Soft pair real and virtual infrared functions in QED

S. Jadach*
The University of Tennessee, Knoxville, Tennessee 37996-1200
and TH Division, CERN, Geneva 23, Switzerland

M. Skrzypek
Department of Physics and Astronomy, The University of Tennessee, Knoxville, Tennessee 37996-1200
and Institute of Nuclear Physics, ul. Kawiory 26a, Krakow, Poland

B. F. L. Ward
Department of Physics and Astronomy, The University of Tennessee, Knoxville, Tennessee 37996-1200; TH Division, CERN, Geneva 23, Switzerland; and SLAC, Stanford University, Stanford, California 94309
(Received 30 June 1993)

We calculate the soft pair production analogues of the Yennie-Frautschi-Suura (YFS) real and virtual infrared functions B_γ and $\tilde{B}_\gamma$, where the latter describe the respective infrared singularities in QED to all orders in α via YFS exponentiation. In our work, we extend the discussion of B_γ and $\tilde{B}_\gamma$ by YFS to treat the case of soft pairs. The respective pair versions of B_γ and $\tilde{B}_\gamma$ are exhibited explicitly. We also discuss some possible applications to high precision Z^0 physics at SLC and/or LEP.

PACS number(s): 12.20.Ds, 11.15.Bt, 13.10.+q, 13.40.Ks

I. INTRODUCTION

The problem of the effect of soft pair production in the radiative corrections to QED processes is a fundamental problem in its own right. Indeed, it is just as fundamental as the analogous problem of soft photon production in such processes. In practice, the fact that the pair production effect occurs first at $O(\alpha^2)$ means that it has only recently [1] come under the kind of detailed investigation of the type effected for soft photons already in [2], for example. Recent improvements in the measurement of the SLAC Linear Collider (SLC) or CERN e^+e^- collider LEP luminosity process $e^+e^- \to e^+e^- + n(\gamma)$ necessitate the extension of the methods in [2] to soft pairs as well. In the following discussion we carry out the first step in the latter extension. Namely, we calculate the soft pair analogues of the real and virtual infrared functions $\tilde{B}_\gamma$ and B_γ, respectively, of [2]. In this way, we prepare the way for the rigorous exponentiation of the Yennie-Frautschi-Suura (YFS) type for the effect of soft pair production in processes such as the SLC and/or LEP luminosity process via the same type of Monte Carlo methods as two of us (S.J. and B.F.L.W.) have realized for multiple photon production in SLC/LEP physics processes in [3], for example. The actual Monte Carlo event generator for multiple soft pair production in the SLC/LEP luminosity process will appear elsewhere [4].

We want to emphasize that the high precision Z^0 physics, in which the experimental error on the all important luminosity process is expected to reach 0.15% in the near future [5], makes it necessary to prove that the attendant theoretical error is below 0.05%. Recently, we have shown [6] that indeed we can control the contribution of soft pairs to this process at the required level of precision. The implementation of the results in [6] into our Bhabha luminosity YFS exponentiated event generator BHLUMI 2.01 [7] would then allow the pair production effect to be treated on an event-by-event basis on equal footing with the respective multiple photon effects, would obviate the need for specialized subtractions for the pair effects, and would allow detailed detector simulation analysis of the soft pair production effect. This would significantly improve the efficiency of the analysis of the pair production effect in the SLC/LEP luminosity process. The latter improvement is the primary motivation for our work in this paper.

Our work is organized as follows. In the next section, we analyze the soft pair analogue of the YFS real photon infrared function $\tilde{B}_\gamma$. In Sec. III, we compute the soft pair analogue of the YFS virtual photon infrared function B_γ. Section IV contains some concluding remarks.

II. THE REAL $\tilde{B}_f$ FOR SOFT PAIR

In the following we calculate $\tilde{B}_f$, the analogue of the YFS real soft photon factor $\tilde{B}_\gamma$ for the emission of a real soft fermion pair. The total QED soft quanta real emission function is then extended to $\tilde{B}_{\rm tot} = \tilde{B}_\gamma + \sum_f \tilde{B}_f$.

