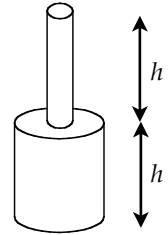


## Exam III

## Solutions

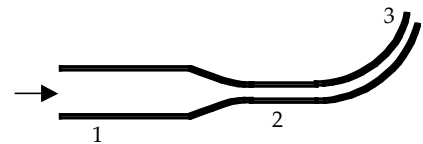
Part A. Multiple choice questions. Check the best answer. Each question carries a value of 4 points.

1. In Pascal's demonstration the barrel shown has height  $h$  and cross-section area  $A$ . The narrow tube fitted into its top also has height  $h$ . Both the barrel and the tube are filled with water.



- The pressure at the bottom of the barrel is twice what it would be without the tube, so the floor under the barrel has to provide twice as much upward force. [The floor supports the total weight, which is not twice as large.]
- The increase in pressure in the barrel is proportional to the volume of water in the tube. [Only the height.]
- The greatest outward pressure on the sides of the barrel is near the bottom, so that is where it is most likely to burst open.
- None of the above is true.

2. The drawing shows a pipe carrying water flowing in from the left and out into the air at point 3. The order of increasing pressure at the



- 1, 2, 3.
- 3, 1, 2.
- 3, 2, 1.
- 2, 3, 1.

points is:

3. Because the available broadcast bands are crowded, AM radio stations are required to filter their programming to restrict sound frequencies to below about 5 kHz, while FM stations have no such restriction. As a consequence:
- Special programming for dogs is limited to FM.
  - A violin and a flute playing a note with fundamental frequency 3 kHz will sound alike on AM.
  - Rush Limbaugh sounds better on AM.
  - AM signals can be received at greater distances than FM.
4. A string of length 2 m is stretched between two walls. It is observed that standing waves can be set up for frequencies 200 and 250 Hz, but no frequency in between. Which of these is wrong?
- The fundamental frequency is 50 Hz.
  - The two frequencies cited are those of the 4th and 5th harmonics.
  - The velocity of waves in the string is 100 m/s. [200]
  - Two of the above are true.

Part B. True-false questions. Check T or F depending on whether the statement is true or false. Each question carries a value of 3 points.

5. For a particle subject to a conservative force, its motion about any point of stable equilibrium, if it has relatively little kinetic energy, is approximately SHM.

T

F

6. For a given frequency of the source, the frequency of sound received will be the same whether the source moves at speed  $v$  toward the receiver at rest or the receiver moves at speed  $v$  toward the source at rest.

T

F

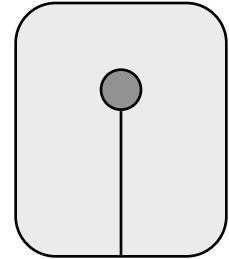
7. The difference in the frequencies of successive harmonics in an organ pipe is equal to the frequency of the fundamental only if the pipe is open at both ends.

T

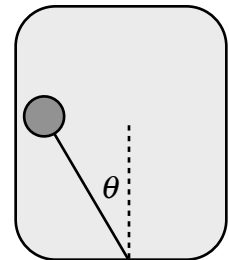
F

Part C. Problems. Work problem in space provided, using extra sheets if needed. Explain your method clearly. Problems carry the point values shown.

1. A closed container is completely filled with water. Attached to the bottom is a string, and the other end is attached to a cork ball of mass  $m$ . The string keeps the ball submerged in the water. When the system is at rest as shown, the tension in the string is half the weight of the ball.



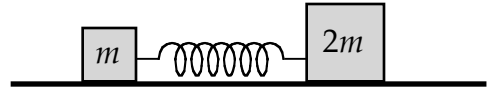
- What is the buoyant force on the ball in this situation?
- What is the ratio  $\rho / \rho_w$  of the densities of the cork and of water?
- Now the whole system is placed on a rotating platform, at distance  $R$  from the axis of rotation. As seen in its rest frame, the situation looks like this, with the rotation axis to the left. What is the angular speed of the rotation, in terms of  $g$ ,  $R$  and  $\theta$ ?
- What is the tension in the string in this case?



(20 points)

- The buoyant force is the only upward force, so it is  $\frac{3}{2}mg$  (weight plus tension).
- We have for the buoyant force  $V\rho_w g = \frac{3}{2}mg = \frac{3}{2}V\rho g$ , so  $\rho / \rho_w = \frac{2}{3}$ .
- The angle is given by  $\tan\theta = a / g$ , so  $a = R\omega^2 = g \tan\theta$ , or  $\omega^2 = (g / R)\tan\theta$ .
- The tension is  $T = \frac{1}{2}mg_{eff} = \frac{1}{2}m\sqrt{g^2 + a^2} = \frac{1}{2}m\sqrt{g^2 + (R\omega^2)^2}$ .

2. Two blocks, of mass  $m$  and  $2m$ , are resting on a frictionless table, attached as shown to an ideal spring of stiffness  $k$ . You push them together to compress the spring and then release them from rest. You are to determine the angular frequency  $\omega$  of the resulting oscillation.



