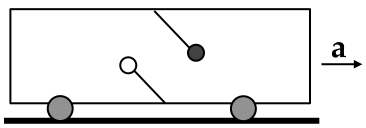
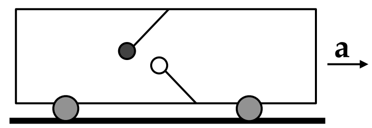
Exam III

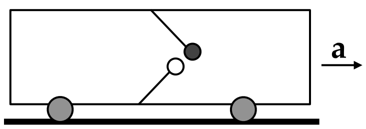
Solutions

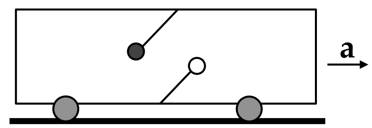
Part A. Multiple choice questions. Check the best answer. Each question carries a value of 4 points.

1. A railroad car accelerating to the right as shown has a mass hanging by a cord from the ceiling, and a helium balloon attached by a string to the floor. Shown are the possibilities for the orientations of those objects. Which one actually occurs?

[ ] 

[ ] 

[ ] 

[√] 

[Both along the line of $\mathbf{g}_{eff}$.]

2. Water flows in a horizontal pipe. When the cross-section area A of the pipe decreases, which of the following does *not* happen?

[√] The pressure P increases. [Decreases.]

[ ] The speed of flow v increases.

[ ] The sum $P+\frac{1}{2}\rho v^2$ remains the same, where ρ is the density.

[ ] The product Av remains the same.

3. The motion of a system is described by $x(t)=A\cos\omega t$. Which of the following is *wrong*?

[ ] If ω is independent of A, the motion is simple harmonic motion.

[ ] At $t=0$ the system is momentarily at rest, but shortly after that it moves in the negative x-direction.

[√] At any time, $a(t)=\omega^2 x(t)$. [Minus this.]

[ ] The potential energy is $U(x)=\frac{1}{2}m\omega^2 x^2$.

4. Two harmonic waves of the same wavelength, frequency and direction are interfering. The waves separately have intensities I_0 and $4I_0$.

☐ The minimum possible total intensity is $3I_0$.

☐ The second wave has amplitude 4 times that of the first.

☑ The maximum possible total intensity is $9I_0$.

☐ None of the above is true.

[Amplitudes are in the ratio 1:2.]

Part B. True-false questions. Check T or F depending on whether the statement is true or false. Each question carries a value of 3 points.

5. The pressure of water as it flows through a nozzle is greater than the pressure in the hose that feeds it.

☐ T ☑ F [Less.]

6. The ride of an automobile is smoothed out by mounting the passenger section on springs with a large amount of damping.

☑ T ☐ F

7. When two sound waves interfere destructively, the energy of the waves is destroyed.

☐ T ☑ F [Energy is conserved. The energy is moved elsewhere.]

Part C. Problems. Work problem in space provided, using extra sheets if needed. Explain your method clearly. Problems carry the point values shown.

1a. A solid object of uniform mass density ρ is floating in water, with fraction x of its volume submerged.

a. Show that $\rho = x \cdot \rho_w$, where ρ_w is the density of water.

b. Will x be different if the whole system is in an elevator accelerating upward? Justify your answer.

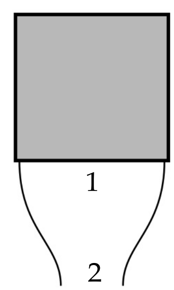
[10 points]

a. Call the volume of the object V. Then its mass is ρV and its weight is $\rho V g$. The water displaced has volume xV and weight $x \cdot \rho_w V g$. By Archimedes, this is equal to the weight of the object, so $x \cdot \rho_w V g = \rho V g$. This proves the claim.

b. The acceleration of the elevator changes the value of g. But g cancelled out of the equation above, so g has no influence on x.

1b. Shown is a water faucet with the stream below it. It is observed that the stream narrows as it falls. Assuming streamline flow, compare the situation at points 1 and 2 to explain this.

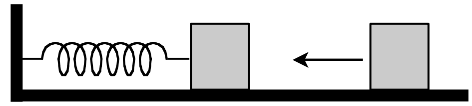
[5 points]



At both points the pressure is air pressure. So $P_0 + \rho g h_1 + \frac{1}{2}\rho {v_1}^2 = P_0 + \rho g h_2 + \frac{1}{2}\rho {v_2}^2$.

Since $h_1 > h_2$, we have $v_2 > v_1$. But by continuity $A_1 v_1 = A_2 v_2$, so $A_2 < A_1$.

2. A block of mass m is attached as shown to a spring of stiffness k. It is at rest on a frictionless floor. An identical block, moving as shown with speed v collides with the first block. Give all answers in terms of m, k, and v.



   a. If the collision is elastic, so that the moving block transfers all its momentum and energy to the one at rest, how far does the spring compress?

   b. How long after the initial collision does it take for the block attached to the spring to come back and collide with the other one a second time?

   c. If the original collision is inelastic and the blocks stick together, what is the angular frequency of the subsequent oscillation?

   d. What is the amplitude of the oscillation?