Repeating exactly the steps leading to $\tilde{B}_\gamma$ we find the following expression for $\tilde{B}_f$ [note that we repeat *only* the construction of $\tilde{B}_\gamma$, and do not make a claim for any complete proof of exponentiation; in other words we show

*Permanent address: Institute of Nuclear Physics, ul. Kawiory 26a, Krakow, Poland.

that, for this particular class of graphs, both leading-log (LL) and infrared (IR) singular contributions can be exponentiated just like one does for photon emission graphs]:

$$\tilde{B}_f = \left(\frac{\alpha}{\pi}\right)^2 \frac{1}{\pi^2} \int \frac{d^3q_1}{2E_{q_1}} \frac{d^3q_2}{2E_{q_2}} \left(\frac{2p-aq}{aq^2-2pq} - \frac{2p'-aq}{aq^2-2p'q}\right)^{\mu} \left(\frac{2p-aq}{aq^2-2pq} - \frac{2p'-aq}{aq^2-2p'q}\right)^{\nu} \frac{4q_1^\mu q_2^\nu - q^2 g^{\mu\nu}}{2q^4}. \tag{1}$$

We work in the s channel, where p, p' are the four-momenta of the incoming electron and antielectron (with masses $p^2 = p'^2 = m^2$), q_1, q_2 are the four-momenta of the additional pair (with masses $q_1^2 = q_2^2 = \mu^2$) and $q = q_1 + q_2$. The parameter $a = 0$ or 1 reflects partly the freedom we have in the definition of the noninfrared, finite part of $\tilde{B}_f$. We recall that the parameter choices $a = 0, 1$ were first introduced in the original YFS[2] analysis, where in their Appendix A, they derive the general photon emission factor (here, k is the respective photon four-momentum) $R_\mu = \left(\frac{2p_\mu - k_\mu}{k^2-2kp+i\epsilon} + \frac{2p'_\mu + k_\mu}{k^2+2kp'+i\epsilon}\right)$, which corresponds to $a = 1$ in Eq. (1), and point out that for real emission one may use the emission factor $\tilde{R}_\mu = (p'_\mu/p'k - p_\mu/pk)$, which corresponds to $a = 0$ in Eq. (1), since the difference between using the two expressions is a term of the type $K(k)$, in their notation [2], which vanishes for $k \to 0$ and does not contribute to the infrared limit. We will see below that our result for $\tilde{B}_f$ will have the same property insofar as a is concerned.

In the first step we rearrange two body phase space[1] $d^3q_1 d^3q_2$. We work temporarily in the $q_1 + q_2$ rest frame (CMS$_q$). Introducing $\delta^4(q - q_1 - q_2)d^4q$, with the help of the identity

$$\int \frac{d^3q_1}{2E_{q_1}} \frac{d^3q_2}{2E_{q_2}} \delta^4(q - q_1 - q_2) = \int \frac{|\mathbf{q}_1| d\cos\theta_1\, d\phi_1}{4\sqrt{q^2}} \tag{2}$$

we get

$$\tilde{B}_f = \frac{-1}{6\pi}\left(\frac{\alpha}{\pi}\right)^2 \int d^4q \sqrt{1 - \frac{4\mu^2}{q^2}} \frac{1}{q^2}\left(1 + \frac{2\mu^2}{q^2}\right)\left(\frac{2p-aq}{aq^2-2pq} - \frac{2p'-aq}{aq^2-2p'q}\right)^2. \tag{3}$$

