

Exam II**Solutions**

Part A. Multiple choice questions. Check the best answer. Each question carries a value of 4 points.

1. Which of these general statements about a system of particles is wrong?

- Any change in total momentum comes from the action of external forces only.
- Any change in total kinetic energy comes from work done by external forces only.
- Any change in total angular momentum comes from the action of external torques only.
- Any change in total mechanical energy comes from work done by non-conservative forces only.

2. A symmetric body rolls without slipping. Which of these is wrong?

- If friction acts on the body, it is static friction.
- The body cannot roll up or down an incline without friction.
- Friction always acts opposite to the direction of the body's CM motion.
- The body's angular acceleration is proportional to its CM acceleration.

3. A rigid body is rotating about an axis with angular velocity ω .
- If the axis passes through the CM then L is proportional to ω .
 - If the axis is a symmetry axis of the body then L is proportional to ω .
 - L is always proportional to ω .
 - None of the above is true.
4. About spring tides and neap tides:
- Each happens once per lunar month.
 - Spring tides result only when the sun and moon are on the same side of the earth.
 - Spring tides happen at the time of full moon.
 - Neap tides result when the sun and moon are on opposite sides of the earth.

Part B. True-false questions. Check T or F depending on whether the statement is true or false. Each question carries a value of 3 points.

5. The total momentum of a system is equal to the total mass times the velocity of the CM.

T

F

6. The total kinetic energy of a system is equal to $\frac{1}{2}Mv_{CM}^2$, where M is the total mass.

T

F

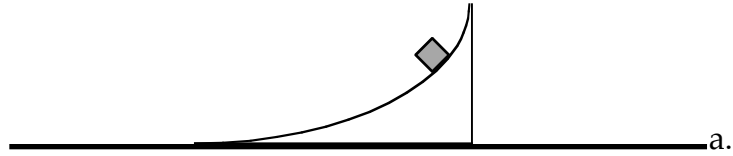
7. If the total torque about the CM of a system is zero the total angular momentum about any point is conserved.

T

F

Part C. Problems. Work problem in space provided, using extra sheets if needed. Explain your method clearly. Problems carry the point values shown.

1. A small block of mass m is at rest at height h on the wedge shaped object of mass $4m$ shown, at rest on a frictionless floor. The CMs of the block and the wedge are on the same vertical line initially. The block is released and slides down without friction, moving horizontally as it leaves the wedge.

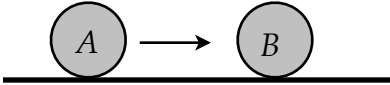


- What quantities are conserved?
- What are the speeds v and V of the block and wedge after they separate?
- When the wedge has moved to the right a distance x , how far has the block moved to the left?
- If the curved track on the wedge had friction, which of your answers to the previous questions would be unchanged? Explain.

(20 points)

- Horizontal momentum and energy.
- Cons. of horizontal momentum: $mv = 4mV$, so $v = 4V$. Cons. of energy:
 $mgh = \frac{1}{2}mv^2 + \frac{1}{2}(4m)V^2$, so $V^2 = gh/10$ and $v^2 = 8gh/5$.
- The CM does not move horizontally, so m has moved distance $4x$.
- The CM would still not move horizontally (internal forces don't affect it), so (c) would still be valid.

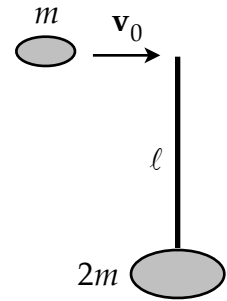
2. Questions about collisions.

- a. On a frictionless horizontal surface, a block of mass m moving with speed v_0 collides elastically with an identical block at rest, head-on, so the situation is one-dimensional. Prove that the first block stops and the second moves off with speed v_0 .
- b. An axially symmetric object (hoop, cylinder or sphere) of mass m , radius R , and moment of inertia I about its symmetry axis is moving across a horizontal surface that has friction. The object's CM speed is v and it rotates at rate ω (but is not necessarily rolling). Show that the quantity $mvR + I\omega$ is constant.
- c. Ball A shown, rolling with CM speed v_0 , collides with identical ball B , initially at rest.  It is a head-on elastic collision; there is no friction between the balls, so they exert no torques on each other during the collision. Immediately after the collision, how are each of the balls moving? [Describe in words.]
- d. The balls in (c) have mass m , radius R and moment of inertia $\frac{2}{5}mR^2$. What are their CM speeds some time after the collision when they are both rolling? [Use the result in (b).]

