Exam I

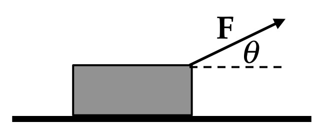
Solutions

Part A: Multiple choice questions. Check the best answer. Each question carries a value of 4 points.

1. Concerning the general principles of mechanics, which of the following is *wrong*?

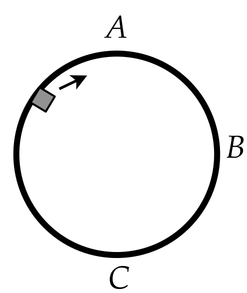
- [ ] In an inertial frame, a body subject to no net force moves with constant velocity.
- [ ] The total force on an object is equal to the rate of change of its momentum.
- [ ] If body A exerts force **F** on body B, then body B also exerts force –**F** on body A.
- [x] The change in total mechanical energy is equal to the work done by external forces. [All forces.]

2. A box is pulled across a rough floor by a force **F**. As the angle θ is increased slowly from zero, with F constant, which of the following is *wrong*?



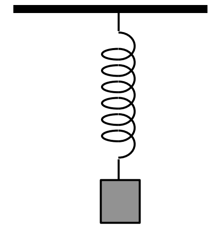
- [ ] The normal force exerted by the floor decreases.
- [x] The kinetic friction force increases. [Decreases, because the normal force decreases.]
- [ ] The horizontal component of **F** decreases.
- [ ] The acceleration of the box increases.

3. A block slides all the way around the *inside* of a vertical circular track as shown. The track is frictionless. Points A, B, and C are at the top, side, and bottom of the track respectively.



- [ ] The minimum possible speed at A is zero.
- [ ] The speed is the same at all three points.
- [ ] The direction of the acceleration is toward the center of the circle at all three points.
- [x] The total mechanical energy of the system is the same at all three points.

4. A mass m is attached to a vertical spring of stiffness k hanging from a ceiling as shown. Initially the mass is held a the height where the spring is at its equilibrium length. Which of the following is *wrong*?



☐ If the mass is lowered slowly, it will hang at rest when the spring has been stretched through distance mg / k.

☐ If the mass is dropped from rest, it will stop momentarily after falling distance $2mg / k$.

☑ If the mass is dropped from rest, it will have its maximum acceleration when it has fallen distance mg / k. [Acceleration is zero at that point.]

☐ If the mass is dropped from rest, it will return to its original height with zero instantaneous velocity.

Part B: True-false questions. Check the correct answer. Each question has a value of 3 points.

1. In a head-on collision between a small car and a large SUV the acceleration of the SUV during the collision has the larger magnitude.

True ☐ False ☑ [Smaller, because its mass is larger and the forces have equal magnitude.]

2. Curves on highways are banked to use a component of the normal force as part of the radial force, reducing the need for friction.

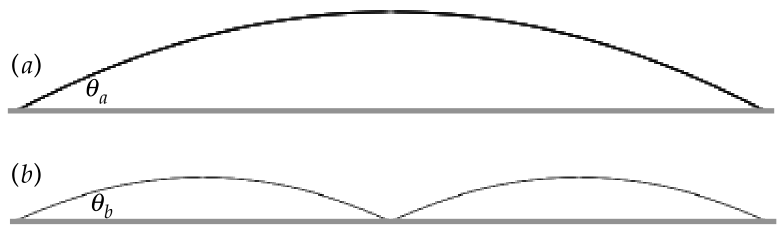
True ☑ False ☐

3. Pushing a box up a frictionless incline to height h requires the same energy input as lifting it straight up to that height, but the force to push it up the incline is less than the force to lift it.

True ☑ False ☐

Part C: Problems. Work problems in the space provided, indicating your method clearly. Problems carry the point values shown.

1. The right fielder needs to throw the ball as quickly as possible to third base, at distance R. He can throw the ball with initial speed v_0 sufficient to get it all the way to the base in the air if he throws it at an elevation angle θ_a, as in (*a*). Or he can throw at the same speed but at angle θ_b, so that the ball bounces off the ground at distance $R/2$, as in (*b*). [Assume a perfect bounce, so the second half of the motion in (*b*) is like the first half.] Call the total times of flight T_a and T_b.



a. Use the vertical motion to show that $T_b / T_a = 2\sin\theta_b / \sin\theta_a$.

b. Use the horizontal motion to show that $T_b / T_a = \cos\theta_a / \cos\theta_b$.

c. Show that the angles are related by $\sin 2\theta_b = \frac{1}{2}\sin 2\theta_a$.

