Exam II

Solutions

In questions or problems not requiring numerical answers, express the answers in terms of the symbols given, and standard constants such as g. If numbers are required, use $g = 10\ \text{m/s}^2$.

Part A: Multiple choice questions. Check the best answer. Each question carries a value of 4 points.

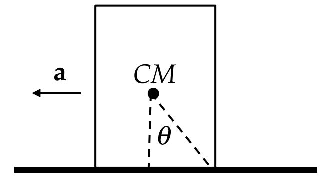
1. You are at one end of a platform on wheels that can roll freely on a horizontal floor. You and the platform are at rest. The platform has length ℓ, and you and the platform have the same mass m. You walk to the other end of the platform and stop. Which of the following is *wrong*?

- [ ] The CM of the system of you plus platform does not move.
- [ ] Total horizontal momentum of the system is conserved.
- [ ] The platform moves across the floor distance $\ell/2$ during your walk.
- [√] You do not move at all relative to the floor. [Only the CM remains fixed.]

2. A system is subject to only two external forces, of *equal* magnitude.

- [ ] If the forces are in opposite directions but not along the same line, there is at least one point about which total angular momentum is conserved. [For points between the lines the torques add; for other points the moment arms are different so they don't cancel.]
- [√] If the forces are in opposite directions and along the same line, total angular momentum is conserved about every point. [Total torque about any point is zero.]
- [ ] If the forces are in the same direction but not along the same line, there is no point about which total angular momentum is conserved. [Point halfway between the lines.]
- [ ] If the forces are in the same direction and along the same line, there is no point about which total angular momentum is conserved. [Point on the line.]

3. A tall crate is on the bed of a truck accelerating to the left. The geometry of the crate is as shown. Assuming it does not slip on the truck bed, the crate will fall over if:



☐ $a < g\tan\theta$.

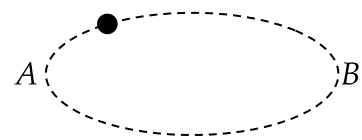
☑ $a > g\tan\theta$. [Line of $\mathbf{g}_{eff}$ passes to the right of the lower right corner.]

☐ $a > g\sin\theta$.

☐ $a > g\cos\theta$.

[Consider the direction of $\mathbf{g}_{eff}$.]

4. A satellite is in an elliptical orbit around the earth. Points *A* and *B* are at opposite ends of the major axis of the ellipse, and the distance between them is 2*a*.



☐ The earth is at the center of the ellipse. [Focus.]

☐ This orbit has less energy than a circular orbit of radius *a*. [Same.]

☐ The speed of the satellite is the same at *A* and *B*. [Larger at end nearer earth.]

☑ None of the above is true.

Part B: Check the correct answer. Each question carries a value of 3 points.

1. Static friction from a stationary surface can provide acceleration to an object, but does no work on it.

True ☑ False ☐

2. To round a curve on a level road, a bicyclist leans the bike so that the line from the CM to the point of contact between tires and road is along $\mathbf{g}_{eff}$.

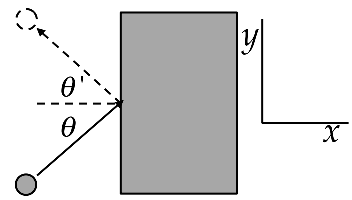
True ☑ False ☐

3. Tides are especially large when the sun and moon are both on one side of the earth, but especially small when they are on opposite sides.

True ☐ False ☑ [Same.]

Part C: Problems. Work problems in the space provided, indicating your method clearly. The problems carry the point values shown.

1a. A small ball of mass m collides *elastically* with the flat surface of a large block of mass M as shown. Its initial momentum $\mathbf{p}$ makes angle θ with the normal to the block's surface, and its final momentum $\mathbf{p}'$ makes angle θ' with the normal. After the collision the block moves with momentum $\mathbf{P}$. The surface of the block is frictionless. [What is the direction of $\mathbf{P}$?]



a. Write the equations for the conservation laws in terms of the variables given, in the coordinate system shown. [Use $K = p^2 / 2m$.]

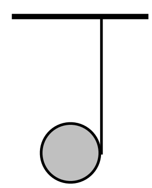
b. Show that if $M >> m$, then $p' \approx p$ and $\theta' \approx \theta$.

