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Parton distribution functions, α_s, and heavy-quark masses for LHC Run II

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We determine a new set of parton distribution functions (ABMP16), the strong coupling constant α_s and the quark masses m_c, m_b and m_t in a global fit to next-to-next-to-leading order (NNLO) in QCD. The analysis uses the $\overline{\mathrm{MS}}$ scheme for α_s and all quark masses and is performed in the fixed-flavor number scheme for $n_f = 3, 4, 5$. Essential new elements of the fit are the combined data from HERA for inclusive deep-inelastic scattering (DIS), data from the fixed-target experiments NOMAD and CHORUS for neutrino-induced DIS, data from Tevatron and the LHC for the Drell-Yan process and the hadro-production of single-top and top-quark pairs. The theory predictions include new improved approximations at NNLO for the production of heavy quarks in DIS and for the hadro-production of single-top quarks. The description of higher twist effects relevant beyond the leading twist collinear factorization approximation is refined. At NNLO, we obtain the value $\alpha_s^{(n_f=5)}(M_Z) = 0.1147 \pm 0.0008$.

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I. INTRODUCTION

Parton distribution functions (PDFs) are indispensable for theory predictions of scattering processes at hadron colliders. Within standard factorization in quantum chromodynamics (QCD), the PDFs are determined by a comparison of theoretical predictions with hard-scattering data covering a broad range of kinematics in the Bjorken variable x and the momentum scale Q^2. Steady progress both in the accumulation and in the analysis of hard-scattering data by experiments at HERA, Tevatron and the Large Hadron Collider (LHC) as well as improvements of the relevant theoretical predictions to next-to-next-to-leading order (NNLO) in perturbative QCD allows for an accurate description of the parton content of the proton in global fits. Such fits provide the proton composition in terms of the gluon and the individual light-quark flavors u, d and s with a good precision. Simultaneously, they are also able to determine the strong coupling constant α_s and the heavy-quark masses m_c, m_b and m_t to NNLO in QCD. These results serve as input to high precision predictions for benchmark processes in the Standard Model (SM) and cross sections for scattering reactions beyond the SM, measured or being searched for in run II of the LHC.

PDF extractions have been carried out by us in the past, with ABM12 [1] being our previous global fit. The present analysis has evolved out of these efforts and results in the new ABMP16 set. It incorporates a number of intermediate updates [2,3], in particular the ABMP15 [3] fit. Moreover, it makes use of improvements in the theoretical description of the hard-scattering processes for the production of heavy quarks in deep-inelastic scattering (DIS) and for the hadro-production of single-top quarks. However, the primary motivation for ABMP16 comes from the wealth of recently published new data for the measurements of electron-induced DIS from HERA [4] as well as W- and Z-boson production at Tevatron and the LHC. These data have great potential to further constrain light-quark PDFs at large and small values of x, to pin down the gluon PDF and to consolidate determinations of α_s using various sets of DIS data published during the last three decades.

In our analysis, the PDFs and all QCD parameters which are often correlated with the PDFs, i.e., $\alpha_s(M_Z)$ and the heavy-quark masses $m_c(m_c)$, $m_b(m_b)$ and $m_t(m_t)$, are determined in the $\overline{\mathrm{MS}}$ scheme with the number of flavors fixed, $n_f = 3, 4, 5$; see, e.g., [5]. The theoretical accuracy is strictly NNLO in QCD. Other PDF sets currently available are CJ15 [6], CT14 [7], HERAPDF2.0 [4], JR14 [8], MMHT14 [9], and NNPDF3.0 [10], all of them accurate to NNLO in QCD except for CJ15, which has limited the precision to next-to-leading order (NLO). None of these PDFs uses all of the latest data considered in the current ABMP16 analysis. A recent benchmarking of those PDFs performed in [11] has shown that differences in the theoretical predictions obtained by using various PDFs are a consequence of specific theory assumptions or underlying physics models used in the fits of some of these PDFs. Therefore, it is essential to provide a detailed account of the theoretical framework used in the PDF analyses.

The paper is organized as follows. We present in Sec. II the set-up of the analysis. In particular, we discuss the various sets of hard-scattering data and their kinematic range in Sec. II A. The improvements in the theory description are given in Sec. II B and include new approximate NNLO QCD predictions for heavy-quark DIS and for single-top quark hadro-production as well as a refined treatment of the higher twist effects. The results for the ABMP16 PDFs are discussed in Sec. III where the quality of the data description is documented, the improvements in the PDFs are discussed and a detailed comparison with the ABM12 fit and other sets is provided. Correlations of the various fit parameters are discussed and particular attention is paid to the value of the strong coupling constant α_s extracted from the global fit and individually from various sets of DIS data. The sensitivity of the value of α_s to higher twist terms for all sets of DIS data is quantified. Furthermore, we report our results on the $\overline{\rm MS}$ heavy-quark masses $m_c(m_c)$, $m_b(m_b)$ and $m_t(m_t)$ and compare with other determinations. Finally, in Sec. IV we present several applications. We compare the second Mellin moment of the nonsinglet quark PDFs to recent lattice measurements and we provide cross-section predictions with the ABMP16 PDFs for relevant LHC processes, such as Higgs boson production in gluon-gluon-fusion and hadro-production of top-quark pairs. In addition, with the measured values of the strong coupling constant α_s and the top-quark mass $m_t(m_t)$ as input we can solve the renormalization group equations for all SM couplings including the scalar self-coupling λ of the SM Higgs boson. This allows us to study the running of λ and to assess whether new physics needs to be invoked in order to stabilize the electroweak vacuum at high scales [12,13]. We also discuss the features of the data grids for the fit results in the format of LHAPDF library (version 6) [14] and conclude in Sec. V.

