HW5

1. (a) $H(q,p) = \frac{1}{2}p^2 + \frac{1}{2}w^2 q^2$

$\dot{q} = \frac{\partial H}{\partial p} = p \; ; \quad \dot{p} = -\frac{\partial H}{\partial q} = -w^2 q$

$\ddot{q} = \dot{p} = -w^2 q \; ; \quad p = \dot{q}$. solve with $p(0) = p_0$, $q(0) = q_0$

$p = p_0 \cos(wt) - w q_0 \sin(wt)$

$q = q_0 \cos(wt) + \frac{p_0}{w} \sin(wt)$

(b) $\{q, p\}_{(q_0, p_0)} = \cos^2 wt + \sin^2 wt = 1$.

(c). $\{q, p\}_{(q_0, p_0)} = 1$. the transformation is canonical.

2. (a) $\frac{\partial S}{\partial t} = -\frac{1}{2} m w^2 \csc^2(wt)(q^2 + p^2) + m w^2 q p \frac{\cos(wt)}{\sin^2(wt)}$

$p = \frac{\partial S}{\partial q} = m w q \cot(wt) - m w p \csc(wt)$

$H(q,p,t) + \frac{\partial S}{\partial t} = 0$; $\quad S(q,p,t)$ is a solution of H-J Eqn

(b) $p = \frac{\partial S}{\partial q} = m w q \cot(wt) - m w p \csc(wt)$

$Q = \frac{\partial S}{\partial p} \Rightarrow q = p\cos(wt) - \frac{Q \sin wt}{m w}$

$\dot{p} = -m w p \sin wt - Q \cos wt = -m w^2 q = -\frac{\partial H}{\partial q}$

$\dot{q} = -w p \sin wt - -\frac{Q}{m} \cos wt = \frac{p}{m} = \frac{\partial H}{\partial p}$

3. H-J Eqn. $\quad \frac{1}{2m}\left(\frac{\partial S}{\partial x}\right)^2 - mAtx + \frac{\partial S}{\partial t} = 0$.

use trial solution. $S(x,t) = f(t)x + g(t)$. we have:

$\frac{1}{2m} f^2(t) - mAtx + f'(t)x + g'(t) = 0$

thus $\begin{cases} f'(t) = mAt \\ \frac{1}{2m} f^2(t) = -g'(t) \end{cases}$ Solve for $g(t)$. $f(t)$ $\quad f = \frac{1}{2} m A t^2 + f_0$

$S(x,t,f_0) = \frac{1}{2} m A t^2 x + f_0 x - \frac{mA^2}{40} t^5 - \frac{Af_0}{6} t^3 - \frac{f_0^2}{2m} t$

$x(t) = V_0 + \frac{A}{6} t^3$, $\quad x(0) = 0$, $\dot{x}(0) = V_0$

$P(t) = \frac{1}{2} m A t^2 / \ldots + m V_0 \ldots$

4. H J . Eqn :

$$\frac{1}{2} f(t) \left[\frac{1}{m} \left(\frac{\partial S}{\partial q}\right)^2 + kq^2 \right] + \frac{\partial S}{\partial t} = 0.$$

choose trial solution $S = W(q) + \tau(t)$.

$$\frac{f(t)}{2} \left[\frac{1}{m} \left(\frac{\partial W}{\partial q}\right)^2 + kq^2 \right] + \frac{\partial \tau}{\partial t} = 0.$$

$$\left(\frac{\partial W}{\partial q}\right)^2 + mkq^2 = -\frac{2m}{f(t)} \tau'(t) = \text{const.} = P$$

P is a constant, momentum. we have

$$\tau(t) = -\frac{P}{2m} \int f(t)\, dt$$

$$W(q) = \int \sqrt{P - mkq^2}\, dq$$

$$S(q,t) = \int \sqrt{P - mkq^2}\, dq - \int \frac{P}{2m} f(t)\, dt.$$

$$Q = \frac{\partial S}{\partial P} = \frac{1}{2\sqrt{mk}} \left[\sin^{-1}\left(q\sqrt{\frac{mk}{P}} \right) - \sqrt{\frac{k}{m}} \int f(t)\, dt \right]$$

$$q = \sqrt{\frac{P}{mk}} \sin\left(2Q\sqrt{mk} + \sqrt{\frac{k}{m}} \int f(t)\, dt \right)$$

$$p = \frac{\partial S}{\partial q} = \sqrt{P} \cos\left(2Q\sqrt{mk} + \sqrt{\frac{k}{m}} \int f(t)\, dt \right)$$

we have phase space ~~trajectory~~

$$\frac{p^2}{P} + \frac{q^2}{P/mk} = 1.$$

$$T = \frac{p^2}{2m} = \frac{P}{2m} \cos^2\left(2Q\sqrt{mk} + \sqrt{\frac{k}{m}} \int f(t)\, dt \right)$$

Three different cases.