   [20 points]

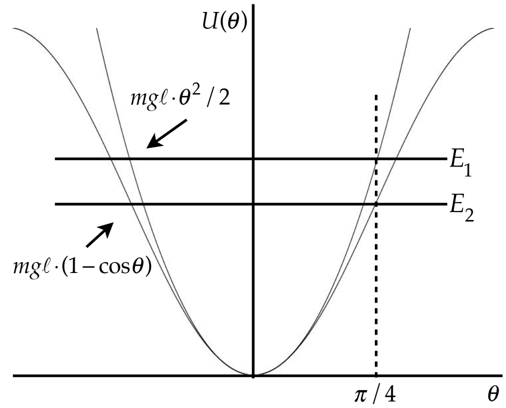
a. The collision adds energy $\frac{1}{2}mv^2$ to the system of spring plus block. This is its total energy, since the system was initially at rest. So $\frac{1}{2}kA^2 = \frac{1}{2}mv^2$ and $A = \sqrt{\frac{m}{k}} \cdot v$. [Can also use $v_{\text{max}} = \omega A$.]

b. The period is $T = 2\pi / \omega = 2\pi\sqrt{\frac{m}{k}}$. But the block attached to the spring will return to hit the other block after only half of the cycle, so the time is $\pi\sqrt{\frac{m}{k}}$.

c. The mass is now $2m$, so $\omega = \sqrt{k / 2m}$.

d. The collision conserves momentum, so the speed of the blocks immediately after the collision is $v / 2$ and the kinetic energy is $K_{\text{max}} = \frac{1}{2}(2m)(v / 2)^2 = \frac{1}{4}mv^2$. This is the total energy, so $\frac{1}{2}kA^2 = \frac{1}{4}mv^2$ and $A = \sqrt{\frac{m}{2k}} \cdot v$.

3a. Shown are plots of $U(\theta)$ for a pendulum, giving the exact function as well as the approximation used for small angle swings. The pendulum starts from rest at $\theta = \pi / 4$.



a. Draw horizontal lines representing the total energy for the two cases. [These intersect the curves at the turning points.]

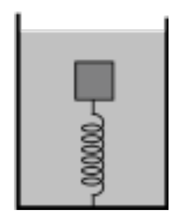
b. Show that the actual period of the pendulum is longer than that predicted by the approximation. [Compare the angular speeds when $\theta = 0$.]

[10 points]

a. Lines shown: E_1 for the approximation; E_2 for the exact function.

b. The turning points (at $\pm\pi / 4$) are the same in both cases, but the energy, and thus the average angular speed in the swing, is greater for the approximation. It therefore has the shorter period.

3b. A wood block of mass *m* is held submerged in water by a spring of stiffness *k*. The downward force exerted by the spring is $\frac{1}{4} mg$.



a. If the system were in an elevator accelerating downward, would the block rest higher in the water, lower, or the same?

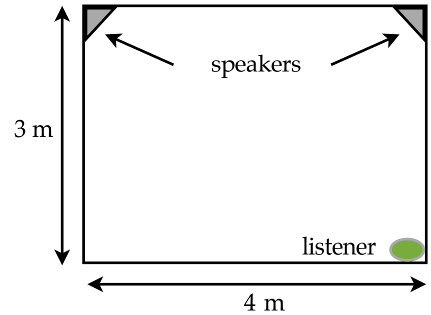
b The block is pushed down and released, executing SHM. Neglecting resistance from the water, what is the angular frequency ω?

[10 points]

a. Reducing the value of *g* reduces the net upward force on the block, because the buoyant force and the weight of the block are both proportional to *g*. The spring stretches less to balance that smaller upward force, so the block rests lower in the water.

b. The value of ω depends only the mass of the object that is moving, not its weight, so the angular frequency is $\omega = \sqrt{k / m}$.

4. A small room has dimensions 4 m by 3 m as shown. In the corners are speakers which emit sound waves starting out in phase with each other. As the frequency of the emitted sound is varied, the listener, in the corner shown, hears the intensity rise and fall.



a. Express the phase difference between the waves as they arrive at the listener, in terms of the wavelength. [What is the path difference of the waves as they arrive at the listener?]

b. What is the largest wavelength for which the listener hears minimum intensity?

c. If the speed of sound is 340 m/s, what is the frequency of that sound?

d. As the listener moves across the back wall (at the bottom in the diagram) where will the intensity be a maximum for all frequencies? Explain.

