Final Exam

Solutions

Part A. Multiple choice questions. Check the best answer. Each question carries a value of 4 points.

1. A particle moves subject to a constant acceleration **a**.

- [ ] Its trajectory must be a straight line along the line of **a**.
- [√] It will not move exactly in a circle. [Its direction must change.]
- [ ] If it starts from rest, the distance it moves in the first second is half the distance it moves in the second one. [One third.]
- [ ] Its speed must increase linearly with time. [Depends on direction of **v**.]

2. A system is subject to zero net external force.

- [ ] Its total momentum and angular momentum are certainly conserved, but its total mechanical energy might not be.
- [ ] Its total momentum, angular momentum and mechanical energy are all certainly conserved.
- [√] Its total momentum is certainly conserved, but its total angular momentum and mechanical energy might not be.
- [ ] None of the three quantities is certainly conserved.

3. Concerning a rigid body rolling without slipping on a surface, which of the following is wrong?

- [ ] If the CM speed is not equal to the angular speed times the radius of the body, it will slip.
- [ ] If the total external force on the body is zero, the total external torque about the CM must be zero in order for it to continue to roll.
- [√] The ratio of kinetic energy of rotation to that of translation varies with the speed.
- [ ] It cannot roll freely without slipping down a frictionless incline.

4. A figure skater starts a slow spin standing on the tip of one skate with the other leg and both arms extended. As she pulls the leg and arms close to her body she spins faster because:

- [ ] Energy is conserved while the moment of inertia about the rotation axis decreases.
- [ ] External torques exerted by the ice cause an angular acceleration.
- [√] Angular momentum about the rotation axis is conserved while the moment of inertia about that axis decreases.
- [ ] None of the above is the reason.

5. Concerning tidal forces acting on the earth, which of the following is wrong?

- [ ] The effect of the moon is stronger than that of the sun despite the fact that the sun is much more massive.
- [ ] The moon’s tidal effect on the side of the earth opposite the moon is approximately equal to its effect on the side nearest the moon.
- [ ] The tidal effect arises because the gravitational fields of the moon and sun are stronger on one side of the earth than on the other.
- [√] Large “spring” tides occur only at full moon. [Also new moon.]

6. A violin string is played simultaneously with a tuning fork of frequency 440 Hz, producing 4 beats per second. If the tension in the string is increased, the beats disappear. The initial frequency of the violin string was:

- ☐ 444 Hz
- ☐ 442 Hz
- ☐ 438 Hz
- ☑ 436 Hz [Increasing tension increases frequency.]

7. Containers A, B, and C have identical amounts of an ideal gas at pressure P and volume V. In all three the pressure is raised to 2P. In A it is done at constant volume, in B adiabatically, and in C isothermally. The entropy changes in the three cases rank, from lowest to highest, as follows: [Consider *dQ*.]

- ☐ A, B, C
- ☐ A, C, B
- ☐ B, C, A
- ☑ C, B, A [For isothermal, $dQ < 0$ because $dW < 0$, for adiabatic $dQ = 0$, for constant volume $dQ > 0$.]

8. A thermos bottle consists of a double walled container with a near vacuum between polished stainless steel walls.

- ☐ The near vacuum inhibits heat transfer by radiation. [Does not.]
- ☐ The steel is polished to make it a good emitter of radiation. [Poor.]
- ☐ The steel is a poor conductor of heat. [Good.]
- ☑ None of the above is true.

Part B. True-false questions. Check T or F depending on whether the statement is true or false. Each question carries a value of 3 points.

1. In a head-on collision between a car and a large truck the acceleration of the car is greater in magnitude than that of the truck.

   [√] T [ ] F [Forces have equal magnitude.]

3. A heavy crate shoved up a rough incline slides back down; the trip down takes longer than the trip up.

   [√] T [ ] F [Average speed is smaller on the way down.]

3. In an accelerating reference frame, the buoyant force is directed opposite to $\mathbf{g}_{eff}$.

   [√] T [ ] F

4. The intensity of sound from a small source doubles as one moves twice as close to the source.

   [ ] T [√] F [Quadruples.]

5. The interior temperature of a closed auto on a cold morning can be much lower than that of the air outside because the auto is a good emitter of radiation.

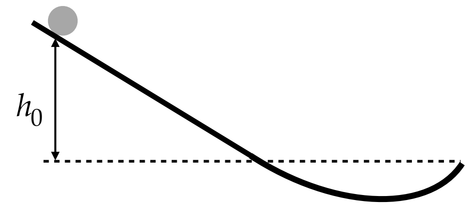
   [√] T [ ] F

6. If you remove a window air conditioner from the window and run it in the room with the windows closed, it will cool the room more effectively.