(a) $f(t) = e^{\alpha t}$, $T(t) \propto \cos^2\left(2Q\sqrt{mk} + \frac{1}{\alpha}\sqrt{\frac{k}{m}}\, e^{\alpha t} \right)$

T oscillates with increasing frequency.

(b) $f(t) = e^{-\alpha t}$, $T(t) \propto \cos^2\left(2Q\sqrt{mk} - \frac{1}{\alpha}\sqrt{\frac{k}{m}}\, e^{-\alpha t} \right)$

T tends to a ~~constant~~ value.

(c) $f(t) = \cos(\Omega t)$, $T(t) \propto \cos^2\left(2Q\sqrt{mk} + \frac{1}{\Omega}\sqrt{\frac{k}{m}} \sin(\Omega t) \right)$

T oscillates w/ periodically varying frequency

5. method of action - angle variables applies when $E<0$.

(bound state)

$$H = \frac{p_r^2}{2m} + \frac{p_\theta^2}{2mr^2} - V_0$$

$$r_{min} = \frac{p_\theta}{\sqrt{2m(V_0 - |E|)}}$$

$$J_r = \int_{r_{min}}^{r_{max}} p_r \, dr = \int_{r_{min}}^{r_{max}} \sqrt{2m(V_0 - E) - \frac{p_\theta^2}{r^2}} \, dr$$

$$\frac{\partial J_r}{\partial E} = \left[\frac{m}{2(V_0 - E)}\right] \left(r_0^2 - \frac{p_\theta^2}{2m(V_0 - E)} \right)^{1/2}$$

$$\nu = \frac{\partial E}{\partial J_r} = \left[\frac{m}{2(V - E)} \left(r_0^2 - \frac{p_\theta^2}{2m(V_0 - E)} \right) \right]^{-1/2}$$

6. $$\frac{p^2}{2m} - \frac{k}{|x|} = E \qquad x_{max} = \frac{k}{E}$$

$$J = \int_{-x_0}^{x_0} p\,dx = \int_0^{-\frac{k}{E}} \sqrt{2m\left(E + \frac{k}{|x|}\right)} \, dx = 2\sqrt{2}\pi\sqrt{m}\frac{k}{\sqrt{|E|}}$$

$$\frac{\partial J}{\partial E} = \pi\sqrt{2m}\, k\, |E|^{-3/2}$$

HW6.

1. original coordinate:

$$L = \frac{m}{2}\left(\dot{q}^2 \sin^2 wt + \dot{q} q w \sin 2wt + q^2 w^2 \right)$$

$$p = \frac{\partial L}{\partial \dot{q}} = m (\dot{q} \sin wt + q w \cos wt) \sin wt$$

$$\dot{q} = \frac{p}{m} \csc^2 wt - q w \cot wt$$

$$H = p\dot{q} - L = \frac{p^2}{2m} \csc^2 wt - p q w \cot wt - \frac{1}{2} m w^2 q^2 \sin^2 wt.$$

the Hamiltonian is not conserved.

in new coordinates $Q = q \sin wt$

$$L = \frac{m}{2}\left[\left(\frac{d}{dt} q \sin wt\right)^2 + (q w \sin wt)^2 \right] = \frac{m}{2}\left[\dot{Q}^2 + w^2 Q^2 \right]$$

$$p = m\dot{Q} \;;\quad \dot{Q} = \frac{p}{m} \;;\quad L = \frac{p^2}{2m} + \frac{1}{2} m w^2 Q^2$$

