

Physics 230
Midterm Exam I - Fall 2008

Wednesday, 8 October 2008, 11:40am - 12:55pm

The midterm exam is out of 50 points. There are 5 bonus points. Any bonus points you earn will be applied towards your homeworks and exams. good luck!

Be sure to write down all your steps and references.

1. Evaluate the integral

$$\int_0^\infty x^m e^{-x^n} dx$$

where m and n are positive integers, in terms of a special function. (5 points)

2. Evaluate the integral

$$\int_0^1 \left(1 - \frac{1}{x}\right)^{2/3} dx$$

in terms of a special function. (5 points)

3. Show that the sequence

$$s_n = \left(\frac{n-1}{n+1}\right)^n$$

converges and find the limit of s_n as $n \rightarrow \infty$. (5 points)

4. Find a power series for $f(x)$, where

$$f(x) = \int_0^x \frac{u du}{1+u^4}$$

that is valid for $|x| < 1$. (5 points)

5. Given that $[x^{-\nu} J_\nu(x)]' = -x^{-\nu} J_{\nu+1}(x)$ and $[x^\nu J_\nu(x)]' = x^\nu J_{\nu-1}(x)$, show that

$$\int x \ln x J_0(x) dx = J_0(x) + x \ln x J_1(x)$$

(10 points)

6. Are $\sinh x$ and $\cosh x$ linearly independent? Prove your answer. (5 points)

7. Find the general solution of

$$x^2 y'' + 3xy' + y = 0$$

You will need to use reduction of order to find the second solution. (10 points)

8. Find the general solution and particular solution of

$$y'' + y = x + e^{-x}$$

(10 points)