   [ ] T [√] F [It will heat the room.]

Part C. Problems. Work problem in space provided, using extra sheets if needed. Explain your method clearly. Problems carry the point values shown.

1. A small disk of mass m, radius R, and moment of inertia $\frac{1}{2}mR^2$ rolls from rest down the rough track shown. It leaves the track moving at angle θ above the horizontal.



   a. What is the speed of its CM as it leaves the track?
   b. What is its rotational kinetic energy at that point?
   c. How high does it rise in its motion through the air?
   d. What would your answers to (a) and (c) be if the track were frictionless?

(20 points)

a. Conservation of E: $mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$. But $\omega = v/R$, and using the value of I we find $mgh = \frac{3}{4}mv^2$, or $v^2 = \frac{4}{3}gh$.

b. We have $\frac{1}{2}I\omega^2 = \frac{1}{4}mv^2 = \frac{1}{3}mgh$.

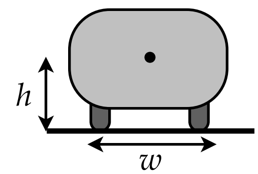
c. The vertical component of CM velocity is $v_y = v\sin\theta$, so at the top of the flight we have $0 = (v\sin\theta)^2 - 2gh'$, or $h' = \dfrac{\frac{4}{3}gh\cdot\sin^2\theta}{2g} = \frac{2}{3}h\cdot\sin^2\theta$.

d. There would be no rotational energy, so the speed at the end of the track would be given by $mgh = \frac{1}{2}mv^2$, or $v^2 = 2gh$. We would still have $0 = (v\sin\theta)^2 - 2gh'$, so $h' = h\cdot\sin^2\theta$.

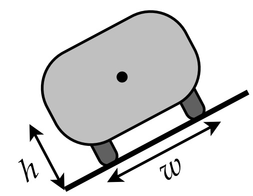
2. A car is moving at speed v around a curve of radius R, with center to the left. We are interested in the maximum safe speed $v_{\max}$ for it to make the curve.

   a. If the road is level and the coefficient of static friction between the road and the tires is μ_s, what is $v_{\max}$ for not sliding?

   b. Shown from behind is the car moving into the page, rounding the curve. If the line along $\mathbf{g}_{eff}$ from the CM (the dot) passes beyond the outside wheel, the car will roll over. Show that $v_{\max}^2 = gR\dfrac{w}{2h}$.



   c. Shown is the car on a banked road. Explain (in words) why the $v_{\max}$ for not sliding is larger.



   d. Explain why the $v_{\max}$ for not rolling is also larger. [Draw $\mathbf{g}_{eff}$ for $v = v_{\max}$ and compare the angle it makes with the vertical with the one in (b).]

(20 points)

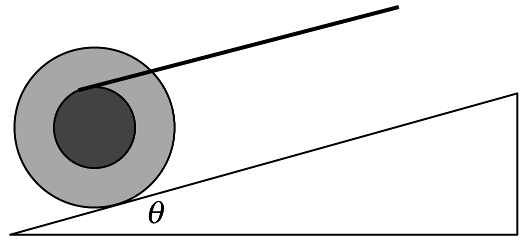
a. Friction must supply the radial force, so $f_s = mv^2 / R$. But $f_s(\max) = \mu_s N = \mu_s mg$, so $mv_{\max}^2 = \mu_s mgR$, or $v_{\max}^2 = \mu_s gR$.

b. The angle between the vertical and the line from the CM to the outer wheel is given by $\tan\theta = w / 2h$. If $\mathbf{g}_{eff}$ is along that line, then $\tan\theta = a / g = v^2 / Rg$. Thus $v_{\max}^2 = Rg \cdot (w / 2h)$ as claimed.

c. Now the horizontal component of the normal force helps supply the radial force, so $v_{\max}$ for not sliding is larger.

d. The angle between the vertical and the line from the CM to the outer wheel is now greater than in (b), so $a / g > w / 2h$, and $v_{\max}$ for not rolling is larger.

3. The heavy wheel shown is being rolled without slipping at *constant* speed up an incline by pulling on a rope wrapped around its axle. The wheel has mass m and radius R; the axle has radius $R/2$. The incline has static friction coefficient μ_s.



a. What direction is the static friction force?

b. What is the tension T in the rope?

c. What is the friction force f_s?

d. What is the minimum value of μ_s for this motion to occur?

[Give answers in terms of m, g, and θ.]