$$H = \frac{p^2}{2m} - \frac{1}{2} m w^2 Q^2$$

the Hamiltonian is conserved

2. in the elevator frame $H = \frac{p^2}{2m} + m(g+a)x = E$

where $E = m(g+a) h_{max}$

$$J = \oint p\,dx = 2\int_0^{h_{max}} \sqrt{2m[E - m(g+a)x]}\,dx$$

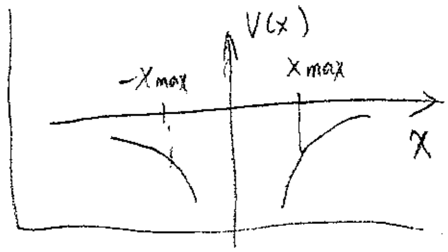
$$= \frac{4}{3} m \sqrt{2(g+a)}\; h_{max}^{3/2}$$

$$h_{max}(t) = h_{max}(0) \cdot \left[\frac{g + a(0)}{g + a(t)} \right]^{1/3}.$$

J is constant

3. (a) $$H = \frac{p^2}{2m} - \frac{k}{|x|} = E < 0.$$

$$\frac{p^2}{2m} \geq 0 \Rightarrow |x| \leq \frac{k}{|E|}$$, bounded



(b) $$H = \frac{1}{2m}\left(\frac{\partial S}{\partial x}\right)^2 - \frac{k}{|x|} + \frac{\partial S}{\partial t} = 0$$

$$J = \oint p\,dq = 4k\sqrt{\frac{2m}{E}} \int_0^{\frac{k}{|E|}} \left[\frac{k}{x} - E\right]^{1/2} dx$$

$$= 2\pi k \sqrt{\frac{2m}{|E|}}$$

$$\nu = \frac{\partial H}{\partial J} = \frac{16 m \pi^2 k^2}{J^3} = \frac{|E|^{3/2}}{\pi k \sqrt{2m}}.$$

(c). $$E = -\frac{8\pi^2 m k^2}{J^2};$$ J is invariant; k increase

$$T = \left(\frac{\partial E}{\partial J}\right)^{-1} = \frac{J^3}{16\pi^2 m k^2}$$

$$|x|_{max} = \frac{k}{|E|} = \frac{J^2}{8\pi^2 m k}$$

E, T, $|x|_{max}$ all of them decrease

4. (a) $L = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) + \frac{k}{r}e^{-r/a}$

eqn. of motion:

$$\frac{d}{dt}(mr^2\dot{\theta}) = 0 \quad , \quad \dot{\theta} = \frac{l}{mr^2}$$

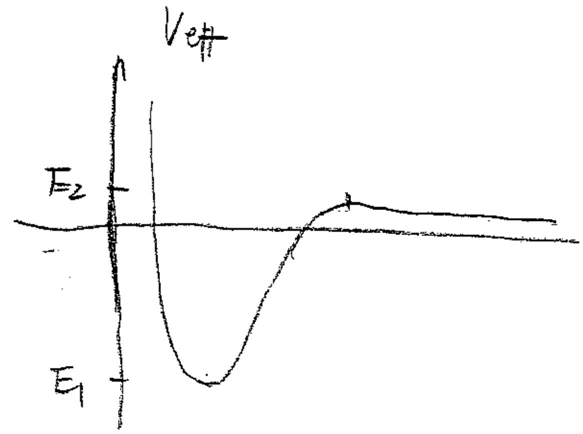
$$m\ddot{r} = mr\dot{\theta}^2 - \frac{k}{r}\left(\frac{1}{a}+\frac{1}{r}\right)e^{-r/a}$$

$$= \frac{l^2}{mr^3} - \frac{k}{r}\left(\frac{1}{a}+\frac{1}{r}\right)e^{-r/a} = f_{eff}(r).$$

effective potential.

$$V_{eff}(r) = -\int f_{eff}(r)\,dr = \frac{l^2}{2mr^2} - \frac{k}{r}e^{-r/a}.$$

E	orbit.
$E = E_1$.	circle
$E_1 < E < 0$	bound.
$0 < E < E_2$	bound or open.
$E > E_2$	open.



(b) $f(r) = -\frac{k}{r}\left(\frac{1}{a}+\frac{1}{r}\right)e^{-r/a}$.

$$\frac{df}{dr} = k\left(\frac{1}{a^2r} + \frac{2}{ar^2} + \frac{3}{r^3}\right)e^{-r/a}$$

$$\frac{r\,df}{f\,dr} = -2 - \frac{r^2}{a(a+r)}$$

$$\beta^2 = 3 + \frac{r}{f}\frac{df}{dr}\Big|_{r=\rho} = 1 - \frac{\rho^2}{a(a+\rho)} \approx 1 - \frac{\rho}{a}$$

$$\beta = 1 - \frac{\rho}{2a}$$

$\beta\theta = 2\pi\left(1 - \frac{\rho}{2a}\right)$. so apsides changes by $\frac{\pi\rho}{a}$.