[15 points]

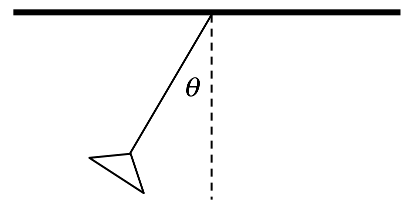
a. The time of flight T is equal to twice the time t to reach maximum height. From $v_y = v_0 \sin\theta - gt$, we get by setting $v_y = 0$: $t = v_0 \sin\theta / g$. So $T = 2t = 2v_0 \sin\theta / g$. Since v_0 is the same in both cases, we get the ratio claimed. [Both parts of (*b*) must be counted, of course.]

b. For the horizontal motion $x = v_0 \cos\theta \cdot t$. So in (*a*) we have $R = v_0 \cos\theta_a \cdot T_a$. In (*b*) we have $R/2 = v_0 \cos\theta_b \cdot T_b$. These lead to the ratio claimed. [This shows that the ball reaches the base faster in (*b*).]

c. Setting the two ratios equal to each other we get $2\sin\theta_b \cos\theta_b = \sin\theta_a \cos\theta_a$. Using the identity (from the formula sheet) $\sin 2\theta = \sin\theta\cos\theta$, we have the relation claimed.

[Example: if $\theta_a = 45°$, then $\theta_b = 15°$, and $T_b / T_a = 0.73$. In practice, the fielder makes the ball bounce closer to the base, for reasons of horizontal accuracy. And of course the bounce is not perfect.]

2a. You are in a moving railroad car and cannot see out. A lamp suspended from the ceiling hangs as shown. There are two explanations: (A) The car is moving at constant speed on an incline at angle θ; (B) The car is on level ground but accelerating.

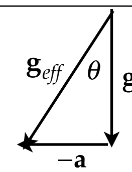


a. If (B) is correct, what is the acceleration (magnitude and direction) in terms of g and θ?

b. What could you measure to decide which explanation is correct?

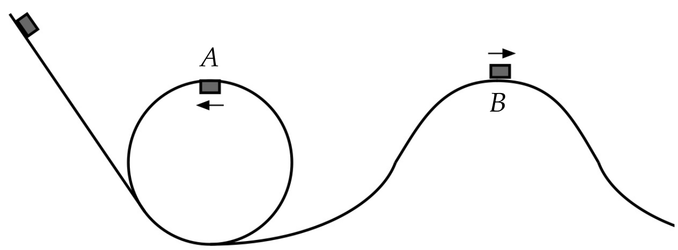
[10 points]

a. The lamp cord is along the line of $\mathbf{g}_{eff}$. Since $\mathbf{g}_{eff} = \mathbf{g} - \mathbf{a}$, we have the vector triangle shown. This shows that $a = g \tan\theta$, and the direction of $\mathbf{a}$ is to the right.



b. Since $g_{eff} > g$, you can measure the tension in the cord to see if it is greater than mg. Or you can measure the time it takes something to fall to the floor and see if the acceleration is greater than g.

2b. A block slides on a frictionless track, starting at the left. At *A* it is sliding on the inside of a circle of radius *R*; at *B* it is sliding at the top of a circular section, also of radius *R*. These points are at the same height. The speed at *A* is the minimum for the block to stay on the track.



a. What is that speed?

b Show that the speed at *B* is the maximum for the block to stay on the track.

[10 points]

a. If the block has the minimum speed, the normal force from the track is zero, so the only force is the weight, and the acceleration toward the center of the circle is g. So we have $v^2 / R = g$, or $v = \sqrt{Rg}$.

b. Since the heights are the same, the speed at *B* will be the same by conservation of energy. The normal force is zero when the speed is its maximum, so the downward acceleration is again g, and the speed is again $v = \sqrt{Rg}$.

3. A block of mass m starts at height h on the left incline, slides down it, across a frictionless horizontal section, and up the right incline. The inclines have $\mu_s = 2/3$, $\mu_k = 1/3$, $\sin\theta = 4/5$, and $\cos\theta = 3/5$.

a. Show that the block will slide down the incline if released from rest.

b. Express the work done by friction as the block slides down the left incline in terms of *m*, *g* and *h*. [Through what distance does it slide?]

c. When the block comes momentarily to rest on the right incline, what is the height *h′* to which it has risen? Give the answer in terms of *h*. [Find the total work done by friction. Be careful about the signs.]

d. Describe its subsequent motion.

[20 points]

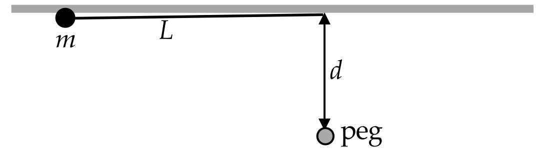
a. Forces down the incline: $mg\sin\theta - f_s$; if the first term is larger the block will slide down. Forces perpendicular to the incline: $N - mg\cos\theta$, so $N = mg\cos\theta$. Maximum friction force: $f_s = \mu_s N = \mu_s mg\cos\theta$. Since $\mu_s\cos\theta = \frac{2}{3}\cdot\frac{3}{5} = \frac{2}{5}$ is less than $\sin\theta = \frac{4}{5}$, the net force down the plane is positive, so the block will slide down.

b. The distance is $d = h/\sin\theta = \frac{5}{4}h$. Using $f_k = \mu_k mg\cos\theta = \frac{1}{3}\cdot\frac{3}{5}mg = \frac{1}{5}mg$, we find for the work $W_f = -f_s d = -\frac{1}{5}mg\cdot\frac{5}{4}h = -\frac{1}{4}mgh$.

