Physics 302
First Midterm Exam

18 October 2010, 10:05-11:20 AM

The Exam is out of 40 points. Each problem is worth 10 points. Show all your work for partial credit.

1. A simple pendulum consists of a massless rigid rod of length l with a bob of mass m at one end. The other end of the rod is attached to a pivot which rises vertically with constant acceleration a. Acceleration due to gravity is g vertically downwards. Assume the pendulum moves in the vertical plane only. Identify the generalized coordinate, write the Lagrangian and find the equation of motion. What is the frequency of small oscillations? Compare your result for the frequency with the result you would get if the pivot were fixed, and explain the result in a few sentences.

2. Suppose it is known experimentally that a particle dropped from rest fell a given distance y_0 in a time $t_0 = \sqrt{2y_0/g}$ in a uniform gravitational field g. The times of fall for other distances are not known, but it is guessed that the functional form is

$$y = at + bt^2$$

Write the Lagrangian for the particle in the vertical dimension. If the constants a and b are adjusted so that the time to fall a distance y_0 is always given by t_0, show directly that the integral

$$\int_0^{t_0} L\, dt$$

is an extremum for real values of the coefficients only when $a = 0$ and $b = g/2$.

3. Show by the use of Poisson brackets that for a one-dimensional harmonic oscillator there is a constant of the motion u defined by

$$u(q, p, t) = \ln(p + im\omega q) - i\omega t$$

where $\omega = \sqrt{k/m}$. What is the physical significance of this constant of the motion?

4. Solve the problem of the motion of a point projectile in a vertical plane with a constant gravitational field g, using the Hamilton-Jacobi method. Find both the equation of the trajectory and the dependence of the coordinates on time, assuming the projectile is fired off at time $t = 0$ from the origin with the velocity v_0, making an angle α with the horizontal.