5. $\frac{d}{dt}(\vec{r}\times\vec{p})$

$$= \vec{r}\times\dot{\vec{p}} + \dot{\vec{r}}\times\vec{p} = 0 + \vec{r}\times\dot{\vec{p}}$$

$$= r\hat{r}\times\dot{\vec{p}} = \frac{qb}{cr}\left[\hat{r}\times(\vec{v}\times\vec{r})\right] = \frac{qb}{c}\dot{\hat{r}}$$

$$\frac{dL}{dt} \neq 0$$

$$\frac{d}{dt}\left(L - \frac{qb}{c}\hat{r}\right) = \frac{d}{dt}D = 0$$

HW7.

1. (a)

$$E = \frac{M}{r}\left(\frac{dr}{dt}\right)^2 + \frac{l^2}{2mr^2} - \frac{\alpha}{r}.$$

$$t = \sqrt{\frac{m}{2}} \int_{r_0}^{r} \frac{1}{\sqrt{\frac{\alpha}{r} - \frac{l^2}{2mr^2} + E}}\, dr.$$

elliptical. motion $\quad a = \frac{r_1 + r_2}{2}$

$$e^2 = m\alpha a(1-e^2)$$

$$t = \sqrt{\frac{M}{2\alpha}} \int_{r_0}^{r} \frac{r\,dr}{\sqrt{r - \frac{r^2}{2a} - \frac{a(1-e^2)}{2}}}, \quad \text{where } r = a(1-e\cos\varphi)$$

$$= \sqrt{\frac{ma^3}{2}} \int_0^{2\pi} (1 - e\cos\varphi)\,d\varphi$$

$$= 2\pi\sqrt{\frac{ma^3}{2}} = 2\pi\sqrt{\frac{a^3}{G(m_1+m_2)}}$$

$$T^2 = \frac{4\pi^2 a^3}{GM}, \quad M = m_1 + m_2$$

(b), $\quad e = \sqrt{1 + \frac{2El^2}{M\alpha^2}} \quad \therefore \quad b = a\sqrt{1-e^2}.$

2. (a) $F = \frac{GMM_E}{r_0^2} + mcr_0 = m\omega_0^2 r.$

$$\omega_0 = \sqrt{\frac{GM}{r_0^3} + c}$$

$$T_0 = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{GM}{r_0^3}+c}}.$$

(b). $\beta^2 = 3 + \frac{r}{f}\frac{df}{dr}\Big|_{r=r_0}.$

$$= 3 - \frac{2-\alpha}{1+\alpha}, \quad \text{where} \quad \alpha = \frac{mr_0^3 c}{k}$$

$$\beta = \sqrt{\frac{\frac{k}{mr_0^3} + 4c}{\frac{k}{mr_0^3} + c}}$$

$$F = cm\vec{r}$$

$$T_r = \frac{T_0}{\beta} \quad \beta = \frac{\omega_r}{\omega_0}$$

(c). $\omega_p = |\omega - \omega_r| = \sqrt{\frac{GM}{r_0^3}}\left|\sqrt{1+\frac{cr_0^3}{GM}} - \sqrt{1+\frac{4cr_0^3}{GM}}\right|.$

small c.

$$\omega_p \approx \sqrt{\frac{GM}{r_0^3}}\left[\left(1+\frac{cr_0^3}{2GM}\right) - \left(1+\frac{2cr_0^3}{GM}\right)\right]$$

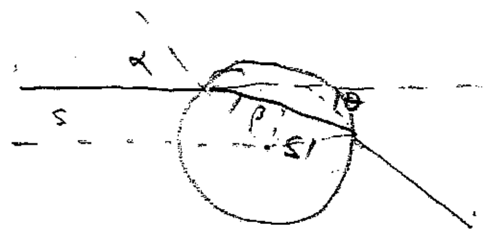
$$= \frac{3}{2}\sqrt{\frac{GM}{r_0^3}}\,\frac{cr_0^3}{GM}$$

3. (a). Conservation of angular momentum

$L = mv_0 s = mv' s'$

$T = \frac{1}{2} m v^2 = E - V$.

