ☐ 5.

Part B. True-false questions. Check T or F depending on whether the statement is true or false. Each question carries a value of 3 points.

1. Balls thrown into the air at the same speed will land at the same level with the same speed, regardless of the elevation angles at which they were thrown.

☑ T ☐ F [Conservation of energy.]

2. Galileo's finding, that an object dropped from rest in a gravitational field has an acceleration independent of its mass, is only valid as an approximation for objects near the earth's surface.

☐ T ☑ F [A general property of gravity.]

3. Because it does no work, static friction cannot cause an object to accelerate.

☐ T ☑ F [It moves you forward in walking.]

4. In the vicinity of a minimum in potential energy, motion with small kinetic energy is approximately simple harmonic motion.

☑ T ☐ F

5. For interference of two waves, each of intensity I_0, if there is a region where the total intensity is greater than $2I_0$, then there must be another region where the total intensity is less than $2I_0$.

☑ T ☐ F [Energy is conserved.]

6. If the difference between the frequencies of successive harmonics of an organ pipe is the frequency of the fundamental, then the pipe is closed at one end.

☐ T ☑ F [Open at both ends.]

Part C. Problems. Work problem in space provided, using extra sheets if needed. Explain your method clearly. Problems carry the point values shown.

1a. Moving at speed v and trying to avoid a fallen tree in front of you, you turn your car sharply to the right in a circular path.

a. If the coefficient of static friction is μ_s, what is the smallest radius R you can use without sliding? [The road is on level ground.]

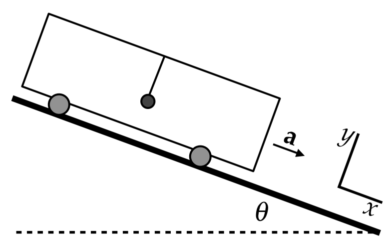
b. Show that you could stop in half that distance by going straight ahead.

[10 points]

a. Friction must provide the radial acceleration $a = v^2 / R$. The maximum friction force is $f_s = \mu_s N = \mu_s mg$. So we have $\mu_s mg = mv^2 / R$, or $R = v^2 / \mu_s g$.

b. The maximum friction force is the same, so the acceleration (backwards) is $\mu_s g$. Using $v_f{}^2 = v^2 - 2ax$, where $v_f = 0$, we find $x = v^2 / 2\mu_s g$ as claimed.

1b. A railroad car is rolling freely down an incline. Hanging from its ceiling is a mass on a string. It is noted that the string is perpendicular to the ceiling.



a. What is the magnitude of the car's acceleration down the incline?

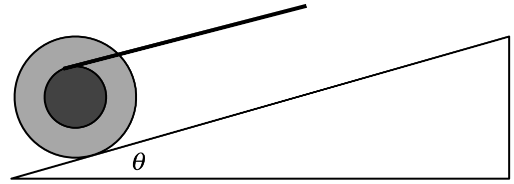
b. Show that the mass is hanging in the direction of $\mathbf{g}_{eff}$. [Use the coordinate system shown and break $\mathbf{g}$ into components.]

[10 points]

a. The normal force and the component of the car's weight normal to the incline cancel. The component of the weight down the incline is $mg\sin\theta$, so the acceleration is $a = g\sin\theta$.

b. Calculate the components of $\mathbf{g}_{eff} = \mathbf{g} - \mathbf{a}$. In the x-direction we have $g\sin\theta - g\sin\theta = 0$. In the y-direction we have only $-g\cos\theta$. So $\mathbf{g}_{eff}$ is in the negative y-direction as claimed, with magnitude $g\cos\theta$.

2. A wheel of mass m and outer radius R is being rolled up an incline at *constant* speed by a rope wrapped around its axle, as shown. The axle has radius $R/2$, and the incline has static friction coefficient μ_s. Give answers in terms of m, g and θ.



a. What direction is the friction force f_s? Explain how you know.

b. What is the tension T in the rope?

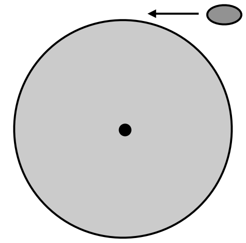
c. What is the minimum value of μ_s for the wheel to roll at constant speed?