(20 points)

- a. Cons. of P : $mv_0 = mv_1 + mv_2$, or $v_0 = v_1 + v_2$. Cons. of K : $\frac{1}{2}mv_0^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$, or $v_0^2 = v_1^2 + v_2^2$. But also $v_0^2 = (v_1 + v_2)^2 = v_1^2 + v_2^2 + 2v_1v_2$. So $v_1v_2 = 0$. If $v_2 = 0$ afterwards then there was no collision, so $v_1 = 0$ and $v_2 = v_0$.
- b. As was shown in the assignment, about a point on the surface the total torque is zero, so L about that point is conserved. But about that point $L = mvR + I\omega$.
- c. By the argument in (a), the CM of A has stopped and B is moving to the right with CM speed v_0 . But A still has rotational motion with angular speed $\omega_0 = v_0 / R$, while B has no rotational motion.
- d. Use the result in (b). For A , $I\omega_0 = mvR + I\omega$, which, with $\omega = v / R$, leads to $v = \frac{2}{7}v_0$. For B , $mv_0R = mvR + I\omega$, which leads to $v = \frac{5}{7}v_0$.

3. A skater of mass m , moving with speed v_0 as shown from above, grabs a massless pole of length ℓ held out by a second skater of mass $2m$, initially at rest. The ice is frictionless.



- What quantities are conserved in the “collision”?
- Describe the subsequent motion and find the velocity of the CM (magnitude and direction).
- What is the kinetic energy of rotation of the skaters about their CM?
- If the pole has mass, which of the quantities in (a) will no longer be conserved when the skater grabs the pole? Explain your choice.

(20 points)

- Linear momentum, angular momentum, and energy.
- The CM (a point on the pole at distance $\ell/3$ from the heavier skater) moves to the right at constant speed, and the skaters rotate around it at constant angular speed.
- Find the speed of the CM: $mv_0 = (3m)v_{CM}$. The kinetic energy of CM motion is $\frac{1}{2}(3m)v_{CM}^2 = \frac{1}{6}mv_0^2$. So the rotational kinetic energy is $\frac{1}{2}mv_0^2 - \frac{1}{6}mv_0^2 = \frac{1}{3}mv_0^2$. [Can also find ω and I , but this is quicker.]
- Energy is lost in grabbing the massive pole (friction to bring the hand to rest on the pole) so energy will not be conserved.

4. An artificial satellite is in a circular orbit around the earth. It has mass $2m$ and the radius of the orbit is R . When it is at the point shown an explosion occurs, splitting it in half, and the rear half is suddenly brought to rest and falls to earth.
- What is the speed of the satellite v_0 before the explosion, in terms of G , the earth's mass M , and R ?
 - What is the total energy of the moving half of the satellite after the explosion, in terms of m and v_0 ? [Express the potential energy in those terms.]
 - Sketch the subsequent trajectory of the moving part on the drawing. Explain your choice.

(15 points)

- a. The energy is $E_0 = -\frac{GM(2m)}{2R}$, and for a circular orbit $K = -E$, so
- $$\frac{1}{2}(2m)v_0^2 = \frac{GM(2m)}{2R}, \text{ and } v_0^2 = \frac{GM}{R}.$$
- b. By conservation of momentum, $v = 2v_0$, so the kinetic energy of this part is
- $$K = \frac{1}{2}m(2v_0)^2 = 2mv_0^2. \text{ The potential energy is } U = -\frac{GMm}{R} = -mv_0^2, \text{ so the total energy is } mv_0^2. \text{ This is positive, so the motion is now unbound.}$$
- c. The trajectory is a hyperbola. See drawing.