Next we introduce also $\delta^4(Q - p - p' + q)d^4Q$. The $d^4q d^4Q$ integration we parametrize in the $p + p'$ rest frame (CMS$_p$). Since

$$\int d^4Q d^4q \delta^4(Q - p - p' + q) = \frac{1}{8s}\int dM_q^2 dM_Q^2 d\cos\theta_q\, d\phi_q \sqrt{\lambda}, \tag{4}$$

we arrive at

$$\tilde{B}_f = \frac{1}{6s}\left(\frac{\alpha}{\pi}\right)^2 \int dM_Q^2 \frac{dM_q^2}{M_q^2}\sqrt{1 - \frac{4\mu^2}{M_q^2}}\left(1 + \frac{2\mu^2}{M_q^2}\right)\left[\frac{(2aM_q^2 - 8m^2)\sqrt{\lambda}}{M_q^2\left[2(1-a)s + 2aM_Q^2\right] + \frac{2m^2}{s}\lambda}\right.$$
$$\left. - \frac{-2aM_q^2 + 4(s - 2m^2)}{\sqrt{1 - \frac{4m^2}{s}}\left(s - M_Q^2 - (2a-1)M_q^2\right)} \ln\frac{s - M_Q^2 - (2a-1)M_q^2 - \sqrt{1 - \frac{4m^2}{s}}\sqrt{\lambda}}{s - M_Q^2 - (2a-1)M_q^2 + \sqrt{1 - \frac{4m^2}{s}}\sqrt{\lambda}}\right]. \tag{5}$$

In the above $s = (p + p')^2$ and $\lambda \equiv 4s|\mathbf{q}_{\mathrm{CMS}_p}|^2 = (s - M_q^2 - M_Q^2)^2 - 4M_q^2 M_Q^2$. For the further reference we also note that $q^0_{\mathrm{CMS}_p} \equiv E_q = (s - M_Q^2 + M_q^2)/(2\sqrt{s})$.

Let us emphasize here that so far the evaluation of Eq. (1) was *rigorous*; i.e., no small mass approximation was made, and all mass terms are kept in Eq. (5).

The definitions of the infrared cutoff and soft phase-space limits depend on the variables used. For the mass variables M_Q^2, M_q^2 we have

$$\Delta^2 > M_q^2 > 4\mu^2, \qquad (\sqrt{s} - M_q)^2 > M_Q^2 > (\sqrt{s} - \Delta)^2. \tag{6}$$

Having in mind the Monte Carlo implementation of this formalism, we would rather have the infrared cutoff defined in terms of E_q, the CMS$_p$ energy of the pair. The soft phase space in variables M_q^2 and E_q is then defined by

$$\Delta^2 > M_q^2 > 4\mu^2, \qquad \Delta > E_q > M_q. \tag{7}$$

To proceed further with the integration of Eq. (5) over the range (7) we will assume that the IR cutoff Δ satisfies the inequalities $\sqrt{s} \gg \Delta \gg 2\mu$, i.e., we will discard all terms of order $\Delta/\sqrt{s}$ and $2\mu/\Delta$. In this limit Eq. (5) becomes

[1]The following phase-space evaluation, up to Eq. (5), is based on the notes of Dr. Z. Wąs.

$$\tilde{B}_f = -\frac{2}{3}\Big(\frac{\alpha}{\pi}\Big)^2 \int_{4\mu^2}^{\Delta^2} \frac{dM_q^2}{M_q^2}\sqrt{1-\frac{4\mu^2}{M_q^2}}\Big(1+\frac{2\mu^2}{M_q^2}\Big)\int_{M_q}^{\Delta}\frac{dE_q}{E_q}\ln\frac{E_q-\sqrt{E_q^2-M_q^2}}{E_q+\sqrt{E_q^2-M_q^2}}\Big(1+O\Big(\frac{\Delta}{\sqrt{s}}\Big)\Big) \quad (8)$$

$$= \frac{2}{3}\Big(\frac{\alpha}{\pi}\Big)^2 \int_{4\mu^2}^{\Delta^2} \frac{dM_q^2}{M_q^2}\sqrt{1-\frac{4\mu^2}{M_q^2}}\Big(1+\frac{2\mu^2}{M_q^2}\Big)\Big[\frac{1}{4}\ln^2 y - \ln y\ln(1+y) - \mathrm{Li}_2(-y) - \frac{\pi^2}{12}\Big]\Big(1+O\Big(\frac{\Delta}{\sqrt{s}}\Big)\Big), \quad (9)$$

where $y = (\Delta - \sqrt{\Delta^2 - M_q^2})/(\Delta + \sqrt{\Delta^2 - M_q^2})$.

