Homework Assignment 8
Physics 302, Classical Mechanics

Fall 2011
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Handed out:	Friday, November 4, 2011
Due time:	Monday, November 14, 2011

Each problem is worth 20 points.

Problems

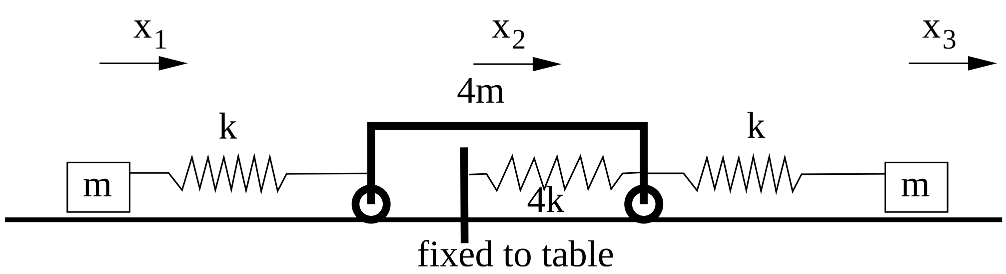
1. A bar of negligible weight and length l had equal mass points m at the two ends. The bar is made to rotate uniformly about an axis passing through the center of the bar and making an angle θ with the bar.
   (a) Find the components along the principle axes of the bar of the torque driving the bar.
   (b) Find the components of the torque along axes fixed in space. Show that these components are consistent with those found in part (a).

2. Consider the following situations in body and inertial coordinates respectively.
   (a) Show that the angular momentum of the torque-free symmetrical top rotates in the body coordinates about the symmetry axis with an angular frequency Ω, and find the expression for Ω.
   (b) Show also that the symmetry axis rotates in space about the fixed direction of the angular momentum with the angular frequency

$$\dot{\phi} = \frac{I_3\,\omega_3}{I_1\,\cos\theta}$$

   where the angular momentum points in the space z direction.

3. Consider the three spring, three mass system shown below. The center post is taken to be fixed to the table.



   (a) Using coordinates referenced to equilibrium, write expressions for the kinetic and potential energies of this system.
   (b) Derive the Lagrangian and Hamiltonian of this system.

(c) Derive the equations of motion.

(d) Find the eigenmodes and corresponding oscillation frequencies. Graph the eigenmodes.

4. An undamped linear oscillator with natural frequency ω_0 is driven at frequency Ω by an external force with the following equation of the motion,

$$\ddot{q} + \omega_0^2 q = f_0 \cos \Omega t.$$

(a) For $\Omega \neq \omega_0$, find the rate at which energy is pumped into the oscillator. Find the maximum energy of such an oscillator starting from rest, i.e. $q(0) = \dot{q}(0) = 0$ at $t = 0$;

(b) For $\Omega = \omega_0$, by going into the complex variables, i.e. replacing $\cos \omega_0 t$ by $e^{i\omega_0 t}$ and setting $q(t) = \alpha(t)e^{i\omega_0 t}$, find the real solution for the driven oscillator;

(c) For an oscillator starting from rest, i.e. $q(0) = \dot{q}(0) = 0$ at $t = 0$, and $\Omega = \omega_0$, find the rate at which energy is pumped into the oscillator and find the energy of the oscillator, averaged over a period, as a function of time.