$\sqrt{2mE}\, s = \sqrt{2m(E+V_0)}\, s'$



$\sin\alpha = \frac{s}{a}, \quad \sin\beta = \frac{s'}{a}.$

$\frac{\sin\alpha}{\sin\beta} = \frac{s}{s'} = \sqrt{\frac{E+V_0}{E}} = n$; identical to light ray refraction

(b). $\theta = 2(\alpha - \beta)$.

$\cos\frac{\theta}{2} = \sqrt{1-\frac{s^2}{a^2}}\sqrt{1-\frac{s^2}{n^2a^2}} + \frac{s}{a}\cdot\frac{s}{na}$

$= \frac{1}{na^2}\left(\sqrt{a^2-s^2}\sqrt{n^2a^2-s^2}+s^2\right).$

So $\left(na^2\cos\frac{\theta}{2}-s^2\right)^2 = (a^2-s^2)(n^2a^2-s^2)$

$s = \frac{na\sin\frac{\theta}{2}}{\sqrt{1+n^2-2n\cos\frac{\theta}{2}}}$.

$\delta(\theta) = \frac{s}{\sin\theta}\frac{ds}{d\theta} = \frac{n^2a^2\left(n\cos\frac{\theta}{2}-1\right)\left(n-\cos\frac{\theta}{2}\right)}{4\cos\frac{\theta}{2}\left(1+n^2-2n\cos\frac{\theta}{2}\right)^2}$

4 mathematica.

(c) using mathematica

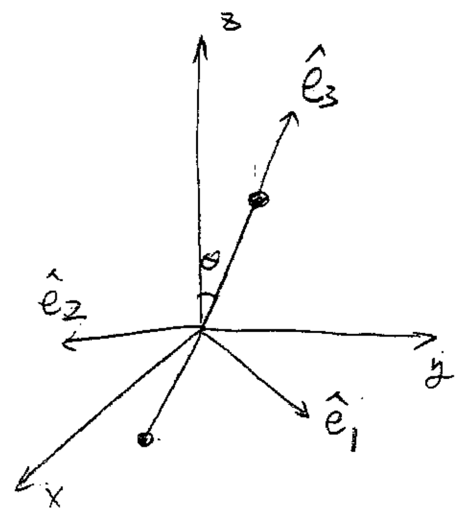
$\sigma = 2\pi\int_0^{\theta_{max}} \sin\theta\,(\delta(\theta)\,d\theta). = \pi a^2$

$\theta_{max} = 2\arcsin\left(\sqrt{\frac{n^2-1}{n^2}}\right)$

HW8

1. (a) $I_1 = 0.$

$I_2 = I_3 = I = 2\times m\left(\frac{l}{2}\right)^2 = \frac{1}{2}ml^2.$



in the pricipal axes

$\omega_1 = \omega\cos\theta;\ \omega_2 = \omega\sin\theta;\ \omega_3 = 0$

$\dot{\omega}_1 = \dot{\omega}_2 = \dot{\omega}_3 = 0.$

$N_1 = I_1\dot{\omega}_1 - \omega_2\omega_3(I_2 - I_3) = 0$

$N_2 = I_2\dot{N}_2 - N_3\omega_1(I_3 - I_1) = 0$

$N_3 = I_3\dot{\omega}_3 - \omega_1\omega_2(I_1 - I_2)$

$= \frac{1}{4}ml^2\omega^2\sin 2\theta.$

(b) $\vec{L} = \sum_i (\vec{r}_i \times \vec{v}_i) = -\frac{1}{2}ml^2\omega\sin\theta\,\hat{e}$

$\frac{d\hat{e}}{dt} = \vec{\omega}\times\hat{e} = -\omega\cos\theta\,(-\sin\phi,\ \cos\phi,\ 0)$

$\vec{N} = \frac{d\vec{L}}{dt} = -\frac{m\omega^2l^2}{2}\sin\theta\cos\theta\,(-\sin\phi,\ \cos\phi,\ 0)$

same magnitude. rotating.