Note that all the dependence of $\tilde{B}_f$ on a has disappeared in the Eq. (8). This is not a completely trivial fact, since, in principle, the solutions for $a=1$ and $a=0$ could differ by some finite constant.[2]

The remaining integration over dM_q^2 can be easily done if the integration range is split into two parts $\Delta^2 > M_q^2 > 2\mu\Delta$ and $2\mu\Delta > M_q^2 > 4\mu^2$. The appropriate approximations in both integrals convert Eq. (9) into

$$\tilde{B}_f = \frac{2}{3}\Big(\frac{\alpha}{\pi}\Big)^2\Big[\int_{2\mu\Delta}^{\Delta^2}\frac{dM_q^2}{M_q^2}\Big(\frac{1}{4}\ln^2 y - \ln y\ln(1+y) - \mathrm{Li}_2(-y) - \frac{\pi^2}{12}\Big)$$
$$+\int_{4\mu^2}^{2\mu\Delta}\frac{dM_q^2}{M_q^2}\sqrt{1-\frac{4\mu^2}{M_q^2}}\Big(1+\frac{2\mu^2}{M_q^2}\Big)\Big(\frac{1}{4}\ln^2\frac{M_q^2}{4\Delta^2}-\frac{\pi^2}{12}\Big)\Big]\Big(1+O\Big(\frac{\Delta}{\sqrt{s}},\frac{\mu}{\Delta}\Big)\Big) \quad (10)$$

and finally

$$\tilde{B}_f(\Delta) = \frac{4}{3}\Big(\frac{\alpha}{\pi}\Big)^2\Big[\frac{1}{3}l_\mu^3 - \frac{5}{6}l_\mu^2 + \Big(\frac{14}{9}-\frac{\pi^2}{6}\Big)l_\mu + \zeta(3) + \frac{5}{36}\pi^2 - \frac{41}{27}\Big]\Big(1+O\Big(\frac{\Delta}{\sqrt{s}},\frac{\mu}{\Delta}\Big)\Big)$$
$$= \frac{4}{3}\Big(\frac{\alpha}{\pi}\Big)^2\Big[\frac{1}{3}\mathcal{L}_\mu^3 + \Big(\frac{31}{36}-\frac{\pi^2}{6}\Big)\mathcal{L}_\mu + \zeta(3) - \frac{197}{324}\Big]\Big(1+O\Big(\frac{\Delta}{\sqrt{s}},\frac{\mu}{\Delta}\Big)\Big) \quad (11)$$

The big logarithms are defined by $l_\mu = \ln(\frac{2\Delta}{\mu})$ and $\mathcal{L}_\mu = l_\mu - \frac{5}{6}$.

The last question to be addressed here is how do we compare to the results in the literature? The correction to the process $e^+e^- \to \mu^+\mu^-$ due to emission of one additional fermion pair has been calculated in Refs. [8, 9]. Apart from the Born cross section, the double mass distributions presented therein coincide in the soft region (6) with our Eq. (8). Since also the IR cut definitions (6) and (7) differ only by terms of $O(\Delta/\sqrt{s})$, the overall result for $\tilde{B}_f$ of Eq. (11) is identical to the corresponding results of Refs. [8, 9]. Let us stress again, however, that this is not an obvious coincidence, but rather a nice but *nontrivial* consequence of the specific convention for the finite part of $\tilde{B}_f$, as defined in Eq. (1).

