Physics 230
Midterm Exam 2 - Fall 2008

18 November 2008, 11:40-12:55

The midterm exam is out of 55 points. There are 5 bonus points. Any bonus points you earn will be applied towards your homeworks and exams. good luck!

Be sure to write down all your steps and references.

1. Find all the critical points of the following equations:

$$\begin{aligned} \dot{x} &= 6x - 2x^2 - xy \\ \dot{y} &= 8y - 3y^2 - 2xy \end{aligned}$$

(10 points)

2. In the above problem, x and y represent the populations of two competing species. Show that the two species can coexist in the long term. (10 points)

3. Show that the Green's function for the equation

$$y''(x) = f(x)$$

with boundary conditions

$$y(0) = y(1) = 0$$

is

$$G(x, z) = x(1 - z)$$

for $0 \leq x < z$ and

$$G(x, z) = z(1 - x)$$

for $z < x \leq 1$. (10 points)

4. Solve the one-dimensional heat equation subject to the conditions $T(0, t) = T(l, t) = 0$ and $T(x, 0) = T_0$. (10 points)

5. Use spherical coordinates to evaluate the integral

$$I = \int\int\int e^{-2r} \cos^2\theta \, dV$$

over all space. (10 points)

6. Show that 2-D polar coordinates form an orthogonal coordinate system by showing that the $\partial\mathbf{r}/\partial u_j$ are orthogonal. (5 points)