Exam I

Solutions

Part A. Multiple choice questions. Check the best answer. Each question carries a value of 4 points.

1. Two projectiles are launched simultaneously with the same speed from the same spot on the ground, one at 30° elevation and the other at 60°.

- [ ] They will rise to the same height.
- [ ] They will reach their maximum height at the same time.
- [√] They will return to the ground at the same distance.
- [ ] They will return to the ground at the same time.

2. A car rounds a curve of radius R at constant speed v but radial acceleration a. Which of these is wrong?

- [ ] $a = v^2 / R$.
- [√] The occupants of the car feel a component of their weight toward the center of the curve. [Away from the center.]
- [ ] The weight of an occupant of mass m is $m\sqrt{g^2 + a^2}$.
- [ ] No work is being done on the car.

3. Which of these statements about work, energy and power is true?

- [ ] Work done by non-conservative forces always decreases the total energy.
- [ ] The potential energy of a mass-spring system is positive if the spring is compressed but negative if it is stretched.
- [ ] If two objects acted on by the same force follow the same path, the power input by the force is the same for both.
- [√] If an object moves in a closed path the work done by gravity is zero.

4. A particle is subject to a single conservative force $F(x)$ for which the potential energy is $U(x)$. The total energy is E. Which of the following is NOT true?

- [ ] The particle cannot be found at a point x for which $E < U(x)$.
- [√] At a point where $dU/dx > 0$ the force $F(x) > 0$. [$F(x) = -dU/dx$.]
- [ ] At points where $E = U(x)$ the particle is at least momentarily at rest.
- [ ] A point where $U(x)$ has a minimum is a point of stable equilibrium.

Part B. True-false questions. Check T or F depending on whether the statement is true or false. Each question carries a value of 3 points.

5. The purpose of a simple machine (such as an inclined plane to lift a weight) is to reduce the power input while keeping the total work done the same.

- [x] T
- [ ] F

[Time to do the work is longer.]

6. A car of mass m and a truck of mass $3m$ collide head on and stick together; during the collision the two vehicles undergo accelerations of the same magnitude.

- [ ] T
- [x] F

[Forces have equal magnitude, not accelerations.]

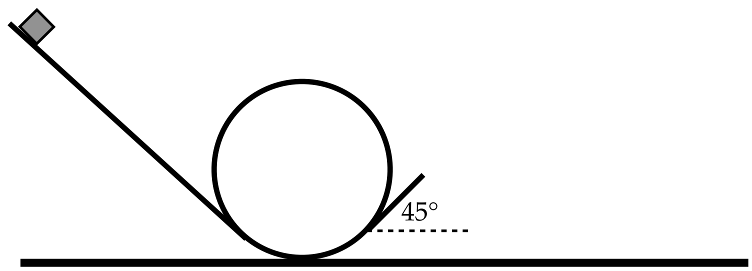
7. A weightlifter grabs a barbell resting on the floor, jerks it up and holds it at rest at chest height; the total work done on the barbell is zero.

- [x] T
- [ ] F

[Kinetic energy is zero initially and finally; no total work done.]

Part C. Problems. Work problem in space provided, using extra sheets if needed. Explain your method clearly. Problems carry the point values shown.

1. A small block sides on the frictionless track shown, starting from rest at height h above the bottom of the circular part, which has radius R.



After sliding around the circular part, the block slides up a straight section at 45° above the horizontal. It leaves that section at height R and flies through the air.

a. What is the minimum speed the block must have at the top of the circle?

b. If it has that minimum speed, what is h?

c. What is its speed when it leaves the end of the track?

d. To what maximum height does it rise in the air?

(20 points)

a. The downward acceleration (radial at that point) is at least g, so the minimum speed occurs when $v^2 / R = g$, or $v = \sqrt{Rg}$.

b. Conservation of energy. Take $U = 0$ at the bottom. Then $mgh = \frac{1}{2}mv^2 + mg(2R) = \frac{1}{2}mgR + 2mgR = \frac{5}{2}mgR$. So $h = \frac{5}{2}R$.

c. Conservation of energy: $mg \cdot \frac{5}{2}R = \frac{1}{2}mv^2 + mgR$. This gives $v = \sqrt{3gR}$.

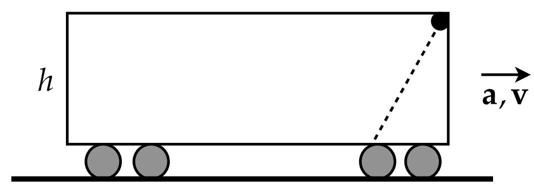
d. The horizontal motion has kinetic energy $K_h = \frac{1}{2}mv_x^2 = \frac{1}{2}m \cdot \frac{3}{2}gR$. At the maximum height y_m the vertical motion has zero kinetic energy. By conservation of energy $\frac{5}{2}mgR = K_h + mgy_m = \frac{3}{4}mgR + mgy_m$. This gives $y_m = \frac{7}{4}R$.

[Can also use projectile motion methods.]