II. SET-UP OF THE ANALYSIS

A. Data

The data used in present analysis have been updated in an essential manner with respect to the ones used in our earlier fits ABM12 [1] and ABMP15 [3]. The changes concern inclusive DIS data as well as data on DIS charm- and bottom-quark production, on the Drell-Yan (DY) process, and on the top-quark hadro-production as follows:

(i) The HERA run I inclusive cross-section data on the neutral-current (NC) and charged-current (CC) $e^{\pm}p$ DIS have been replaced with the final combination of the run I + II results [4]. This input provides improved constraints on the small-x gluon and sea-quark PDFs and significant benefits for the separation of the up- and down-quark PDFs by virtue of the precise CC data.

(ii) The data on production of bottom quark in $e^{\pm}p$ DIS obtained by the H1 [15] and ZEUS [16] Collaborations are added. These data are particularly useful for the determination of the bottom-quark mass and are also sensitive to the small-x PDFs.

(iii) New data on the charm-quark production in CC neutrino-nucleon DIS collected by the NOMAD [17] and CHORUS [18] experiments are added in order to improve the strange sea determination, cf. Ref. [3] for details.

(iv) The latest data on $W^{\pm}$- and Z-boson production from LHC and Tevatron are added in order to provide an improved determination of the light-quark PDFs over a wide range of the parton momentum fractions x and to disentangle distributions for quarks and anti-quarks. The data include rapidity distributions for $W^{\pm}$- and Z-boson production in the forward region at the collision energies of $\sqrt{s} = 7$ and 8 TeV obtained by LHCb [19–21], DØ data on the electron charge asymmetry, which also probes forward kinematics [22], DØ data on the muon charge asymmetry in the central region [23], new CMS rapidity distributions for the $W^{\pm}$-boson recorded at $\sqrt{s} = 7$ and 8 TeV using the muon decay channel [24,25] and the cross section of $W^{\pm}$- and Z-boson production at $\sqrt{s} = 13$ TeV in the fiducial volume obtained by ATLAS [26].

(v) A collection of the recent t-quark data from the LHC [27–52] and Tevatron [53] added to the present analysis provides additional constraints on the gluon PDF and allows to perform a consistent determination of the top-quark mass with full account of its correlations with the gluon PDF and the strong coupling α_s.

With the new measurements included in the present study, the theoretical framework has been updated correspondingly to account for the best possible precision and consistency of the PDF fit as discussed in Sec II B. In the following, the DIS, DY, and heavy-quark production data sets used in our fit are described in detail.

1. Inclusive DIS

The recent HERA inclusive NC and CC DIS data set [4] includes a combination of all published H1 and ZEUS measurements performed in the runs I and II of the HERA collider. The data were collected at the proton beam energies $E_p = 920$, 820, 575 and 460 GeV which correspond to the center-of-mass energies $\sqrt{s} = 320$, 300, 251 and 225 GeV, respectively. The combined HERA data [4] cover the range of momentum transfer squared Q^2 up to 50000 GeV^2 and are the most precise measurements of ep DIS over that wide kinematic range. The high-statistics HERA II data used in the new combination improve the accuracy at high Q^2, as compared to the HERA I inclusive combination, in particular for the NC e^-p and the CC e^+p (e^-p) data. The latter impose improved constraints on the valence down-(up-)quark distributions in the proton and in combination with the new DY collider data added to our fit they allow to avoid using the fixed-target DIS data [54–60] collected by the SLAC, BCDMS, and NMC experiments with a deuteron target. Previously, those samples have been employed in the ABM12 fit and our earlier analyses in order to constrain the down-quark distributions at the expense of having to deal also with nuclear effects. Now, with the extended DIS and DY input the experimental uncertainties in the down-quark PDFs do not deteriorate as compared to the ABM12 PDFs even in the absence of deuteron data, while any additional uncertainty caused by the modeling of nuclear effects has been eliminated.