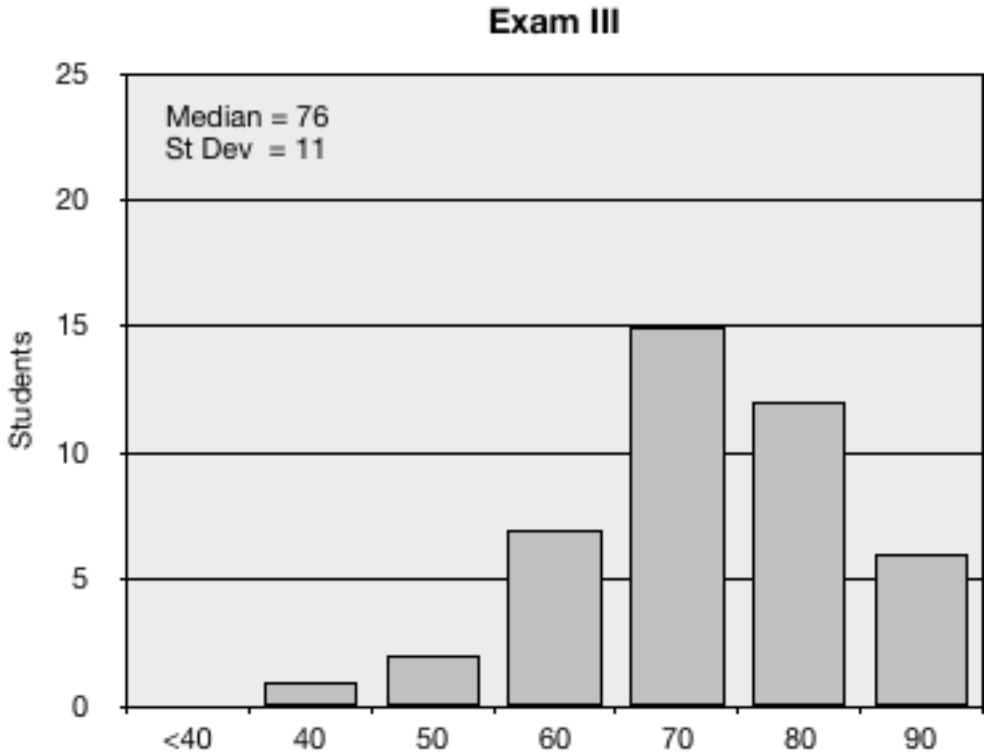
[20 points]

a. The distance from the speakers to the listener are 3 m and 5 m, so the path difference is 2 m. This gives $\delta = \dfrac{2\pi}{\lambda}\Delta x = \dfrac{4\pi}{\lambda}$, with λ in meters.

b. The smallest value for δ to give destructive interference is π, so we have $\lambda = 4\ \text{m}$.

c. Since $v = f\lambda$ we find $f = 85\ \text{Hz}$.

d. At the center of the wall, where the distance to the speakers is the same. There $\delta = 0$ regardless of the value of λ.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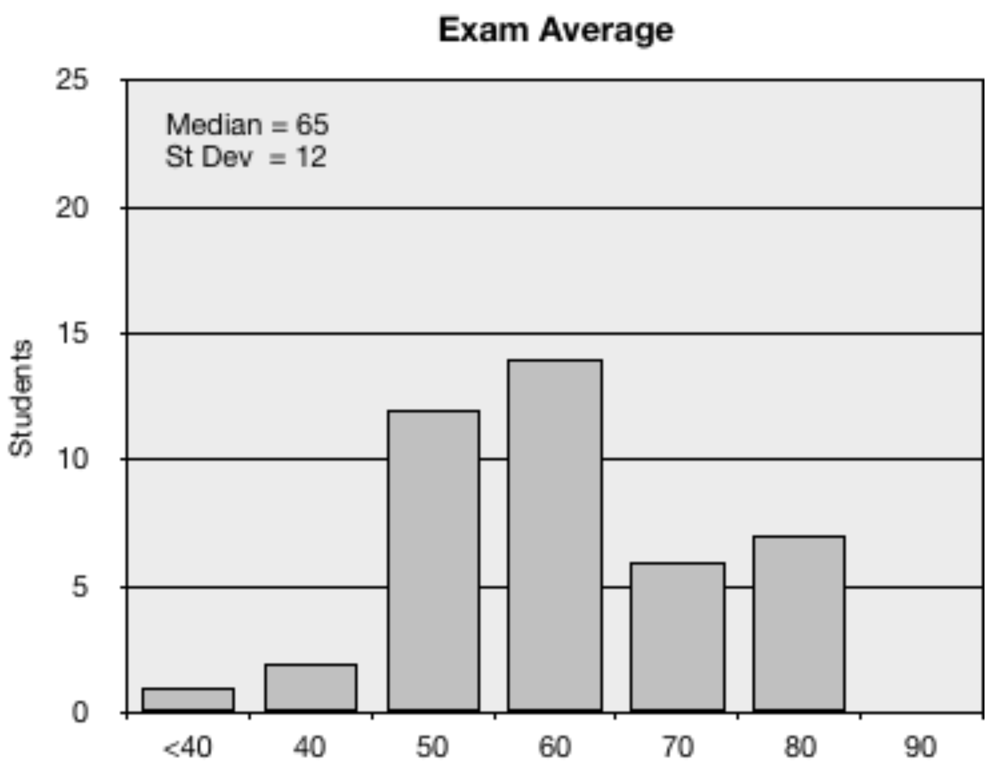
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