- Let the amplitude of the oscillation of the heavier block be  $A$ . What is the amplitude of the oscillation of the lighter block? [Consider the motion of the CM of the system.]
- At the instant when the two blocks are farthest apart from each other and at rest momentarily, what is the amount by which the spring is stretched, in terms of  $A$ ? What is the potential energy at that instant?
- Find the maximum speed of each block in terms of its amplitude and  $\omega$ , and express the maximum total kinetic energy of the two blocks in terms of  $A$ ,  $m$ , and  $\omega$ .
- Use your answers to (b) and (c) to determine  $\omega$ .

(20 points)

- The CM must remain at rest, so if the heavier block moves distance  $A$  the lighter one moves distance  $2A$ , which will be the amplitude of its oscillation.
- Their total distance apart is  $3A$ , so the spring's potential energy is  $\frac{1}{2}k(3A)^2 = \frac{9}{2}kA^2$ .
- For each,  $v_{\max} = \omega A$ , so we have  $K_{\max} = \frac{1}{2}m(2\omega A)^2 + \frac{1}{2}(2m)(\omega A)^2 = 3m(\omega A)^2$ .
- The maximum kinetic energy is equal to the maximum potential energy, so  $\frac{9}{2}kA^2 = 3m\omega^2 A^2$ , or  $\omega = \sqrt{\frac{3k}{2m}}$ .

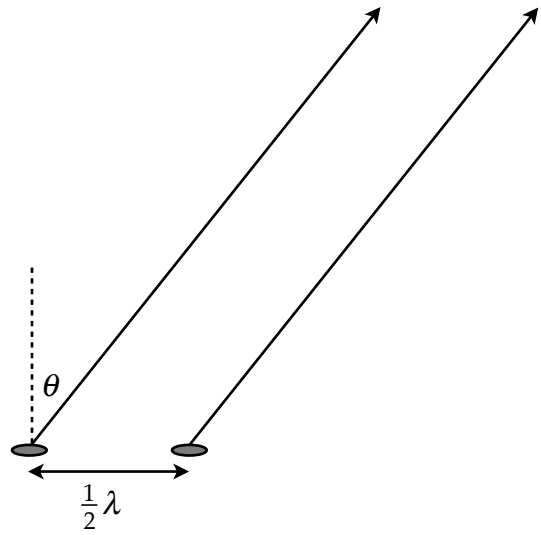
3. An ambulance is approaching from behind you, and you have stopped your car to let it pass. The ambulance is traveling at  $1/20$  the speed of sound, and its horn emits sound of frequency 200 Hz. Ahead of you is a large building, and you hear sound both directly from the ambulance horn and from its reflection off the building. Call the frequency you hear from the ambulance  $f_1$  and the frequency of the echo  $f_2$ . You listen for beats produced by these two sounds.
- What is the beat frequency as the ambulance approaches you?
  - What is the beat frequency as the ambulance recedes in front of you but approaches the building?
  - Would your answer to (a) be different if you had only slowed down but not stopped? Explain.

[Use the binomial approximation  $\frac{1}{1 \pm \alpha} \approx 1 \mp \alpha$  for  $\alpha \ll 1$ .]

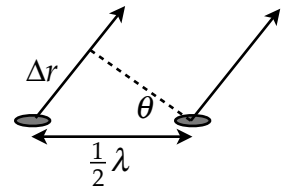
(15 points)

- Zero. You and the building receive the same frequency.
- Now  $f_1 = 200 \frac{1}{1 + 1/20}$  and  $f_2 = 200 \frac{1}{1 - 1/20}$ , so  $f_2 - f_1 \approx 200 \cdot (2/20) = 20$  Hz.
- Yes. Now you and the building receive receive different frequencies. Yours is lower than the building's. Also you are now moving toward the building, so you hear a higher frequency from the echo.

4. Two radio antennas, shown from above, send out waves of wavelength  $\lambda$ , starting in phase. We are interested in the intensity when the two waves arrive at a distant receiver. Each antenna alone would result in intensity  $I_0$  at the receiver. The receiver is far away, so the two lines to it from the antennas (shown) are practically parallel.



- Find the difference in distance  $\Delta r$  from the antennas to the receiver, as a function of  $\theta$ . [See the drawing below.]
- Show that constructive interference occurs only for  $\theta = 0$ . [We are interested only in  $0 \leq \theta \leq \pi/2$ .]
- What is the intensity for  $\theta = \pi/2$ ?
- For what value of  $\theta$  is the intensity half of its value for  $\theta = 0$ ? [Give  $\sin \theta$ .]



(20 points)

- The right triangle in the drawing gives  $\Delta r = \frac{1}{2} \lambda \cdot \sin \theta$ .
- The phase difference is  $\phi = k \cdot \Delta r = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} \cdot \sin \theta = \pi \sin \theta$ . This means  $0 \leq \phi \leq \pi$ . The only angle in this range that gives constructive interference is zero, which means  $\theta = 0$ .
- This gives  $\phi = \pi$ , so the intensity is zero.
- This requires  $\cos \phi = 0$ , or  $\phi = \pi/2$ . This happens if  $\sin \theta = \frac{1}{2}$ , or  $\theta = \pi/6$ .