Homework Assignment 7
Physics 302, Classical Mechanics

Fall 2011
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Handed out:	Monday, October 24, 2011
Due time:	Friday, November 4, 2011

Problems

Each problem is out of 20 points.

1. Consider the Kepler problem. The energy conservation equation is given by

$$E = \frac{\mu}{2}\left(\frac{dr}{dt}\right)^2 + \frac{l^2}{2\mu r^2} - \frac{\alpha}{r},$$

where $\alpha = G\mu M = Gm_1m_2$, l is the angular momentum, and E is the total energy of the system.

   (a) Prove the Kepler's third law for elliptical orbits

$$T^2 = \frac{4\pi^2}{G\,M}\,a^3,$$

   where $M = m_1 + m_2$ is the total mass, $a = \frac{1}{2}(r_{min} + r_{max})$ is the semi-major axis, and T is the period of the orbit.

   (b) Show that the semi-minor axis is given by

$$b = a\,\sqrt{1-\epsilon^2},$$

   where $\epsilon = \sqrt{1 + \frac{2El^2}{\mu\alpha^2}}$ is the eccentricity of the orbit. We notice that when $\epsilon = 0$, $b = a$, the orbit is a circle.

2. A uniform distribution of dust in the solar system adds to the gravitional attaction of the Sun on a planet an additional force

$$\vec{F} = -mC\vec{r}$$

where m is the mass of the planet, C is a constant proportional to the gravitational constant and the density of the dust, and $\vec{r}$ is the radius vector from the Sun to the planet (both considered as points). This additional force is very small compared to the direct Sun-planet gravitational force.

   (a) Calculate the period for a circular orbit of radius r_0 of the planet in this combined field.

   (b) Calculate the period of radial oscillations for slight disturbances from this circular orbit.

   (c) Show that nearly circular orbits can be approximated by a precessing ellipse and find the precession frequency. Is the precession in the same or opposite direction to the orbital angular velocity?

3. A central force potential that is frequently encountered in nuclear physics is the rectangular well, defined by the potential

$$V = \begin{cases} -V_0, & r \le a, \\ 0, & r > a. \end{cases}$$

Show that the scattering produced by such a potential in classical mechanics is identical with the refraction of light rays by a sphere of radius a and relative index of refraction

$$n = \sqrt{\frac{E + V_0}{E}}.$$

(This equivalence demonstrates why it was possible to explain refraction phenomena both by Huygens's waves and by Newton's mechanical corpuscles).

Show also that the differential cross section is

$$\sigma(\theta) = \frac{n^2 a^2 (n \cos\frac{\theta}{2} - 1)(n - \cos\frac{\theta}{2})}{4 \cos\frac{\theta}{2} (1 + n^2 - 2n \cos\frac{\theta}{2})}.$$

Calculate the total cross section explicitly. You should find that it has a value that is easily explained. If you use Mathematica to do the integral (which is OK), please turn in a page showing the relevant commands.