III. THE VIRTUAL B_f FOR SOFT PAIR

In this section we define and calculate the virtual factor B_f arising from the virtual soft fermion pair contribution. It is the analogue of the YFS virtual photonic B_γ factor of Ref. [2]. The total QED virtual infrared function will then be given by $B_{\rm tot} = B_\gamma + \sum_f B_f$. Following closely Ref. [2] we define B_f as

$$B_f^U(s) = -e^2\int\frac{d^4k}{(2\pi)^4}\frac{iP(k^2)}{k^4}\Big(\frac{2p+k}{k^2+2pk}\Big)\Big(\frac{-2p'+k}{k^2-2p'k}\Big), \quad (12)$$

with the vacuum polarization $P(k^2)$, due to virtual fermion pair with arbitrary mass μ, given in Ref. [10]:

$$P(k^2) = \frac{\alpha\mu^2}{3\pi}\Big[-\frac{22}{3}+\frac{5}{3}\Big(\zeta+\frac{1}{\zeta}\Big)$$
$$+\Big(\zeta+\frac{1}{\zeta}-4\Big)\frac{1+\zeta}{1-\zeta}\ln\zeta\Big],$$
$$\frac{k^2}{\mu^2} = -\frac{(1-\zeta)^2}{\zeta},\quad k^2<0. \quad (13)$$

Here we note that $P(k^2)$ is already renormalized and that B_f as given by Eq. (12) is free of infrared divergences. As in the real case we work in the s channel with p, p' being the incoming electron and positron four-momenta with masses $p^2 = p'^2 = m^2$ and $s = (p+p')^2$. The index U in Eq. (12) refers to the fact that so defined B_f^U requires

[2]Let us stress that even though the mass m disappeared also from Eq. (8), an arbitrary choice of $\mu \geq m$ is all the times legitimate in Eq. (8). In other words no additional approximation such as $\mu \gg m$ was taken in Eq. (8). This is consistent with the requirements of the Lee-Kinoshita-Nauenberg theorem. An analogous behavior for the dependence of our soft pairs virtual infrared function on m must hold due to the same theorem.

further UV renormalization. Like in the photonic case of Ref. [2] it can be done by the direct replacement

$$\left(\frac{2p+k}{k^2+2pk}\right)\left(\frac{-2p'+k}{k^2-2p'k}\right) \to -\frac{1}{2}\left(\frac{2p+k}{k^2+2pk}-\frac{-2p'+k}{k^2-2p'k}\right)^2 \qquad (14)$$

in Eq. (12). Instead, we prefer to use the method of dispersion relations to evaluate Eq. (12), cf. e.g., Refs. [10–12]. As always only the real part of B_f^U is UV divergent, and the imaginary part of B_f is given by the formula

$$2i\mathrm{Im}B_f(s) = e^2\int\frac{d^4k}{(2\pi)^2}\frac{iP(k^2)}{k^4}(2p+k)(-2p'+k) \times\delta((p+k)^2-m^2) \times\delta((p'-k)^2-m^2)\theta(p_0+k_0). \qquad (15)$$

The dispersion relation with one subtraction leads to the renormalized real part of B_f, compatible with the replacement (14):

$$\mathrm{Re}B_f(s) = \frac{s}{\pi}\mathrm{P}\int_{4m^2}^{\infty}\frac{ds'}{s'}\frac{\mathrm{Im}B(s')}{s'-s}. \qquad (16)$$

The P stands for the principal value of the integral.