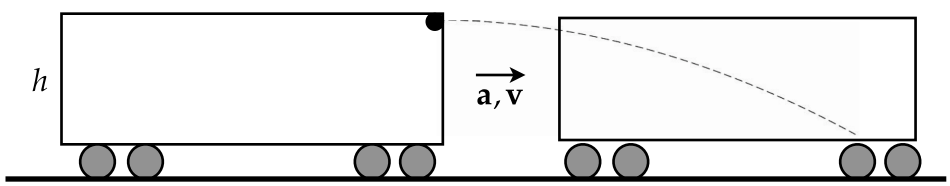
2. A railroad car is moving to the right, its speed increasing at rate $a = g/2$. At $t = 0$ a small object located at the top front corner of the car drops to the floor, a vertical distance h. You are to analyze the fall of the object as observed by someone (A) at rest in the car, and by someone (B) at rest on the ground.

   a. Sketch on the drawing the path of the fall of the object as seen by A.

   b. Where on the floor (relative to the front) does the object land, in terms of h?



   c. The other drawing shows the car at the instant the object drops, as seen by B. Make another drawing of the car to show it when the object lands on the floor, and sketch the object's trajectory as seen by B. Be sure the location of the point where it lands on the floor is consistent with your answers to (a) and (b).



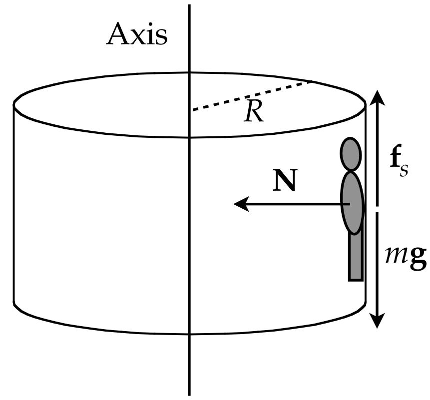
(15 points)

a. See drawing. Trajectory is a straight line along $\mathbf{g}_{eff}$.

b. Since $a = g/2$, the angle made with the vertical by $\mathbf{g}_{eff}$ is $\tan^{-1}(1/2)$, so the object lands at distance $h/2$ behind the front wall of the car.

c. See drawing. In this frame the initial velocity of the object is the velocity of the car, which is not zero. The curve is a parabola.

3. An amusement park ride consists of a large hollow cylinder of radius R mounted to rotate about its symmetry axis. Riders stand on a floor at the bottom of the cylinder as it begins rotating, increasing its angular speed ω. When ω reaches a certain value the floor folds away, leaving the riders supported only by friction from the wall of the cylinder.



   a. Draw a free body diagram (in an inertial frame fixed on the ground) showing the forces on a rider after the floor has folded away. [Use the figure in the drawing above .]

   b. What minimum normal force from the wall will support a rider of mass m if the coefficient of static friction is μ_s?

   c. What is the minimum angular speed ω of the cylinder?

   d. Draw a free body diagram in a non-inertial frame moving with the rider, showing the forces that keep him at rest. [Use the drawing below.]

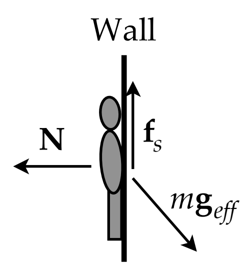
(20 points)

a. See drawing above.

b. We require $f_s = mg$, so $mg \le \mu_s N$, or $N \ge mg / \mu_s$.

c. Since $N = mR\omega^2$ we have $\omega^2 \ge g / \mu_s R$.

d. See drawing below. The forces add to zero.



4. The potential energy function for a mass attached to a faulty spring is $U(x) = x^2 - \frac{1}{4}x^4$.

a. What is the force $F(x)$? [Use $\frac{dx^n}{dx} = nx^{n-1}$.]

b. For what values of x, other than $x = 0$, is $F(x) = 0$? Call them x_1 and x_2, where $x_1 > x_2$. Specify whether each is a point of stable or unstable equilibrium.

c. What is $U(x_1)$? What is $U(x_2)$?

d. Let the total energy the system be E. In the following cases, is the motion bound or unbound? Give your reasons.

1. $E < U(x_1)$ and $x_2 < x < x_1$.

2. $E < U(x_1)$ and $x > x_1$.

3. $E > U(x_1)$.

(20 points)

a. We have $F(x) = -\frac{dU}{dx} = -2x + x^3$.

b. Setting $F = 0$ we find (dividing by x): $x^2 = 2$, or $x_1 = +\sqrt{2}$, $x_2 = -\sqrt{2}$. They are both maxima ($d^2U/dx^2 < 0$), thus unstable equilibrium points.

c. Both are 1.

d. 1. This is the region between the two maxima in *U*, so there will be two turning points. The motion is bound.

2. This is to the right of the maximum at x_1. From that point on *U* decreases, so there is only a turning point near the maximum. The motion is unbound.

3. Now $E > U$ everywhere, so there are no turning points. The motion is unbound.