

Homework Assignment 7

Physics 761, Classical Mechanics

A. V. Kotwal

Handed out: Monday, November 12, 2012

Due time: Monday, November 19, 2012

Problems

Each problem is out of 20 points.

1. Consider the Kepler problem. The energy conservation equation is given by

$$E = \frac{\mu}{2} \left(\frac{dr}{dt} \right)^2 + \frac{l^2}{2\mu r^2} - \frac{\alpha}{r},$$

where $\alpha = G\mu M = Gm_1m_2$, l is the angular momentum, and E is the total energy of the system.

- (a) Prove the Kepler's third law for elliptical orbits

$$T^2 = \frac{4\pi^2}{GM} a^3,$$

where $M = m_1 + m_2$ is the total mass, $a = \frac{1}{2}(r_{min} + r_{max})$ is the semi-major axis, and T is the period of the orbit.

- (b) Show that the semi-minor axis is given by

$$b = a \sqrt{1 - \epsilon^2},$$

where $\epsilon = \sqrt{1 + \frac{2El^2}{\mu\alpha^2}}$ is the eccentricity of the orbit. We notice that when $\epsilon = 0$, $b = a$, the orbit is a circle.

2. A uniform distribution of dust in the solar system adds to the gravitational attraction of the Sun on a planet an additional force

$$\vec{F} = -mC\vec{r}$$

where m is the mass of the planet, C is a constant proportional to the gravitational constant and the density of the dust, and \vec{r} is the radius vector from the Sun to the planet (both considered as points). This additional force is very small compared to the direct Sun-planet gravitational force.

- (a) Calculate the period for a circular orbit of radius r_0 of the planet in this combined field.
 - (b) Calculate the period of radial oscillations for slight disturbances from this circular orbit.
 - (c) Show that nearly circular orbits can be approximated by a precessing ellipse and find the precession frequency. Is the precession in the same or opposite direction to the orbital angular velocity?
3. A central force potential that is frequently encountered in nuclear physics is the rectangular well, defined by the potential

$$V = \begin{cases} -V_0, & r \leq a, \\ 0, & r > a. \end{cases}$$

Show that the scattering produced by such a potential in classical mechanics is identical with the refraction of light rays by a sphere of radius a and relative index of refraction

$$n = \sqrt{\frac{E + V_0}{E}}.$$

(This equivalence demonstrates why it was possible to explain refraction phenomena both by Huygens's waves and by Newton's mechanical corpuscles).

Show also that the differential cross section is

$$\sigma(\theta) = \frac{n^2 a^2 (n \cos \frac{\theta}{2} - 1)(n - \cos \frac{\theta}{2})}{4 \cos \frac{\theta}{2} (1 + n^2 - 2n \cos \frac{\theta}{2})}.$$

Calculate the total cross section explicitly. You should find that it has a value that is easily explained. If you use Mathematica to do the integral (which is OK), please turn in a page showing the relevant commands.