[10 points]

a. The force on M is only normal to the surface, so $\mathbf{P}$ has no y-component. We have: x-component of momentum: $p\cos\theta = P - p'\cos\theta'$; y-component of momentum: $p\sin\theta = p'\sin\theta'$; kinetic energy: $\dfrac{p^2}{2m} = \dfrac{p'^2}{2m} + \dfrac{P^2}{2M}$.

b. We see that P is comparable in size to p and p'. From the energy equation we see that if $M >> m$ then $p' \approx p$. From the y-components we then have $\theta' \approx \theta$.

1b. A cylinder (mass m, radius R, I about the symmetry axis $\frac{1}{2}mR^2$) is rolling down a string as shown.



a. Write the equations for the vertical acceleration and the angular acceleration about the CM.

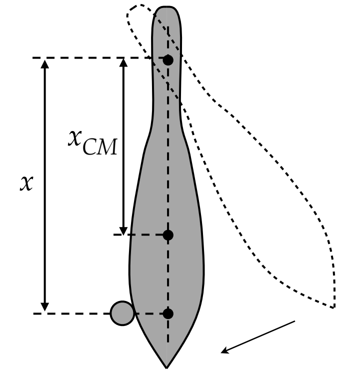
b. Find the acceleration in terms of g.

[10 points]

a. The forces are gravity and the string tension, so $mg - T = ma$. For torques about the CM we have $RT = I\alpha$.

b. The cylinder is rolling on the string, so $\alpha = a / R$. Eliminating T we find $mg = (m + I / R^2)a = \frac{3}{2}ma$ and $a = \frac{2}{3}g$.

2. An object, shown from above, rotates clockwise about a frictionless pivot. It has mass M, moment of inertia I about the pivot, and CM at distance x_{CM} from the pivot. When it is as shown its angular speed is ω_0, and It collides with a ball of mass m at rest at distance x from the pivot. After the collision the object has angular speed ω and the ball moves to the left with speed v.



a. Find v in terms of ω and the given quantities.
[What is conserved, regardless of whether the collision is elastic?]

b. If the collision is elastic, and $M >> m$ so that $\omega \approx \omega_0$, what is v?
[Remember $x^2 - y^2 = (x-y)(x+y)$ and use $\omega + \omega_0 \approx 2\omega_0$.]

c. The external force from the pivot ordinarily prevents linear momentum from being conserved. But there is a value of x for which the horizontal force from the pivot is zero. Find that value of x. [Require conservation of horizontal momentum. The linear momentum of the object is $M\mathbf{v}_{CM}$.]

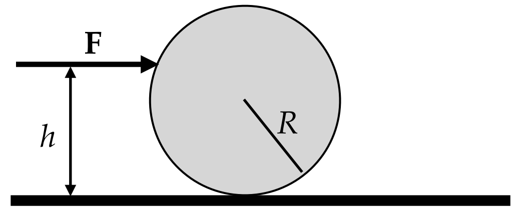
[15 points]

a. There are no external torques about the pivot, so angular momentum about that point is conserved. We have $I\omega_0 = I\omega + mxv$, or $v = \dfrac{I}{mx}(\omega_0 - \omega)$.

b. Kinetic energy is conserved, so $\frac{1}{2}I\omega_0^2 = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$, or $v^2 = \dfrac{I}{m}(\omega_0^2 - \omega^2)$. Using the hint, we write this as $v^2 = \dfrac{I}{m}(\omega_0 + \omega)(\omega_0 - \omega)$. Using the other part of the hint, $v^2 \approx \dfrac{I}{m}(2\omega_0)(\omega_0 - \omega)$. Substituting from (a) we find $v \approx 2x\omega_0$.

c. If horizontal momentum is conserved, we have, using $v_{CM} = x_{CM}\omega$, $Mx_{CM}\omega_0 = Mx_{CM}\omega + mv$. Using (a) we have $Mx_{CM}(\omega_0 - \omega) = \dfrac{I}{x}(\omega_0 - \omega)$. This gives $x = \dfrac{I}{Mx_{CM}}$. If the ball is struck at this distance from the pivot, there is no horizontal force exerted by or on the pivot. [This point is called the center of percussion. In sports such as baseball or tennis, it is called the "sweet spot", because if the ball is hit at this distance from the hand holding the bat or racket there is no reaction force back on the hand.]