The unprecedented precision achieved for the HERA run I + II data facilitates an accurate calibration of the earlier fixed-target DIS experiments' normalization. Therefore, we introduce a normalization factor for each remaining fixed-target data set, SLAC, NMC and BCDMS, and fit these factors simultaneously with the PDF parameters. The fitted values of normalization factors are determined with an uncertainty of $\mathcal{O}(1\%)$, cf. Table I. Such a reevaluation of the normalization is entirely justified for the SLAC and NMC experiments, as it was determined in those experiments in a similar way, using, however, less accurate data sets for calibration. It is also relevant for the BCDMS data [61], which were not subject to an additional re-normalization in our earlier ABM12 and ABMP15 fits based on the HERA I data. Indeed, the BCDMS normalization uncertainty determined in present analysis is much smaller than the one of 3% provided by BCDMS itself. In general, the normalization factors obtained in the present analysis are comparable to unity within the normalization uncertainties quoted by respective experiment. However, for the SLAC-E89a experiment [55] the normalization factor deviates from unity by ~5%. Besides, the data description quality achieved for the SLAC-E89a data is significantly worse than for other SLAC experiments. It is also worth noting that the SLAC-89a experiment is kinematically separated from other SLAC measurements. Therefore, having no possibility to clarify the issue of its normalization, we do not use SLAC-E89a data in the final version of the present analysis.

2. DIS charm- and bottom-quark production

In addition to the HERA inclusive combination [4], we include into the fit the semi-inclusive HERA data on NC DIS charm-quark production obtained by a combination of the corresponding H1 and ZEUS results [63]. Those data provide a complementary constraint on the low-x gluon and sea-quark distributions, cf. Ref. [63], and have already been employed in our earlier ABM12 and ABMP15 analyses.

The CC DIS charm-quark production, which is mostly relevant for disentangling the strange sea distribution, is routinely measured by detecting di-muons produced in neutrino-nucleon interaction. Two data sets of such kind, obtained by the CCFR and NuTeV experiments [64], were used in our earlier ABM12 and ABMP15 fits. For the present analysis we add the recent precision measurement of di-muon production in ν-Fe DIS performed by the NOMAD experiment [17], which allows to improve the strange sea determination at large x, cf. Ref. [2]. One more new measurement of the CC charm-quark production was performed by the CHORUS Collaboration [18] using an emulsion target. As a benefit of this technique, the charmed hadrons are detected directly by their hadronic decays, therefore the CHORUS data are less sensitive to the details of the charm fragmentation modeling. Likewise, the data on the charmed-hadron production rates from the emulsion experiment FNAL-532 [65] help to constrain the charmed-hadron semileptonic branching ratio, which is required for the analysis of the CCFR, NuTeV, and NOMAD di-muon data, cf. Ref. [2].

Finally, the bottom-quark DIS production cross sections measured by the H1 [15] and ZEUS [16] collaborations are also included into the present analysis. This allows to determine the value of the bottom-quark mass.

TABLE I. The values of fitted normalization factors for the fixed-target DIS data sets used in the present analysis with the uncertainties quoted in parentheses.

Experiment	Process	Beam energy (GeV)	Reference	Normalization
SLAC-49a	$ep \to eX$	$7 \div 20$	[54,62]	1.001(11)
SLAC-49b	$ep \to eX$	$4.5 \div 18$	[54,62]	1.010(15)
SLAC-87	$ep \to eX$	$8.7 \div 20$	[54,62]	1.012(11)
SLAC-89b	$ep \to eX$	$6.5 \div 19.5$	[56,62]	1.000(11)
BCDMS	$\mu p \to \mu X$	$100 \div 280$	[61]	0.976(7)
NMC	$\mu p \to \mu X$	90	[60]	0.993(13)
		120		1.011(12)
		200		1.022(12)
		280		1.012(12)

3. Drell-Yan process

The data on the hadro-production of $W^\pm$- and Z-bosons and the DIS data sets discussed above are mutually complementary in the context of disentangling the light-quark PDFs. In particular, the high statistics data from LHC and Tevatron on $W^\pm$-production in the forward region allow to improve the determination of the up- and down-quark distributions down to $x \sim 10^{-4}$, cf. Ref. [3]. For the present analysis we select the most recent and statistically significant data sets on the $W^\pm$- and Z-boson production collected by the ATLAS, CMS, and LHCb experiments at the LHC and the DØ experiment at Fermilab, cf. Table II. The updated analysis of ATLAS data [66] collected at $\sqrt{s} = 7$ TeV was released after completion of our fit. These data are in a good agreement with the predictions based on the ABM12 PDFs and therefore should be smoothly accommodated into a future release.

The data on $W^\pm$-production in Table II are given in form of pseudo-rapidity distributions for the decay electron or muon. The DØ data on $W^\pm$-distributions obtained by unfolding the charged-lepton ones are also available [70]. Since those data are sensitive to the details of the modeling of the $W^\pm$-decay and, in particular, to the PDFs used, they

TABLE II. The list of DIS and DY data used in the current analysis with the collider data listed first. The top-quark production data are detailed in Tables III and IV.