[15 points]

a. The string produces a clockwise torque about the axle. To move at constant speed (and constant angular speed, since it rolls) the net torque must be zero, so friction must produce a counter-clockwise torque. So the friction force is up the incline.

b. For the two torques to cancel we have $T \cdot R/2 = f_s \cdot R$, so $f_s = T/2$. The forces along the incline must cancel to have constant speed, so we have $T + f_s - mg\sin\theta = 0$, or $\frac{3}{2}T = mg\sin\theta$, so $T = \frac{2}{3}mg\sin\theta$.

c, For the forces normal to the incline we have $N - mg\cos\theta = 0$. The maximum friction force is thus $\mu_s N = \mu_s mg\cos\theta$. This must be equal to $\frac{1}{2}T = \frac{1}{3}mg\sin\theta$. So we find $\mu_s = \frac{1}{3}\tan\theta$ as the minimum value.



3. As shown from above, a child of mass m runs at velocity $\mathbf{v}_0$ tangent to the rim of a playground turntable of radius R at rest, and jumps on. She stays at rest on the rim for a while, then jumps off backward in such a way that her final velocity relative to the ground is zero. Give answers in terms of m, R and v_0. [Rotational kinetic energy can be written as $L^2 / 2I$.]

   a. What is conserved in both jumps?

   b. Call the child's initial kinetic energy K_0 and her initial angular momentum relative to the turntable axle L_0. What are these?

   c. The turntable has moment of inertia $I = 9mR^2$ about its axle. What is the kinetic energy K_1 of the turntable plus child when she is at rest on it?

   d. What is the kinetic energy K_2 of the turntable after she has jumped off?

   [20 points]

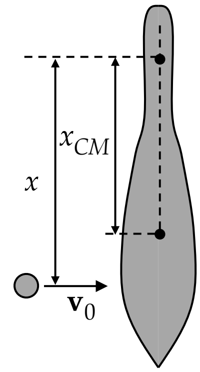
a. Angular momentum about the axle. The axle exerts an external sideways force during the jumps, so linear momentum is not conserved. Energy is lost by internal forces when the child comes to rest on the turntable and gained when she jumps off again.

b. $K_0 = \frac{1}{2}mv_0^{\,2}$, $L_0 = mRv_0$.

c. The total moment of inertia is $I_{tot} = 9mR^2 + mR^2 = 10mR^2$. Since L is constant, the new kinetic energy is $K_1 = L_0^{\,2} / 2I_{tot} = (mRv_0)^2 / 20mR^2 = \frac{1}{20}mv_0^{\,2}$. [Can also find the angular speed ω from $L_0 = I_{tot}\omega$ and use $K_1 = \frac{1}{2}I_{tot}\omega^2$.]

d., Her kinetic energy is now zero and the moment of inertia returns to that of the turntable alone, so $K_2 = L_0^{\,2} / 2I = (mRv_0)^2 / 18mR^2 = \frac{1}{18}mv_0^{\,2}$. [The energy gain comes from her muscles in the jump.]

4. An object hangs at rest from a frictionless pivot as shown. It has mass M, moment of inertia I about the pivot, and CM at distance x_{CM} from the pivot. A ball of mass m moving horizontally with speed v_0 strikes the object at distance x below the pivot. The collision is elastic and the ball bounces straight back with speed v.



   a. What quantities are conserved in the collision, regardless of the value of x?

   b. Find the angular speed ω of the object just after the collision, in terms of v_0 and x. [Use $a^2 - b^2 = (a+b)(a-b)$.]

   c. There is a value of x for which the horizontal force from the pivot is zero, so that horizontal linear momentum is also conserved. Find that value of x, in terms of I, M and x_{CM}. [Linear momentum of the object is $M\mathbf{v}_{CM}$.]

[15 points]

a. Angular momentum about the pivot and (because the collision is elastic) kinetic energy.

b. Conservation of angular momentum about the pivot: $mv_0 x = -mvx + I\omega$.

