Homework Assignment 5
Physics 302, Classical Mechanics

Fall, 2011
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Handed out:	Tuesday, October 4, 2011
Due in class on:	Monday, October 17, 2011

Problems

1. Consider the Hamiltonian of a one-dimensional simple harmonic oscillator,
$$H(q,p) = \frac{1}{2}p^2 + \frac{1}{2}\omega^2 q^2.$$
   (a) Use Hamilton's equations of motion to solve for $q(t)$ and $p(t)$ in terms of initial values (q_0, p_0).
   (b) Evaluate the Poisson bracket $\{q(t), p(t)\}$ with respect to (q_0, p_0).
   (c) Show that the relationship between $(q(t), p(t))$ and (q_0, p_0) represent a canonical transformation.

2. Show that the function
$$S(q,P,t) = \frac{m\omega}{2}(q^2 + P^2)\cot\omega t - m\omega q\, P \csc\omega t$$
is a solution of the Hamilton-Jacobi for Hamilton's principal function for the simple harmonic oscillator with
$$H = \frac{1}{2m}(p^2 + m^2\omega^2 q^2).$$
Show that this function generates a correct solution to the motion of the harmonic oscillator.

3. Suppose the potential in a problem of one degree of freedom is linearly dependent upon time, such that the Hamiltonian has the form
$$H(x,p,t) = \frac{p^2}{2m} - m\,A\,t\,x,$$
where A is a constant. Solve the dynamical problem by means of Hamilton's principal function, under the initial conditions $t = 0$, $x = 0$, $p = mv_0$.

4. Solve the Hamilton-Jacobi equation for the Hamiltonian
$$H = f(t)(p^2/m + kq^2)/2$$
where m and k are constants and $f(t)$ is an integrable function. Find $q(t)$ and $p(t)$ and the phase-space trajectory. Find the kinetic energy as a function of the time for the three special cases (a) $f(t) = e^{\alpha t}$, (b) $f(t) = e^{-\alpha t}$, and (c) $f(t) = \cos\Omega t$, where $\alpha > 0$ and Ω are constants. Describe the motion of these three cases.

5. A particle of mass m moves in a plane in a square-well potential
$$\begin{aligned} V(r) &= -V_0 \quad 0 < r < r_0 \\ &= 0 \quad\quad\; r > r_0 \end{aligned}$$
Under what conditions can the method of action-angle variables be applied? Assuming these conditions hold, use the method of action-angle variables to find the frequencies of the motion.

6. A particle of mass m moves in one dimension under a potential $V = -k/|x|$. For energies that are negative, the motion is bounded and oscillatory. By the method of action-angle variables, find an expression for the period of motion as a function of the particle's energy.