3. A billiard ball (mass m, radius R, moment of inertia about its CM $I = \frac{2}{5}mR^2$) is at rest on a horizontal table. It is struck a sharp horizontal blow by a cue stick, at height h above the table as shown. The large average force F, which acts for a very short time Δt, gives the ball an initial horizontal linear momentum $mv_0 = F \cdot \Delta t$. The same force produces, for time Δt, a torque about the point of contact between the ball ball and the table. Call that point A.



a. What initial angular momentum (in terms of m, h, and v_0) does the torque give the ball about A?

b. The ball then moves across the table, perhaps not rolling at first. Show that angular momentum about A is conserved.

c. When it finally does roll, what is the ratio of its CM speed v to v_0? [Answer in terms of h and R.]

d. For what value of h will it roll from the start?

[20 points]

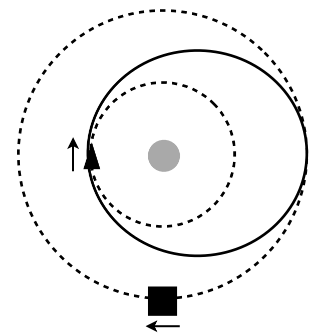
a. The torque of the force F about point A is $\tau = hF$, and in time Δt it produces angular momentum $L_0 = hF\Delta t = hmv_0$ about A.

b. Forces on the ball: weight, normal force, friction. The weight and normal force are equal and opposite and along the same line, so they give zero torque about any point. Friction acts along the surface, so it has no moment arm about A. The total external torque about A is zero, so angular momentum about A is conserved.

c. Use conservation of angular momentum about A, where $L = L(\text{of CM}) + L(\text{about CM})$. About A, $L(\text{of CM}) = mvR$ and $L(\text{about CM}) = I\omega$. Using (a) for the initial angular momentum: $mv_0h = mvR + I\omega$. When the ball rolls, $\omega = v/R$. Using the value of I, we find $mv_0h = \frac{7}{5}mvR$ and $\frac{v}{v_0} = \frac{5h}{7R}$.

d. To have $v_0 = v$ we need $h = \frac{7}{5}R$. Struck at this point, the ball will roll without ever slipping on the surface.

4. A space shuttle of mass m is in a circular orbit of radius R around the earth. In a larger circular orbit of radius $2R$ is the space station. The shuttle is to change its orbit so it can reach the space station. Give answers in terms of G, earth's mass M, m, and R.



   a. When the shuttle is at the point shown, the engines are fired briefly to send it into an elliptical orbit with apogee distance $2R$. How much energy must the engines supply?

   b. The engines provide acceleration along the line of the velocity (the arrow shown). Sketch carefully the elliptical orbit on the drawing.

   c. What is the speed (find v^2) of the shuttle when it is at apogee (i.e., distance $2R$)? [What are E and U?]

   d. Find v^2 for the space station. What must be done if the shuttle is to dock at the space station?

[20 points]

a. The original energy was $E_0 = -\frac{GMm}{2R}$. For the new orbit, $2a = 3R$, so the energy is $E_1 = -\frac{GMm}{3R}$. The energy input is $\Delta E = E_1 - E_0 = \frac{GMm}{6R}$.

b. The angular momentum is *increased* by the torque (about the earth) provided by the engines, so the new orbit runs *outside* the original one, as shown.

c. At $r = 2R$ the potential energy is $U = -\frac{GMm}{2R}$. The kinetic energy is $K = E_1 - U = \frac{GMm}{R}(-\frac{1}{3} + \frac{1}{2}) = \frac{1}{6}\frac{GMm}{R}$. Set this equal to $\frac{1}{2}mv^2$ and find $v^2 = \frac{1}{3}\frac{GM}{R}$.

d. Call the space station mass m'. Its energy is $E_s = -\frac{GMm'}{4R}$ and its potential energy is $U_s = -\frac{GMm'}{2R}$, so its kinetic energy is $K_s = E_s - U_s = \frac{GMm'}{R}(-\frac{1}{4} + \frac{1}{2}) = \frac{1}{4}\frac{GMm'}{R}$. Set this equal to $\frac{1}{2}m'v'^2$ and find $v'^2 = \frac{1}{2}\frac{GM}{R}$. This is larger than the shuttle's speed, so the shuttle must fire its engines again to catch up. (This maneuver involves delicate timing, and is turned over to computers.)