Conservation of kinetic energy: $\frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$. So we have (1) $v_0 + v = \dfrac{I\omega}{mx}$ and (2) $v_0^2 - v^2 = \dfrac{I\omega^2}{m}$. Dividing (2) by (1) we find $v_0 - v = x\omega$. Adding this to (1) we get $2v_0 = \omega(x + I/mx)$, so $\omega = \dfrac{2v_0/x}{1 + I/mx^2}$. [One can also solve for v.]

c. Require conservation of horizontal momentum: $mv_0 = Mv_{CM} - mv$, where $v_{CM} = x_{CM}\omega$. We have $m(v_0 + v) = Mx_{CM}\omega$. Using (1) from (b) we see that this is $m \cdot \dfrac{I\omega}{mx} = Mx_{CM}\omega$, or $x = \dfrac{I}{Mx_{CM}}$. [Since (1) follows from conservation of angular momentum, this value of x is independent of whether the collision is elastic.]

5a. Spring tides are unusually high tides caused by the action of the moon and sun.

a. Make a drawing showing the relative position of those three objects when spring tides occur.

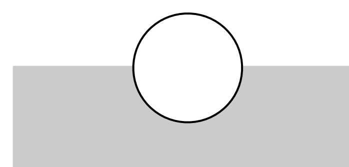
b. Such tides occur two weeks apart. Explain why, perhaps with a second drawing.

[10 points]

a. One configuration in which both bodies produce high tides on the same two sides of the earth:

b. Another, two weeks later:

5b. A spherical ball of volume V is floating in water, exactly half submerged as shown.



a. Express the ball's mass m in terms of V and the density ρ_w of water.

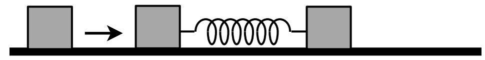
b. If you push the ball slightly deeper into the water and then release it, will its vertical oscillation be simple harmonic motion? Explain.

[10 points]

a. The ball's weight is supported by the buoyant force, which is the weight of a volume of water equal to the volume submerged. So $mg = (V/2)\rho_w g$ and

$m = (V/2)\rho_w$.

b. It will not be SHM. The horizontal cross-section of the ball is not constant, so the water displaced is not simply proportional to the depth of the ball. This means the buoyant force is not proportional to the extra depth, so it is not of the form $F = -(const.)\cdot x$



6. Two blocks of mass m attached to a spring of stiffness k are at rest on a horizontal frictionless surface. A third block of the same mass, moving with speed v_0, collides elastically with the left block of the pair, transferring to that block all its energy and momentum. You are to determine properties of the motion of the first two blocks after the collision, in terms of the given quantities.

   a. What is the kinetic energy of the CM motion?

   b. What is the total energy of the oscillation of the two blocks about the CM?

   c. Call A the amplitude of the oscillation for each block, and call $v_{\max}$ its maximum speed (relative to the CM) of that oscillation. Find A and $v_{\max}$.

   d. Use $v_{\max} = \omega A$ to find ω.

   [20 points]

a. The total momentum is $P_{tot} = mv_0$ so the speed of the CM is $v_{CM} = P_{tot}/2m = v_0/2$. The kinetic energy is $K_{CM} = \frac{1}{2}(2m)(v_0/2)^2 = \frac{1}{4}mv_0{}^2$.

b. The total energy of the system is that of the original block, $\frac{1}{2}mv_0{}^2$. Subtracting K_{CM}, we find the energy of motion about the CM to be $\frac{1}{4}mv_0{}^2$.

c. When both masses are at rest relative to the CM, each has stretched (or compressed) the spring by the amount A. The total extension (or compression) of the spring is thus $2A$ and the maximum potential energy is $\frac{1}{2}k(2A)^2 = 2kA^2$. At this time the blocks are at rest relative to the CM, so we have $2kA^2 = \frac{1}{4}mv_0{}^2$, or $A^2 = \frac{m}{8k}v_0{}^2$. When the spring is relaxed, the total kinetic energy relative to the CM is $2 \cdot \frac{1}{2}mv_{\max}{}^2 = \frac{1}{4}mv_0{}^2$, or $v_{\max}{}^2 = \frac{1}{4}v_0{}^2$.

d. Comparing these results we see that $v_{\max}{}^2 = \frac{2k}{m}A^2$, so $\omega^2 = 2k/m$.