The evaluation of Eqs. (15) and (16) proceeds as follows. The simple identity (cf. Ref. [10])

$$\int d^4k\delta\Big((p+k)^2-m^2\Big)\delta((p'-k)^2-m^2)\theta(p_0+k_0) = \frac{1}{4s}\left(1-\frac{4m^2}{s}\right)^{-1/2}\int_{-s+4m^2}^{0}dk^2d\phi \qquad (17)$$

leads to

$$\begin{aligned}\mathrm{Im}B_f(s) &= -\frac{\alpha}{4}\frac{1}{s}\left(1-\frac{4m^2}{s}\right)^{-1/2}\int_y^1 d\zeta\frac{1+\zeta}{(1-\zeta)^3}\left[2\frac{s-2m^2}{\mu^2}-\frac{(1-\zeta)^2}{\zeta}\right]P(\zeta)\\ &= -\frac{\alpha^2\mu^2}{12\pi}\frac{1}{s}\left(1-\frac{4m^2}{s}\right)^{-1/2}\left[\frac{2(s-2m^2)}{3\mu^2(y-1)^3}\Big((y+1)(5y^2-14y+5)\ln y+8y(y-1)\Big)\right.\\ &\quad\left.-\frac{s-2m^2}{\mu^2}\ln^2 y+\frac{(y+1)^3}{y(y-1)}\ln y-\frac{8}{3}\frac{y^2+y+1}{y}-\frac{56}{9}\frac{s-2m^2}{\mu^2}\right],\end{aligned} \qquad (18)$$

where $y = -(\sqrt{s-4m^2}-\sqrt{s-4m^2+4\mu^2})/(\sqrt{s-4m^2}+\sqrt{s-4m^2+4\mu^2})$, or alternatively $s = \mu^2(1-y)^2/y+4m^2$. Consequently, introducing the new variable y' related to s' as y is related to s, the dispersion relation (16) becomes

$$\mathrm{Re}B_f(s) = \frac{sy}{\pi}\mathrm{P}\int_0^1\frac{dy'}{y'}\frac{1-y'^2}{y-y'}\frac{1}{1-yy'}\frac{\mathrm{Im}B(y')}{s'(y')}. \qquad (19)$$

So far the evaluation of B_f was valid for any choice of mass μ; i.e., no small mass approximation of any kind was made. At this point we have to consider separately the cases $\mu = m$ and $\mu \gg m$, as well as to take the appropriate small mass limits.

A. The case $\mu = m$

In this case the direct integration of Eq. (19), with $s' = m^2(1+y')^2/y'$ and $y \simeq m^2/s$, leads to

$$\mathrm{Re}B_m(s) = -\frac{\alpha^2}{12\pi^2}\left[\frac{1}{3}L_m^3-\frac{13}{6}L_m^2+\left(\frac{80}{9}-\frac{2}{3}\pi^2\right)L_m+\frac{2}{3}\pi^2-\frac{74}{9}\right]\left(1+O\Big(\frac{m^2}{s}\Big)\right), \qquad (20)$$

with $L_m = \ln(s/m^2)$.

B. The case $\mu \gg m$

As far as this case is concerned, we found it convenient to divide the integration range in Eq. (16) into two parts: (I) $4m^2 < s' < 4m\mu$ and (II) $4m\mu < s' < \infty$. One can easily show that the entire integral I is of $O(m/\mu)$ and therefore can be discarded in our approximation. With the same accuracy we can set in the range (II) $s' \simeq s'-2m^2 \simeq s'-4m^2 = \mu^2(1-y')^2/y'[1+O(m/\mu)]$ and $y'(s'=4m\mu) = 1-2\sqrt{m/\mu}+O(m/\mu)$. Integrating Eq. (16) with those simplifications we arrive at[3]

$$\mathrm{Re}B_\mu(s) = -\frac{\alpha^2}{12\pi^2}\left[\frac{1}{3}L_\mu^3-\frac{13}{6}L_\mu^2+\left(\frac{80}{9}-\frac{2}{3}\pi^2\right)L_\mu+4\zeta(3)+\frac{13}{9}\pi^2-\frac{484}{27}\right]\left(1+O\Big(\frac{m}{\mu},\frac{m^2}{s},\frac{\mu^2}{s}\Big)\right), \qquad (21)$$

with $L_\mu = \log(s/\mu^2)$.