Experiment	Beam (E_b) or center-of-mass energy ($\sqrt{s}$)	$\mathcal{L}$ (1/fb)	Process	Kinematic cuts used in the present analysis (cf. orginal references for notations)	Ref.
			DIS		
HERA I + II	$\sqrt{s} = 0.225 \div 0.32$ TeV	0.5	$e^{\pm}p \to e^{\pm}X$	$2.5 \leq Q^2 \leq 50000$ GeV^2, $2.5 \times 10^{-5} \leq x \leq 0.65$	[4]
			$e^{\pm}p \to \overset{(-)}{\nu} X$	$200 \leq Q^2 \leq 50000$ GeV^2, $1.3 \times 10^{-2} \leq x \leq 0.40$	
BCDMS	$E_b = 100 \div 280$ GeV		$\mu^+ p \to \mu^+ X$	$7 < Q^2 < 230$ GeV^2, $0.07 \leq x \leq 0.75$	[61]
NMC	$E_b = 90 \div 280$ GeV		$\mu^+ p \to \mu^+ X$	$2.5 \leq Q^2 < 65$ GeV^2, $0.009 \leq x < 0.5$	[60]
SLAC-49a	$E_b = 7 \div 20$ GeV		$e^- p \to e^- X$	$2.5 \leq Q^2 < 8$ GeV^2, $0.1 < x < 0.8$, $W \geq 1.8$ GeV	[54] [62]
SLAC-49b	$E_b = 4.5 \div 18$ GeV		$e^- p \to e^- X$	$2.5 \leq Q^2 < 20$ GeV^2, $0.1 < x < 0.9$, $W \geq 1.8$ GeV	[54] [62]
SLAC-87	$E_b = 8.7 \div 20$ GeV		$e^- p \to e^- X$	$2.5 \leq Q^2 < 20$ GeV^2, $0.3 < x < 0.9$, $W \geq 1.8$ GeV	[54] [62]
SLAC-89b	$E_b = 6.5 \div 19.5$ GeV		$e^- p \to e^- X$	$2.5 \leq Q^2 \leq 19$ GeV^2, $0.17 < x < 0.9$, $W \geq 1.8$ GeV	[56] [62]
			DIS heavy-quark production		
HERA I + II	$\sqrt{s} = 0.32$ TeV		$e^{\pm}p \to e^{\pm}cX$	$2.5 \leq Q^2 \leq 2000$ GeV^2, $2.5 \times 10^{-5} \leq x \leq 0.05$	[63]
H1	$\sqrt{s} = 0.32$ TeV	0.189	$e^{\pm}p \to e^{\pm}bX$	$5 \leq Q^2 \leq 2000$ GeV^2, $2 \times 10^{-4} \leq x \leq 0.05$	[15]
ZEUS	$\sqrt{s} = 0.32$ TeV	0.354	$e^{\pm}p \to e^{\pm}bX$	$6.5 \leq Q^2 \leq 600$ GeV^2, $1.5 \times 10^{-4} \leq x \leq 0.035$	[16]
CCFR	$87 \lesssim E_b \lesssim 333$ GeV		$\overset{(-)}{\nu} N \to \mu^{\pm}cX$	$1 \leq Q^2 < 170$ GeV^2, $0.015 \leq x \leq 0.33$	[64]
CHORUS	$\langle E_b \rangle \approx 27$ GeV		$\nu N \to \mu^+ cX$		[18]
NOMAD	$6 \leq E_b \leq 300$ GeV		$\nu N \to \mu^+ cX$	$1 \leq Q^2 < 20$ GeV^2, $0.02 \lesssim x \leq 0.75$	[17]
NuTeV	$79 \lesssim E_b \lesssim 245$ GeV		$\overset{(-)}{\nu} N \to \mu^{\pm}cX$	$1 \leq Q^2 < 120$ GeV^2, $0.015 \leq x \leq 0.33$	[64]
			DY		
ATLAS	$\sqrt{s} = 7$ TeV	0.035	$pp \to W^{\pm}X \to l^{\pm}\nu X$	$p_T^l > 20$ GeV, $p_T^{\nu} > 25$ GeV, $m_T > 40$ GeV	[67]
			$pp \to ZX \to l^+l^-X$	$p_T^l > 20$ GeV, $66 < m_{ll} < 116$ GeV	
	$\sqrt{s} = 13$ TeV	0.081	$pp \to W^{\pm}X \to l^{\pm}\nu X$	$p_T^{\nu} > 25$ GeV, $m_T > 50$ GeV	[26]
			$pp \to ZX \to l^+l^-X$	$p_T^l > 25$ GeV, $66 < m_{ll} < 116$ GeV	
CMS	$\sqrt{s} = 7$ TeV	4.7	$pp \to W^{\pm}X \to \mu^{\pm}\nu X$	$p_T^{\mu} > 25$ GeV	[24]
	$\sqrt{s} = 8$ TeV	18.8	$pp \to W^{\pm}X \to \mu^{\pm}\nu X$	$p_T^{\mu} > 25$ GeV	[25]
DØ	$\sqrt{s} = 1.96$ TeV	7.3	$\bar{p}p \to W^{\pm}X \to \mu^{\pm}\nu X$	$p_T^{\mu} > 25$ GeV, $\not{E}_T > 25$ GeV	[23]
		9.7	$\bar{p}p \to W^{\pm}X \to e^{\pm}\nu X$	$p_T^{e} > 25$ GeV, $\not{E}_T > 25$ GeV	[22]
LHCb	$\sqrt{s} = 7$ TeV	1	$pp \to W^{\pm}X \to \mu^{\pm}\nu X$	$p_T^{\mu} > 20$ GeV	[19]
			$pp \to ZX \to \mu^+\mu^-X$	$p_T^{\mu} > 20$ GeV, $60 < m_{\mu\mu} < 120$ GeV	
	$\sqrt{s} = 8$ TeV	2	$pp \to ZX \to e^+e^-X$	$p_T^{e} > 20$ GeV, $60 < m_{ee} < 120$ GeV	[21]
		2.9	$pp \to W^{\pm}X \to \mu^{\pm}\nu X$	$p_T^{\mu} > 20$ GeV	[20]
			$pp \to ZX \to \mu^+\mu^-X$	$p_T^{\mu} > 20$ GeV, $60 < m_{\mu\mu} < 120$ GeV	
FNAL-605	$E_b = 800$ GeV		$pCu \to \mu^+\mu^-X$	$7 \leq M_{\mu\mu} \leq 18$ GeV	[68]
FNAL-866	$E_b = 800$ GeV		$pp \to \mu^+\mu^-X$	$4.6 \leq M_{\mu\mu} \leq 12.9$ GeV	[69]
			$pD \to \mu^+\mu^-X$		

