

Homework Assignment 3

Physics 302, Classical Mechanics

Fall, 2011
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Handed out: Friday, September 16, 2011
Due in class on: Wednesday, September 28, 2011

Problems

(Ten points for each problem unless noted otherwise.)

1. Use the Lagrange multiplier method to solve the following problem: A particle in a uniform gravitational field is free to move without friction on a paraboloid of revolution whose symmetry axis is vertical (opening upward). Obtain the force of constraint. Prove that for a given energy and angular momentum about the symmetry axis there is a minimum and a maximum height to which the particle will go.
2. A bead is constrained to move without friction on a helix whose equation in cylindrical polar coordinates is $\rho = b$, $z = a\phi$, with the potential $V = \frac{1}{2}k(\rho^2 + z^2)$. Use the Lagrange multiplier method to find the constraint forces.
3. Consider a three-dimensional one-particle system whose potential energy in cylindrical polar coordinates ρ, θ, z is of the form $V(\rho, k\theta + z)$, where k is a constant. Find a symmetry of the Lagrangian and use Noether's theorem to obtain the constant of the motion associated with it.
4. A point mass is constrained to move on a massless hoop of radius a fixed in a vertical plane that rotates about its vertical symmetry axis with constant angular speed ω . Obtain the Lagrange equations of motion assuming the only external forces arise from gravity. What are the constants of motion? Show that if ω is greater than a critical value ω_0 , there can be a solution in which the particle remains stationary on the hoop at a point other than the bottom, but if $\omega < \omega_0$, the only stationary point for the particle is at the bottom of the hoop. What is the value of ω_0 ?
5. Suppose a particle moves in space subject to a conservative potential $V(\mathbf{r})$ but is constrained to always move on a surface whose equation is $\sigma(\mathbf{r}, t) = 0$. The explicit dependence of σ on t indicates that the surface may be moving. The instantaneous force of constraint is taken as always perpendicular to the surface. Show analytically that the energy of the particle is not conserved if the surface moves in time. What physically is the reason for nonconservation of the energy under this circumstance?
6. A particle of mass m and electric charge e moves in a plane under the influence of a central force potential $V(r)$ and a constant uniform magnetic field \mathbf{B} , perpendicular to the plane, generated by a static vector potential

$$\mathbf{A} = \frac{1}{2}\mathbf{B} \times \mathbf{r}$$

Find the Hamiltonian using coordinates in the observer's inertial system. Next, find the Hamiltonian using coordinates rotating relative to the previous coordinate system about an axis perpendicular to the plane with an angular rate of rotation:

$$\omega = -\frac{eB}{2m}$$