7a. Two waves of equal wavelength and frequency and moving in the same direction interfere. The waves have separate intensities I_1 and I_2.

a. Show that the maximum resultant intensity is $(\sqrt{I_1}+\sqrt{I_2})^2$.

b. Show that the minimum resultant intensity is $(\sqrt{I_1}-\sqrt{I_2})^2$.

[10 points]

a. I is proportional to A^2, so we can write $A = K\sqrt{I}$, where K is a constant. For constructive interference the total amplitude is the sum of the amplitudes, so $A_{tot} = K\left(\sqrt{I_1}+\sqrt{I_2}\right)$. Using $I = A^2/K^2$ we find $I = \left(\sqrt{I_1}+\sqrt{I_2}\right)^2$ as claimed.

b. The procedure is the same, but the interference is destructive, so $A_{tot} = K\left(\sqrt{I_1}-\sqrt{I_2}\right)$, leading to the result claimed.

7b. Consider three wave functions describing waves in a certain medium:

$$y_1 = A\cos(20\pi x - 2000\pi t)$$
$$y_2 = A\cos(20.1\pi x - 2010\pi t)$$
$$y_3 = A\cos(20\pi x + 2000\pi t)$$

a. Which pair will combine to give a standing wave, and how far apart are the nodes?

b. Which pair will combine to give beats, and what is the beat frequency?

[10 points]

a. The waves must move in opposite directions, so it is the pair y_1 and y_3. The nodes are $\lambda/2$ apart, where $k = 2\pi/\lambda = 20\pi$. Thus $\lambda = 0.1$ m and the nodes are 0.05 m apart.

b. The waves must go in the same direction, with different wavelength and frequency, so it is the pair y_1 and y_2. The frequencies are found from $\omega = 2\pi f$, so they are $f_1 = 1000$ Hz and $f_2 = 1005$ Hz. The beat frequency is $|f_1 - f_2| = 5$ per second.

8a. While driving along a highway you hear the horn of an emergency vehicle behind you, so you pull over and stop. As the vehicle passes you the frequency you hear from the horn drops to 6/7 of what you heard when it was behind you.

a. Call the ratio of the vehicle's speed to the speed of sound α and find it.

b. If the frequency you heard when the vehicle was behind you was 520 Hz, what is the frequency of the horn?

[10 points]

a. When the vehicle is behind, you hear $f_1 = f_0 \frac{1}{1-\alpha}$. When it is in front you hear $f_2 = f_0 \frac{1}{1+\alpha}$. So $\frac{f_2}{f_1} = \frac{6}{7} = \frac{1-\alpha}{1+\alpha}$. This gives $\alpha = 1/13$.

b. We have $520 = f_0 \frac{1}{1-1/13} = f_0 \frac{13}{12}$. This gives $f_0 = 480$ Hz.

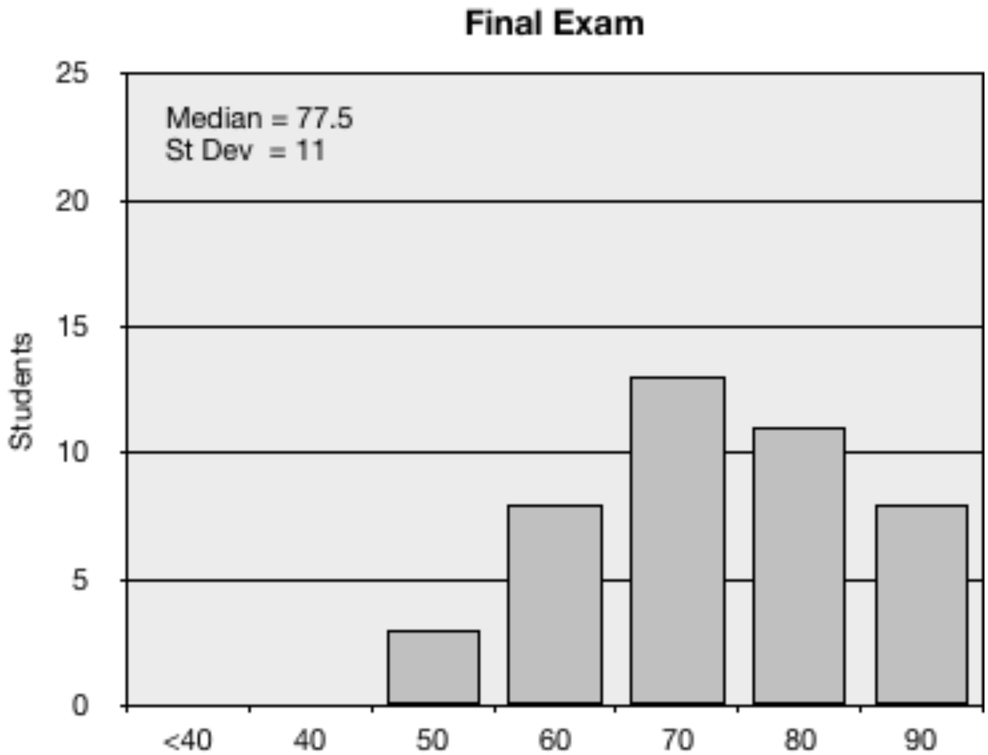
8b. Ordinary bugle calls are based on four of the harmonics of the instrument. For a particular bugle the frequencies are 330, 440, 550 and 660 Hz.