2. (a) symmetrical top $\qquad I_1 = I_2 \neq I_3$

$\omega_3 =$ constant.

$$\dot{\omega}_1 = -\left(\frac{I_3 - I}{I}\omega_3\right)\omega_2$$

$$\dot{\omega}_2 = -\left(\frac{I_3 - I}{I}\omega_3\right)\omega_1,$$

$$\Omega = \frac{I_3 - I}{I}\omega_3, \quad \text{solve.}$$

$$(\dot{\omega}_1 + i\dot{\omega}_2) - i\Omega(\omega_1 + i\omega_2) = 0.$$

one gets.

$$\omega_1(t) = A\cos\Omega t.$$

$$\omega_2(t) = A\sin\Omega t.$$

the angular velocity vector processes around the body x_3 axis with ~~constant~~ angular velocity

$$\Omega = \frac{I_3 - I}{I}\omega_3$$

(b) $$L = \frac{I_1}{2}(\dot{\theta}^2 + \dot{\phi}^2\sin^2\theta) + \frac{I_3}{2}(\dot{\psi} + \dot{\phi}\cos\theta)^2.$$

equation of motion for θ:

$$\frac{d}{dt}(I_1\dot{\theta}) = I_1\dot{\phi}^2\sin\theta\cos\theta - I_3(\dot{\psi} + \dot{\phi}\cos\theta)\dot{\phi}\sin\theta.$$

and $\quad \dot{\psi} + \dot{\phi}\cos\theta = \omega_3$

we have

$$I_1\dot{\phi}^2\sin\theta\cos\theta + I_3\omega_3(-\dot{\phi}\sin\theta) = 0$$

$$\dot{\phi} = \frac{I_3\omega_3}{I_1\cos\theta}$$

3. (a) $T = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} \cdot 4m \cdot \dot{x}_2^2 + \frac{1}{2} m \dot{x}_3^2$.

$= \frac{1}{2} m \dot{x}_1^2 + 2m \dot{x}_2^2 + \frac{1}{2} m \dot{x}_3^2$

$V = \frac{1}{2} k (x_2 - x_1)^2 + \frac{1}{2}(4k) x_2^2 + \frac{1}{2} k (x_3 - x_2)^2$.

$= \frac{1}{2} k (x_1^2 + 6x_2^2 + x_3^2 - 2x_1 x_2 - 2x_2 x_3)$

(b). $L = T - V = \frac{1}{2} m (\dot{x}_1^2 + 4\dot{x}_2^2 + \dot{x}_3^2) - \frac{1}{2} k (x_1^2 + 6x_2^2 + x_3^2 - 2x_1 x_2 - 2x_2 x_3)$.

$H = T + V = \frac{1}{2} m (\dot{x}_1^2 + 4\dot{x}_2^2 + \dot{x}_3^2) + \frac{1}{2} k (x_1^2 + 6x_2^2 + x_3^2 - 2x_1 x_2 - 2x_2 x_3)$.

(c). $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_i}\right) - \frac{\partial L}{\partial x_i} = 0$.

$$\begin{cases} m\ddot{x}_1 + k(x_1 - x_2) = 0. \\ 4m\ddot{x}_2 + k(6x_2 - x_1 - x_3) = 0. \\ m\ddot{x}_3 + k(x_3 - x_2) = 0. \end{cases}$$

(d). $$\begin{vmatrix} k - \omega^2 m & -k & 0 \\ -k & 6k - \omega^2(4m) & -k \\ 0 & -k & k - \omega^2 m \end{vmatrix} = 0.$$

$\Rightarrow \omega_1 = \sqrt{\frac{k}{2m}}$, $\omega_2 = \sqrt{\frac{k}{m}}$, $\omega_3 = \sqrt{\frac{2k}{m}}$.

ω_1: $a_1 = 2a_2$, $a_3 = 2a_2$

ω_2: $a_2 = 0$, $a_1 = -a_3$.

ω_3: $a_1 = a_3 = -a_2$.

m_1 m_2 m_3.

4. (a) $\frac{dE}{dt} = (mf_0 \cos\Omega t)\, \dot{q}$

$q = A\cos\omega t + B\sin\omega t + \frac{f_0}{\omega_0^2 - \Omega^2}\cos\Omega t$

$\dot{q} = -A\omega\sin\omega t + B\omega\cos\omega t - \frac{f_0\Omega}{\omega_0^2 - \Omega^2}\sin\Omega t$

$\frac{dE}{dt} = (mf_0\cos\Omega t)\left(-A\omega\sin\omega t + B\omega\cos\omega t - \frac{f_0\Omega}{\omega_0^2 - \Omega^2}\sin\Omega t\right)$

for $q(0) = \dot{q}(0) = 0$ at $t=0$, $B=0$, $A = \frac{-f_0}{\omega_0^2 - \Omega^2}$