TABLE III. The data on the $t\bar{t}$-production cross section from the LHC used in the present analysis. The errors given are combinations of the statistical and systematic ones. An additional error of 1.4, 3.3, 4.2 and 12 pb due to the beam energy uncertainty applies to all entries for the collision energy of $\sqrt{s} = 5$, 7, 8 and 13 TeV, respectively. The quoted values are rounded for the purpose of a compact presentation.

	Cross section (pb)						
$\sqrt{s}$ (TeV)	5	7		8		13	
Experiment	CMS	ATLAS	CMS	ATLAS	CMS	ATLAS	CMS
Decay mode: dilepton + b-jet(s)		183 ± 6 [36]		243 ± 8 [36]		818 ± 36 [37]	792 ± 43 [38]
dilepton + jets		181 ± 11 [33]	174 ± 6 [34]		245 ± 9 [34]		746 ± 86 [35]
lepton + jets			162 ± 14 [39]	260 ± 24 [40]	229 ± 15 [39]		836 ± 133 [41]
lepton + jets, $b \to \mu\nu X$		165 ± 38 [42]					
lepton + $\tau \to$ hadrons		183 ± 25 [43]	143 ± 26 [44]		257 ± 25 [51]		
jets + $\tau \to$ hadrons		194 ± 49 [46]	152 ± 34 [47]				
all-jets		168 ± 60 [48]	139 ± 28 [49]		276 ± 39 [45]		834^{+123}_{-109} [50]
eμ	82 ± 23 [52]						

are not included into our analysis in order to avoid a bias due to a mismatch between the PDFs used in the DØ analysis and ours; see also the discussion in Ref. [3].

When available [19–21,25,67], the absolute measurements of the lepton pseudorapidity distributions are used. In other cases [22–24], we employ the lepton charge asymmetries. However, as a cross-check we also compare our predictions with the LHCb [19,20] and CMS [25] data on the lepton charge asymmetry, although the absolute measurements are used in the fit, cf. Sec. III A. The recent ATLAS measurements of the $W^{\pm}$- and Z-boson cross sections in the fiducial volume at $\sqrt{s} = 13$ TeV [26] used in our analysis are separated for the electron- and muon-decay channels taking into account correlations between these measurements. This gives six data points in total for our $\sqrt{s} = 13$ TeV ATLAS data set.

The fixed-target Drell-Yan data provide information on the quark PDFs in the high-x region and allow to separate the sea and valence quark distributions. In the present analysis, two data sets of this kind are employed: the ratio of the proton-proton and proton-deuterium cross sections from the FNAL-866 experiment [69] and the proton-copper data from the FNAL-605 experiment [68]. Both sets have been used in our earlier fits, cf. Ref. [71].