(20 points)

a. Because the wheel moves at constant speed, the total force and total torque are both zero. Since the rope gives a clockwise torque, friction must give a counter-clockwise torque, so it must be directed up the incline.

b. Total force: $T + f_s - mg\sin\theta = 0$. Total torque: $(R/2)T - Rf_s = 0$. Thus $f_s = T/2$, and $T = \frac{2}{3}mg\sin\theta$.

c. $f_s = \frac{1}{3}mg\sin\theta$.

d. Since $N = mg\cos\theta$ and $f_s \le \mu_s N$, we have $\mu_s \ge f_s / N = \frac{1}{3}\tan\theta$.

4. A small mouse of mass m is sitting on the rim of a horizontal turntable of mass $10m$, radius R, and moment of inertia $5mR^2$. Originally everything is at rest. The mouse begins to run clockwise (as seen from above) around the rim, reaching final speed v (relative to the ground).

   a. Analyze the total energy, linear momentum, and angular momentum of the system, stating what is conserved and why the others are not.

   b. What is the final angular speed of the turntable, and what direction (clockwise or counter-clockwise, as seen from above) does it rotate?

   c. How much work does the mouse do in reaching the final speed?

   d. The mouse jumps off the moving turntable is such a manner that his velocity (relative to the ground) is directly away from the axis of rotation. What is the final angular speed of the turntable after he jumps? [What is conserved and why?]

   [Give answers in terms of m, v and R.]

   (20 points)

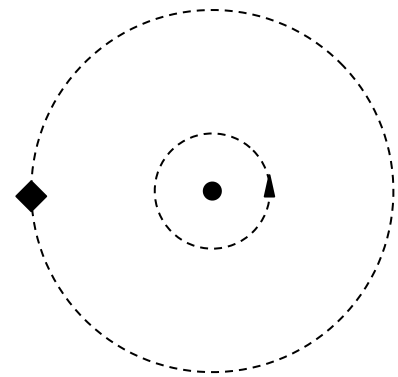
a. Energy is not conserved because the forces involving work done by the mouse's muscles are non-conservative. Linear momentum is not conserved because the axle can exert external forces. Angular momentum about the axle is conserved because there are no *external* forces that result in torques about the axle.

b. The initial angular momentum is zero. Take clockwise as positive. The angular momentum of the mouse is mvR so the angular momentum of the turntable must be $-mvR$. Thus $I\omega = mvR$ (counter-clockwise). This gives $\omega = mv/5mR^2 = v/5R$.

c. The work done by the mouse is responsible for all the final kinetic energy, which is $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{3}{5}mv^2$.

d. Angular momentum about the axle is still conserved because there are still no *external* torques about it. The mouse now has zero angular momentum, and the total is also zero, so the turntable is brought to rest by the mouse's jump.

5a. The space shuttle is in a circular orbit of radius R around the earth. The space station is in another circular orbit of radius $3R$. The shuttle crew wishes to effect a rendezvous with the space station. Let the shuttle's mass be m and express all answers in terms of R, m, the earth's mass M, and G.



a. The shuttle first fires its engines to go into an orbit that has apogee $3R$. How much energy must the engines add to do this? [Sketch the new orbit.]

b. The engines are then fired again to go into a circular orbit of radius $3R$. How much energy must the engines contribute this time?

(10 points)

a. The original energy of the shuttle is $E_0 = -\dfrac{GMm}{2R}$. The energy of the new orbit is $E_1 = -\dfrac{GMm}{4R}$, so the engines added energy $\Delta E = E_1 - E_0 = +\dfrac{GMm}{4R}$.

b. The energy of the final orbit is $E_2 = -\dfrac{GMm}{6R}$, so the engines added energy $\Delta E = E_2 - E_1 = +\dfrac{GMm}{12R}$.

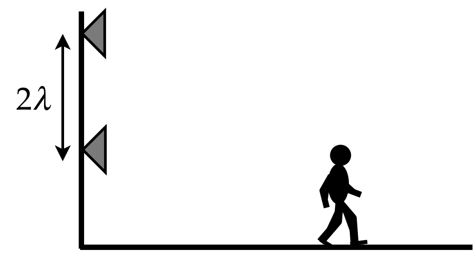
5b. A person can barely breathe lying on a floor with a 40 kg weight on his chest. Take the chest area to be $0.1\ \text{m}^2$. Suppose the person is lying on the bottom of a pool of water with the depth at his chest equal to d. Take the density of water to be $1000\ \text{kg/m}^3$ and find the maximum value of d for which this person could breathe through an air tube to the surface.

(5 points)

The pressure (above air pressure) at depth d is $\rho g d$, and the total force on the chest is $F = \rho g d \cdot A$. For this to be equal to the weight of a 40 kg mass, we have (using the numbers) $1000 \cdot 0.1 \cdot g \cdot d = 40g$, or $d = 0.4$ m. [This is quite small, which is why special apparatus is needed to breathe under water.]