$E = T + V$

$= \frac{1}{2}m(A^2\omega^2\sin^2\omega t - 2A^2\Omega\omega\sin\omega t\sin\Omega t + A^2\Omega^2\sin^2\Omega t)$

$+ \frac{1}{2}m(A^2\omega^2\cos^2\omega t - 2A^2\omega^2\cos\omega t\cos\Omega t + A^2\omega^2\cos^2\Omega t)$

for maximum E, $\frac{dE}{dt} = 0$

$(mf_0\cos\Omega t)(-A\omega\sin\omega t + A\Omega\sin\omega t) = 0$

when $\omega < \Omega$. $E_{max} = \frac{mf_0^2}{2(\omega - \Omega)^2}$.

when $\omega > \Omega$. $E_{max} = \frac{2mf_0^2\omega^2}{(\omega^2 - \Omega^2)2}$

(b) $q(t) = \alpha(t)\, e^{i\omega_0 t}$.

$\dot{q}(t) = \alpha'(t)e^{i\omega_0 t} + i\omega_0\alpha(t)e^{i\omega_0 t}$.

$\ddot{q}(t) = \alpha''(t)e^{i\omega_0 t} + i(2\omega_0)\alpha'(t)e^{i\omega_0 t} - \omega_0^2\alpha(t)e^{i\omega_0 t}$

Let $\beta(t) = \alpha'(t)$

$\beta'(t) + i(2\omega_0)\beta(t) = f_0$

$\beta(t) = \frac{f_0 - e^{-i(2\omega_0 t + \delta)}}{i \cdot 2\omega_0}$.

$\alpha(t) = \frac{f_0 t}{i(2\omega_0)} - \frac{e^{-i(2\omega_0 t + \delta)}}{(2\omega_0)^2} + c$.

$q(t) = \mathrm{Re}(\alpha(t)e^{-i\omega_0 t}) = \frac{f_0 t\sin\omega t}{2\omega_0} - \frac{\cos(\omega_0 t + \delta)}{(2\omega_0)^2} + c\cos\omega t$

4 (c). $t=0,\ q=0.\ \dot{q}=0.$

$$q(t) = \frac{f_0 t \sin wt}{2w}.$$

$$\dot{q}(t) = \frac{f_0}{2w_0}\left(\sin w_0 t + w_0 t \cos w_0 t\right)$$

$$T = \frac{m}{2}\left(\dot{q}(t)\right)^2.$$

$$= \frac{m}{2}\left(\frac{f_0}{2w_0}\right)^2\left(\sin^2 w_0 t + 2w_0 t \sin w_0 t \cos w_0 t + w_0^2 t^2 \cos^2 w_0 t\right)$$

$$V = \frac{m}{2} w_0^2 q^2(t) = \frac{m}{2}\left(\frac{f_0}{2w_0}\right)^2\left(w_0^2 t^2 \sin^2 w_0 t\right)$$

$$E = T + V = \frac{m}{2}\left(\frac{f_0}{2w_0}\right)^2\left(\sin^2 w_0 t + w_0 t \sin 2w_0 t + w_0^2 t^2\right)$$

$$\frac{dE}{dt} = \frac{m}{2}\left(\frac{f_0}{2w_0}\right)^2\left(2w_0 \sin 2w_0 t + 2w_0^2 t \cos 2w_0 t + 2w_0^2 t\right).$$

$$\bar{E} = \frac{1}{T}\int_{t_0}^{t_0+T} E\,dt. \qquad T = \frac{2\pi}{w}$$

$$= \frac{1}{T}\frac{m}{2}\left(\frac{f_0}{2w_0}\right)^2\int_{t_0}^{t_0+T}\left(\sin^2 w_0 t + w_0 t \sin 2w_0 t + w_0^2 t^2\right)dt$$

$$= \frac{mf_0}{16\pi w_0}\left(2\pi w_0 t_0^2 + 4\pi^2 t_0 + \frac{8\pi^2}{3w} + \frac{\pi}{w} - \frac{\pi}{w}\cos(2\right.$$

when $t_0 \gg 1$.

$$E \approx \frac{mf_0}{16\pi w_0} 2\pi w_0 t_0^2 \approx \frac{mf_0 t^2}{8}.$$