4. Top-quark production

Measurements of top-quark production at the LHC and Tevatron provide a powerful tool for the study of the gluon distribution at large x and of α_s at large renormalization scales. However, due the strong sensitivity to the value of m_t, the accuracy achieved in such a study is essentially limited by the uncertainty in m_t. To take into account this interplay we fit the value of m_t simultaneously with the PDF parameters and α_s, cf. Sec. III. The t-quark data included into the present analysis comprise the $t\bar{t}$-production cross sections measured with various analysis techniques and for different decay modes at the center-of-mass energies $\sqrt{s} = 5$, 7, 8, and 13 TeV by ATLAS and CMS, cf. Table III, and those at $\sqrt{s} = 1.96$ TeV obtained at Tevatron [72]. In addition, single-top production data in the s- and t-channel from Tevatron and in the t-channel from the LHC are considered, cf. Table IV. Single-top production is mediated by the electroweak interactions at leading order and thus not particularly sensitive to α_s and the gluon distribution. Therefore, the latter input dampens the correlation between the gluon PDF, α_s and m_t, which emerges in the analysis of the $t\bar{t}$-data.

Due to specifics of the experimental analyses for the t-quark detection as well as necessary extrapolations in

TABLE IV. The data on single-top production in association with a light quark q or $\bar{b}$-quark from the LHC and Tevatron used in the present analysis. The errors given are combinations of the statistical, systematic, and luminosity ones.

Experiment	ATLAS			CMS			CDF&DØ
$\sqrt{s}$ (TeV)	7	8	13	7	8	13	1.96
Final states	tq	tq	tq	tq	tq	tq	$tq, t\bar{b}$
Reference	[27]	[28]	[29]	[30]	[31]	[32]	[53]
Luminosity (1/fb)	4.59	20.3	3.2	2.73	19.7	2.3	9.7×2
Cross section (pb)	68 ± 8	82.6 ± 12.1	247 ± 46	67.2 ± 6.1	83.6 ± 7.7	232 ± 30.9	$3.30^{+0.52}_{-0.40}$ (sum)

TABLE V. The values of χ^2 obtained in the present analysis for the data on inclusive DIS, the fixed-target DY process, and on heavy-quark production. The collider DY data are listed in Table VI.

Experiment	Process	Reference	NDP	χ^2
	DIS			
HERA I + II	$e^{\pm}p \to e^{\pm}X$	[4]	1168	1510
	$e^{\pm}p \to \overset{(-)}{\nu}X$			
BCDMS	$\mu^+p \to \mu^+X$	[61]	351	411
NMC	$\mu^+p \to \mu^+X$	[60]	245	343
SLAC-49a	$e^-p \to e^-X$	[54,62]	38	59
SLAC-49b	$e^-p \to e^-X$	[54,62]	154	171
SLAC-87	$e^-p \to e^-X$	[54,62]	109	103
SLAC-89b	$e^-p \to e^-X$	[56,62]	90	79
	DIS heavy-quark production			
HERA I + II	$e^{\pm}p \to e^{\pm}cX$	[63]	52	62
H1	$e^{\pm}p \to e^{\pm}bX$	[15]	12	5
ZEUS	$e^{\pm}p \to e^{\pm}bX$	[16]	17	16
CCFR	$\overset{(-)}{\nu}N \to \mu^{\pm}cX$	[64]	89	62
CHORUS	$\nu N \to \mu^+cX$	[18]	6	7.6
NOMAD	$\nu N \to \mu^+cX$	[17]	48	59
NuTeV	$\overset{(-)}{\nu}N \to \mu^{\pm}cX$	[64]	89	49
	DY			
FNAL-605	$pCu \to \mu^+\mu^-X$	[68]	119	165
FNAL-866	$pp \to \mu^+\mu^-X$	[69]	39	53
	$pD \to \mu^+\mu^-X$			
	Top-quark production			
ATLAS, CMS	$pp \to tqX$	[27–32]	10	2.3
CDF&DØ	$\bar{p}p \to tbX$	[53]	2	1.1
	$\bar{p}p \to tqX$			
ATLAS, CMS	$pp \to t\bar{t}X$	[33–52]	23	13
CDF&DØ	$\bar{p}p \to t\bar{t}X$	[53]	1	0.2

a relative deviation of the experimental data off the resulting fit predictions do not demonstrate any statistically significant trend and no additional improvements can evidently be achieved by further releasing the PDF shape, cf. Sec. III B. A detailed breakdown of the values of χ^2 for the separate processes and data sets is given in Tables V and VI and discussed in the following.

1. DIS data

The data sets newly included into the present analysis are smoothly accommodated in general, while keeping the quality of the fixed-target BCDMS, NMC and SLAC data included in the earlier ABM12 fit. In particular, this applies to the HERA inclusive DIS data obtained from the combination of the statistics of run I and II [4]. No trend can be observed in the pulls of this sample plotted in Figs. 4–6, although the fluctuations in the central values of the data extend somewhat beyond the published uncertainties. As a result, these fluctuations prevent a statistically ideal description of the inclusive HERA data yielding values of χ^2/NDP slightly bigger than one. However, the fit cannot be improved in any essential way by further relaxing the fitted PDF shape.