a. What number harmonics are these?

b. What is the fundamental frequency of the instrument?

a. The difference between successive harmonics is 110 Hz and they are all multiples of this, so they are numbers 3, 4, 5, and 6.

b. The fundamental is 110 Hz.


Final Exam
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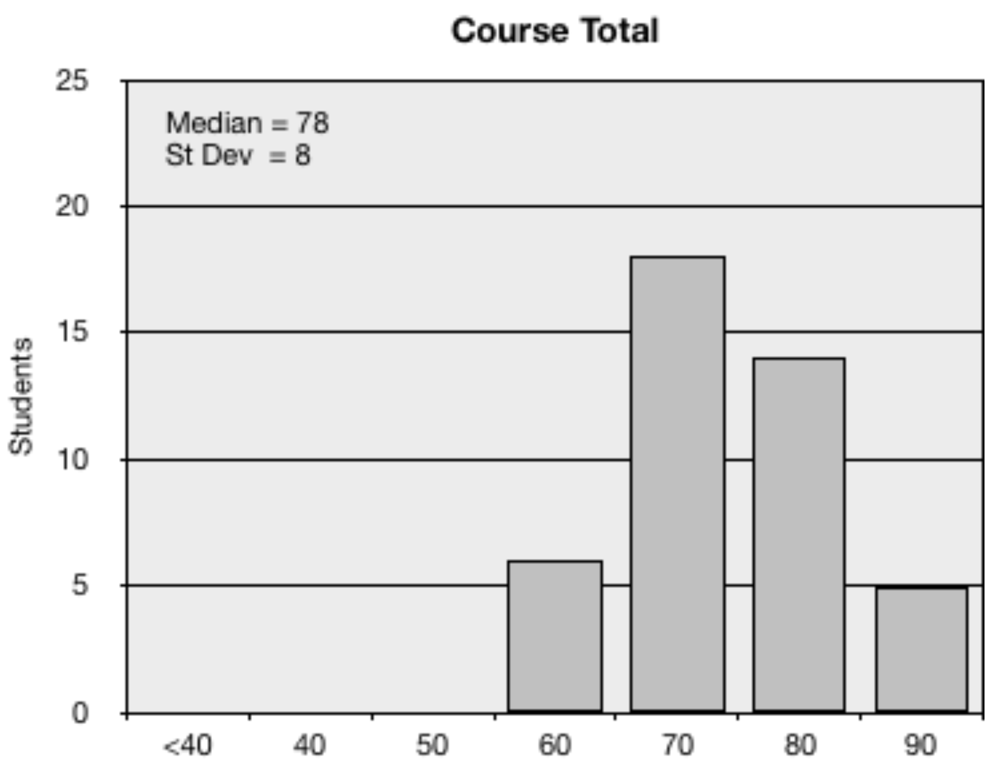
10
5
0
<40
40
50
60
70
80
90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St Dev = 8
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20
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10
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0
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40
50
60
70
80
90