How do Eqs. (20) and (21) compare to the literature? The corrections to the electron form factors due to vacuum polarization by an arbitrary fermion pair have been presented various limits in Refs. [12, 13]. We find imme-

[3] A little care is required to show that terms of $O(\sqrt{m/\mu})$ actually cancel in the final result.

diately that the leading terms (L^3) of Eqs. (20) and (21) are, as expected, the same as in Refs. [12, 13]. The other, "noninfrared," terms differ, however, as in the photonic case of the original paper [2].

IV. CONCLUSIONS

In this paper, we have extended the computation of the YFS photon real and virtual infrared functions $\tilde{B}_\gamma$ and B_γ to include the effect of soft fermion pair emission. The resulting complete QED soft quanta emission functions $\tilde{B}_{\rm tot}$ and $B_{\rm tot}$ allow one to treat the LL and soft pair emission effects by the same YFS exponentiation methods as one uses for photons. We note that $\tilde{B}_{\rm tot}$ and $B_{\rm tot}$ are fundamental properties of the soft and LL limits of the QED theory and therefore are of theoretical interest in their own right.

Our primary objective in deriving the complete QED soft quanta emission functions lies in their application to our [7] high precision YFS exponentiated calculation of the SLC/LEP luminosity process on an event-by-event basis via Monte Carlo methods. With these complete QED soft quanta emission functions, we may now incorporate the effect of soft pairs into our luminosity simulations on equal footing with soft photons. The resulting version of our luminosity event generator BHLUMI 2.01 [7] will appear elsewhere [4].

ACKNOWLEDGMENTS

This work was supported in part by the U.S. Department of Energy Contracts Nos. DE-FG05-91ER40627 and DE-AC03-76ER00515, and by the Polish Government Grants Nos. KBN 223729102 and KBN 203809101. The authors are grateful to Professor C. Prescott, Professor M. Breidenbach, and Professor J. Ellis for the respective hospitality of the SLAC Groups A and SLD and the CERN TH Division while part of this work was completed. The authors have benefited from helpful discussions with Dr. Z. Wąs.

[1] S. Jadach, M. Skrzypek, and M. Martinez, Phys. Lett. B **280**, 129 (1992).
[2] D.R. Yennie, S.C. Frautschi, and H. Suura, Ann. Phys. (N.Y.) **13**, 379 (1961).
[3] S. Jadach and B.F.L. Ward, Comput. Phys. Commun. **56**, 351 (1990).
[4] S. Jadach *et al.* (unpublished).
[5] J. Le Francois (private communication).
[6] S. Jadach, M. Skrzypek, and B.F.L. Ward, Phys. Rev. D **47**, 3733 (1993).
[7] S. Jadach, E. Richter-Wąs, B.F.L. Ward, and Z. Wąs, Comput. Phys. Commun. **70**, 305 (1992); S. Jadach, E. Richter-Wąs, B.F.L. Ward, and Z. Wąs, Phys. Lett. B **268**, 253 (1991); **260**, 438 (1991); **253**, 469 (1991).
[8] F.A. Berends, W.L. van Neerven, and G.J.H. Burgers, Nucl. Phys. **B297**, 429 (1988); **B304**, 921 (1988).
[9] B.A. Kniehl, M. Krawczyk, J.H. Kühn, and R.G. Stuart, Phys. Lett. B **209**, 337 (1988).
[10] E.M. Lifshitz and L.P. Pitaevskii, *Relativistic Quantum Theory* (Pergamon, New York, 1973), Pt. 2.
[11] R. Barbieri, J.A. Mignaco, and E. Remiddi, Nuovo Cimento **11A**, 824, 865 (1972).
[12] G.J.H. Burgers, Phys. Lett. B **164**, 167 (1985).
[13] B.A. Kniehl, Phys. Lett. B **237**, 127 (1990).