c. The distance along the right incline is $d' = h'/\sin\theta = \frac{5}{4}h'$. The friction force is the same, so we have $W_f' = -f_k d' = -\frac{1}{5}mg\cdot\frac{5}{4}h' = -\frac{1}{4}mgh'$. The total work by friction is $W_f + W_f' = -\frac{1}{4}mg(h+h')$. The change in kinetic energy is zero (the block starts and ends at rest). The change in potential energy is $U' - U = mgh' - mgh$. We have $W_f + W_f' = U' - U$, or $-\frac{1}{4}mg(h+h') = mg(h'-h)$. This leads to $h' = \frac{3}{5}h$.

d. The block slides back and forth, going $\frac{3}{5}$ as high on each incline as it did on the previous incline.

4. A ball of mass m is attached to a massless string L, which is attached to a ceiling. Sticking out from the wall at distance d below the ceiling is a peg. The ball is released from rest at the ceiling. At the bottom of the swing, the string encounters the peg; the upper part of the string stops moving, and the ball swings about the peg in a circle of radius $L-d$.



a. If d is large enough the ball will swing in a complete circle around the peg. What is the minimum speed it must have at the top of that circle, in terms of g, L and d? [What is the string tension at that point?]

b. Find the value of d for which that case occurs. [Take gravitational potential energy to be zero at the bottom of the swing.]

c. For this value of d, at the bottom of the swing just before the string hits the peg, what is the tension in the string? [What is the ball's speed?]

d. What is the tension in the bottom part of the string just after it hits the peg, when the ball begins to move in the smaller circle?

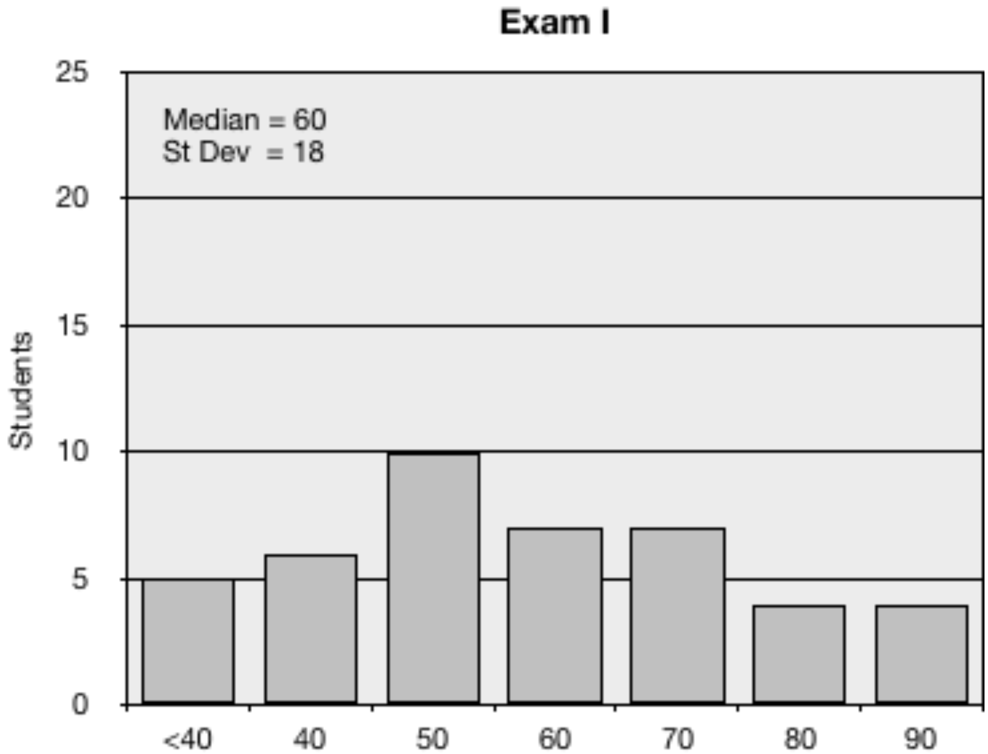
[20 points]

a. At the minimum speed, the string tension is zero and gravity provides the radial acceleration, so $mg = mv^2/(L-d)$, or $v^2 = g(L-d)$.

b. Conservation of energy, comparing the situation when the ball is released to that when it is at the top of the small circle. $0 + mgL = \frac{1}{2}mv^2 + mg \cdot 2(L-d)$. Using the result in (a) we find $d = \frac{3}{5}L$.

c. Conservation of energy to get the speed at the bottom of the swing: $mgL = \frac{1}{2}mv_b{}^2$, or $v_b{}^2 = 2gL$. The net upward force is the tension minus the weight, so $T - mg = mv_b{}^2/L = 2mg$, and we find $T = 3mg$.

d. The speed is the same but the circle radius is now $L-d = \frac{2}{5}L$, so we have $T - mg = mv_b{}^2/(2L/5) = 5mg$, and $T = 6mg$.


Exam I
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St Dev = 18
Students
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