We have also checked the combined HERA inclusive data with varying cuts on Q^2. Due to bigger errors in the data at large Q^2 the value of χ^2/NDP is smaller for the variants of the fit with more stringent cuts on Q^2. We find $1350/1092 = 1.24$ and $1225/1007 = 1.22$ for the cuts of $Q^2 > 5$ GeV^2 and $Q^2 > 10$ GeV^2, respectively. The same conclusion was drawn in a previous QCD analysis [4], however, the values of χ^2 reported in Ref. [4] are somewhat smaller than ours due to the limited number of data sets employed in that analysis.

2. Drell-Yan data

Due to large amount of the DY data from Tevatron and the LHC a precision determination of the light-quark PDFs in a wider kinematic range in x than ever before becomes possible; see also [3]. The quality of the ABMP16 fit for the Drell-Yan data description is summarized in Table VI. Data sets of lower accuracy, which have become obsolete and data sets superseded are not listed there. Instead, we refer to the review [11] for further comparisons concerning the status of Drell-Yan data in PDF fits.

In general, the data sets in Table VI with a total $NDP = 172$ can be smoothly accommodated, although the values of χ^2/NDP obtained for individual data sets are bigger than one in some places. This is the case, for instance, for the CMS data on the muon charge asymmetry collected at the collision energy of $\sqrt{s} = 7$ TeV shown in Fig. 7, which yields a value of $\chi^2/NDP \sim 2$. Similar CMS data for $\sqrt{s} = 8$ TeV, however, are much smoother, cf. the pulls in Figs. 7 and 8, and a good value of χ^2/NDP is achieved in this case. Therefore, the observed fluctuations in the CMS data for $\sqrt{s} = 7$ TeV should rather be attributed to experimental systematic effects than to any shortcomings in the fitted PDFs.

The pulls for the LHCb data on the muon charge asymmetry collected at $\sqrt{s} = 7$ TeV [19] are shown in Fig. 9. They display an irregularity at pseudo-rapidity $\eta_\mu = 3.275$, which is not confirmed by the LHCb data at $\sqrt{s} = 8$ TeV [20]. Moreover, this spike at $\eta_\mu = 3.275$ coincides with fluctuations in the correction for final-state radiation which has been applied to the LHCb data, cf. Fig. 5 in Ref. [3]. The two data points for W^+- and W^--boson production corresponding to this spike contribute about 13 units to the value of χ^2 and, in line with our earlier analysis [3], we discard these two data points from the fitted set. This has only marginal impact on the fit results.

The pulls for the LHCb data on the W-production at $\sqrt{s} = 8$ TeV [20] are displayed in Fig. 10. They exhibit an excess at $\eta_\mu = 2.125$ both in μ^+ and μ^- channels, while the muon charge asymmetry remains smooth. Since these two

TABLE VI. Compilation of precise data on W- and Z-boson production in pp and $p\bar{p}$ collisions and the χ^2 values obtained for these data sets in different PDF analyses using their individual definitions of χ^2. The NNLO fit results are quoted as a default, while the NLO values are given for the CJ15 [6] and `HERAFitter` [201] PDFs. Missing table entries indicate that the respective data sets have not been used in the analysis.

Experiment	ATLAS		CMS		DØ		LHCb		
$\sqrt{s}$ (TeV)	7	13	7	8	1.96		7	8	
Final states	$W^+ \to l^+\nu$ $W^- \to l^-\nu$ $Z \to l^+l^-$	$W^+ \to l^+\nu$ $W^- \to l^-\nu$ $Z \to l^+l^-$	$W^+ \to \mu^+\nu$ $W^- \to \mu^-\nu$ (asym)	$W^+ \to \mu^+\nu$ $W^- \to \mu^-\nu$	$W^+ \to \mu^+\nu$ $W^- \to \mu^-\nu$ (asym)	$W^+ \to e^+\nu$ $W^- \to e^-\nu$ (asym)	$W^+ \to \mu^+\nu$ $W^- \to \mu^-\nu$ $Z \to \mu^+\mu^-$	$Z \to e^+e^-$	$W^+ \to \mu^+\nu$ $W^- \to \mu^-\nu$ $Z \to \mu^+\mu^-$
Cut on the lepton P_T	$P_T^l > 20$ GeV	$P_T^e > 25$ GeV	$P_T^\mu > 25$ GeV	$P_T^\mu > 25$ GeV	$P_T^\mu > 25$ GeV	$P_T^e > 25$ GeV	$P_T^\mu > 20$ GeV	$P_T^e > 20$ GeV	$P_T^\mu > 20$ GeV
Luminosity (1/fb)	0.035	0.081	4.7	18.8	7.3	9.7	1	2	2.9
Reference	[67]	[26]	[24]	[25]	[23]	[22]	[19]	[21]	[20]
NDP	30	6	11	22	10	13	31	17	32
χ^2 Present analysis[a]	31.0	9.2	22.4	16.5	17.6	19.0	45.1	21.7	40.0
CJ15 [6]	...	...	...	...	20	29	...	...	...
CT14 [7]	42	...	...[b]	...	...	34.7	...	...	...
JR14 [8]	...	...	...	...	...	...	...	...	...
`HERAFitter` [201]	...	...	...	...	13	19	...	...	...
MMHT14 [9]	39	...	...	...	21	...	...	...	...
NNPDF3.0 [10]	35.4	...	18.9	...	...	...	...	...	...