6. Two questions concerning sound waves. (10 points each)

a. Two loudspeakers emitting sound of wavelength λ in phase are on a vertical pole. One is at the level of the listener's ears, the other at height 2λ above that. The listener starts at the base of the pole and walks away from it. For what finite positive distance x from the pole will the listener hear constructive interference in the sound? [There is only one such distance.]



When the listener is at distance x from the pole the path difference to the speakers is $\Delta x = \sqrt{x^2 + 4\lambda^2} - x$. For constructive interference Δx must be an integer multiple of λ (the phase difference ϕ must be a multiple of 2π). We start with $\Delta x = 0$, which is satisfied by $x = 0$, but we are looking for a non-zero value of x. We try $\Delta x = \lambda$ and find $x = \frac{3}{2}\lambda$. This is the solution we want. (You can verify that larger values of Δx give a negative value of x.)

b. You and an ambulance are driving in opposite directions on a divided highway, both at fraction α of the speed of sound. The ambulance is blowing its horn. What is the ratio of the frequency you hear when it is approaching you to the frequency when it is receding from you?

When the ambulance is coming toward you, the frequency you hear is $f_1 = f_0 \dfrac{1+\alpha}{1-\alpha}$.

When it is receding from you, the received frequency is $f_2 = f_0 \dfrac{1-\alpha}{1+\alpha}$. The ratio is

$$\frac{f_1}{f_2} = \frac{(1+\alpha)^2}{(1-\alpha)^2}.$$

7. An engine using one mole of an ideal monatomic gas ($c_V = \frac{3}{2}R$) runs on the following cycle:

(1) Starting at temperature T_0 the gas is heated at constant volume to temperature $2T_0$.

(2) The gas expands adiabatically until the temperature returns to T_0

(3) The gas is compressed isothermally back to the original state.

a. Plot the cycle on a P-V diagram.

b. How much heat is taken in during step 1?

c. How much work is done in steps 1 and 2? [Consider the overall effect.]

d. The efficiency of the engine is 0.2. How much heat is expelled in step 3?

[Give answers in terms of R and T_0. No integrals are required.]

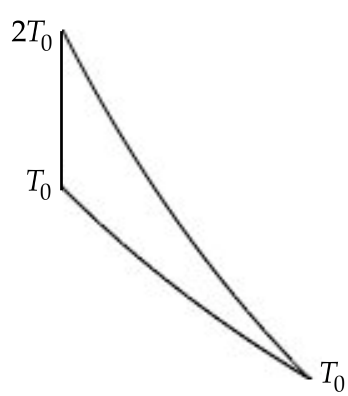
(20 points)

a. Drawing below

b. Since no work is done, $Q_1 = c_V \Delta T = \frac{3}{2} R T_0$.

c. There is no overall change in T, so no change in E_{int}. Thus $W = Q$. But in step 2 $Q = 0$, so $W = Q_1 = \frac{3}{2} R T_0$.

d. The total work done is $W_{tot} = e \cdot Q_{in} = 0.2 \cdot Q_1 = 0.3 R T_0$. Then $Q_{out} = Q_{in} - W = 1.2 R T_0$.



8a. The freezing point of a substance is T_C, and its latent heat of fusion is L. A refrigerator freezes mass m of the substance, expelling heat into a room at temperature 1.2 T_C.

a. If the work required to run this process is $0.5mL$, what is the entropy change of the universe?

b. If the refrigerator runs on a Carnot cycle, what is the work required?

[Give answers in terms of m, L and T_C.]

(10 points)

a. The heat removed is $Q_C = mL$; this is done at temperature T_C, so the entropy change is $-mL/T_C$. The heat expelled is $Q_H = W + Q_C = 1.5mL$; this is done at temperature $1.2T_C$, so the entropy change is $+1.5mL/1.2T_C = 5mL/4T_C$. The total entropy change is $\Delta S = \frac{mL}{T_C}\left(\frac{5}{4} - 1\right) = \frac{1}{4}\frac{mL}{T_C}$.

b. We would have $\frac{Q_C}{T_C} = \frac{Q_H}{T_H}$, or $\frac{Q_C}{T_C} = \frac{W + Q_C}{1.2T_C}$. This gives $W = 0.2Q_C = 0.2mL$.

8b. Show that the flow of an amount of heat Q from a body at temperature T_1 to a body at temperature $T_2 < T_1$ is irreversible. [Consider the entropy change.]

(5 points)

The entropy change of the hotter body is $-Q/T_1$; that of the colder body is $+Q/T_2$. The total entropy change is $\Delta S = Q\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$. This is positive, so the process is irreversible.