[a]The ABM12 [1] analysis has used older data sets from CMS and LHCb.
[b]For the statistically less significant data with the cut of $P_T^\mu > 35$ GeV the value of $\chi^2 = 12.1$ was obtained.

TABLE XV. Table XIII continued.

	b_{ss}	A_{ss}	a_g	b_g	$\gamma_{1,g}$	$\alpha_s^{(n_f=3)}(\mu_0)$	$m_c(m_c)$	$m_b(m_b)$	$m_t(m_t)$
a_u	−0.1186	−0.1013	0.0046	0.2662	0.2008	0.1083	−0.0006	0.0661	−0.1339
b_u	−0.0480	−0.0411	−0.0374	0.3141	0.2274	−0.0607	0.0170	0.0554	−0.2170
$\gamma_{1,u}$	−0.1532	−0.1458	0.1109	0.1579	0.0706	0.0848	−0.0104	0.0605	−0.0816
$\gamma_{2,u}$	0.1549	0.1802	−0.1934	−0.0050	0.0876	−0.0250	0.0206	−0.0367	0.0081
$\gamma_{3,u}$	−0.1536	−0.1625	0.1653	−0.0207	−0.0835	0.0765	−0.0201	0.0287	0.0250
a_d	0.0486	0.1216	−0.0288	0.0973	0.0919	0.0763	−0.0123	−0.0116	−0.0616
b_d	0.1508	0.1678	−0.0122	0.0870	0.0574	−0.0306	−0.0161	0.0029	−0.0813
$\gamma_{1,d}$	0.0267	0.0924	0.0053	0.0646	0.0493	0.0725	−0.0114	−0.0074	−0.0491
$\gamma_{2,d}$	−0.1161	−0.1196	0.0059	−0.0666	−0.0364	0.0243	0.0108	−0.0051	0.0736
$\gamma_{3,d}$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
a_{us}	0.2197	0.3627	−0.2570	−0.1419	−0.0241	0.0954	0.0704	−0.0183	0.0641
b_{us}	0.0643	0.0261	0.0001	0.1266	0.0332	−0.2866	−0.0093	−0.0132	−0.1841
$\gamma_{-1,us}$	−0.4479	−0.6319	0.2197	0.0694	−0.0226	−0.0341	−0.0034	0.0044	−0.0408
$\gamma_{1,us}$	0.1286	0.0102	0.0039	0.2648	0.1296	−0.3493	−0.0462	0.0209	−0.2635
A_{us}	0.1193	0.2412	−0.2493	−0.1715	−0.0489	0.1110	0.1182	−0.0298	0.0755
a_{ds}	−0.1579	−0.2688	−0.2190	−0.0515	−0.0137	−0.0604	0.0849	−0.0006	−0.0573
b_{bs}	−0.0260	−0.0180	−0.0454	0.0917	0.0503	−0.1265	0.0547	0.0332	−0.1067
$\gamma_{1,ds}$	0.0169	−0.0960	−0.1031	0.2130	0.1409	−0.1811	0.0413	0.0695	−0.2003
A_{ds}	−0.0896	−0.1797	−0.2571	−0.0469	0.0022	−0.1330	0.1193	−0.0432	−0.0869
a_{ss}	0.6522	0.9280	0.0626	−0.0092	−0.0279	−0.0841	−0.0728	−0.0159	0.0169
b_{ss}	1.0	0.6427	−0.0179	0.1967	0.1164	−0.2390	−0.0965	0.0169	−0.1675
A_{ss}	0.6427	1.0	−0.0211	0.1403	0.0997	−0.1385	0.0216	0.0072	−0.1109
a_g	−0.0179	−0.0211	1.0	−0.5279	−0.8046	0.1838	−0.2829	0.0076	0.3310
b_g	0.1967	0.1403	−0.5279	1.0	0.8837	−0.5124	0.1438	0.1255	−0.7275
$\gamma_{1,g}$	0.1164	0.0997	−0.8046	0.8837	1.0	−0.2511	0.1829	0.0814	−0.5180
$\alpha_s^{(n_f=3)}(\mu_0)$	−0.2390	−0.1385	0.1838	−0.5124	−0.2511	1.0	−0.1048	0.0423	0.6924
$m_c(m_c)$	−0.0965	0.0216	−0.2829	0.1438	0.1829	−0.1048	1.0	0.0328	−0.1577
$m_b(m_b)$	0.0169	0.0072	0.0076	0.1255	0.0814	0.0423	0.0328	1.0	−0.0900
$m_t(m_t)$	−0.1675	−0.1109	0.3310	−0.7275	−0.5180	0.6924	−0.1577	−0.0900	1.0

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