A New Precise Measurement of the W Boson Mass at CDF

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Motivation for Precision Electroweak Measurements

- The electroweak gauge sector of the standard model, defined by \((g, g', \nu)\), is constrained by three precisely known parameters
  - \(\alpha_{\text{EM}}(M_Z) = 1 / 127.918(18)\)
  - \(G_F = 1.16637 (1) \times 10^{-5} \text{ GeV}^{-2}\)
  - \(M_Z = 91.1876 (21) \text{ GeV}\)

- At tree-level, these parameters are related to other electroweak observables, e.g. \(M_W\)
  - \(M_W^2 = \pi \alpha_{\text{EM}} / \sqrt{2} G_F \sin^2 \vartheta_W\)
    - where \(\vartheta_W\) is the Weinberg mixing angle, defined by \(\cos \vartheta_W = M_W / M_Z\)
Motivation for Precision Electroweak Measurements

- Radiative corrections due to heavy quark and Higgs loops and exotica

Motivate the introduction of the $\rho$ parameter: $M_W^2 = \rho [M_W(\text{tree})]^2$
with the predictions $\Delta \rho = (\rho - 1) \sim M_{\text{top}}^2$ and $\Delta \rho \sim \ln M_H$

- In conjunction with $M_{\text{top}}$, the $W$ boson mass constrains the mass of the Higgs boson, and possibly new particles beyond the standard model
Contributions from Supersymmetric Particles

- Radiative correction depends on mass splitting ($\Delta m^2$) between squarks in SU(2) doublet
- After folding in limits on SUSY particles from direct searches, SUSY loops can contribute 100 MeV to $M_W$
Progress on $M_{\text{top}}$ at the Tevatron

- From the Tevatron, $\Delta M_{\text{top}} = 0.9$ GeV $\Rightarrow \Delta M_H / M_H = 8\%$
- equivalent $\Delta M_W = 6$ MeV for the same Higgs mass constraint
- Current world average $\Delta M_W = 23$ MeV
  - progress on $\Delta M_W$ has the biggest impact on Higgs constraint
Motivation

- Generic parameterization of new physics contributing to W and Z boson self-energies: \( S, T, U \) parameters (Peskin & Takeuchi)

\[ M_W \] and Asymmetries are the most powerful observables in this parameterization

Additionally, \( M_W \) is the only measurement which constrains \( U \)

(\( M_H \sim 120 \text{ GeV} \)
\( M_H > 600 \text{ GeV} \))

(from P. Langacker, 2012)
W Boson Production at the Tevatron

Quark-antiquark annihilation dominates (80%)

Lepton $p_T$ carries most of $W$ mass information, can be measured precisely (achieved 0.01%)

Initial state QCD radiation is $O(10 \text{ GeV})$, measure as soft 'hadronic recoil' in calorimeter (calibrated to $\sim0.5\%$)
Quadrant of Collider Detector at Fermilab (CDF)

- Central electromagnetic calorimeter
- Central hadronic calorimeter
- Drift chamber provides precise lepton track momentum measurement
- EM calorimeter provides precise electron energy measurement
- Calorimeters measure hadronic recoil particles

Select W and Z bosons with central (|\eta| < 1) leptons
Analysis Strategy

Maximize the number of internal constraints and cross-checks

Driven by two goals:

1) **Robustness**: constrain the same parameters in as many different ways as possible

2) **Precision**: combine independent measurements after showing consistency
Internal Alignment of COT

- Use a clean sample of $\sim 400k$ cosmic rays for cell-by-cell internal alignment

- Fit COT hits on both sides simultaneously to a single helix (A. Kotwal, H. Gerberich and C. Hays, NIMA 506, 110 (2003))
  - Time of incidence is a floated parameter in this 'dicosmic fit'
Custom Monte Carlo Detector Simulation

- A complete detector simulation of all quantities measured in the data
- First-principles simulation of tracking
  - Tracks and photons propagated through a high-granularity 3-D lookup table of material properties for silicon detector and COT
  - Ionization energy loss according to complete Bethe-Bloch formula
  - Generate bremsstrahlung photons down to 4 MeV, using detailed cross-section and spectrum calculations
  - Simulate photon conversion and Compton scattering
  - Propagate bremsstrahlung photons and conversion electrons
  - Simulate multiple Coulomb scattering, including non-Gaussian tail
Tracking Momentum Scale

Set using $J/\psi \rightarrow \mu\mu$ and $\Upsilon \rightarrow \mu\mu$ resonance and $Z \rightarrow \mu\mu$ masses

- Extracted by fitting $J/\psi$ mass in bins of $1/p_T(\mu)$, and extrapolating momentum scale to zero curvature
- $J/\psi \rightarrow \mu\mu$ mass independent of $p_T(\mu)$ after 4% tuning of energy loss

\[ \int L \, dt = 2.2 \text{ fb}^{-1} \]

Scale correction $= (-1.299 \pm 0.022) \times 10^{-3}$
Slope $= (0.8 \pm 6.4) \times 10^{-5}$ GeV

Default energy loss $\times 1.04$

$\Delta p/p = (-1.284 \pm 0.024_{\text{stat}}) \times 10^{-3}$
$\chi^2/\text{dof} = 95/86$

$J/\psi \rightarrow \mu\mu$ mass fit (bin 5)
Tracking Momentum Scale

$\Upsilon \rightarrow \mu\mu$ resonance provides

- Momentum scale measurement at higher $p_T$
- Validation of beam-constaining procedure (upsilons are promptly produced)
- Cross-check of non-beam-constrained (NBC) and beam-constrained (BC) fits

$$\int L \, dt = 2.2 \, \text{fb}^{-1}$$

$$\Delta p/p = (-1.335 \pm 0.025^{\text{stat}}) \times 10^{-3}$$

$$\chi^2/\text{dof} = 59 / 48$$

NBC $\Upsilon \rightarrow \mu\mu$ mass fit
**Z → µµ**  Mass Cross-check & Combination

- Using the J/ψ and ϒ momentum scale, performed “blinded” measurement of Z mass

  - Z mass consistent with PDG value (91188 MeV) (0.7σ statistical)

  - \[ M_Z = 91180 \pm 12_{\text{stat}} \pm 9_{\text{momentum}} \pm 5_{\text{QED}} \pm 2_{\text{alignment}} \text{ MeV} \]

\[ \int L \, dt = 2.2 \, \text{fb}^{-1} \]

\[ M_Z = (91180 \pm 12_{\text{stat}}) \text{ MeV} \]

\[ \chi^2/\text{dof} = 30/30 \]

\[ \text{Data} \quad \text{Simulation} \]
Tracker Linearity Cross-check & Combination

- Final calibration using the $J/\psi$, $\Upsilon$ and $Z$ bosons for calibration

- Combined momentum scale correction:

$$\Delta p/p = (-1.29 \pm 0.07_{\text{independent}} \pm 0.05_{\text{QED}} \pm 0.02_{\text{align}}) \times 10^{-3}$$

\[\int L \, dt = 2.2 \text{ fb}^{-1}\]

$$\Delta M_W = 7 \text{ MeV}$$
EM Calorimeter Scale

- E/p peak from $W \rightarrow \nu$ decays provides measurements of EM calorimeter scale and its ($E_T$-dependent) non-linearity

$$\Delta S_E = (9_{\text{stat}} \pm 5_{\text{non-linearity}} \pm 5_{\text{X0}} \pm 9_{\text{Tracker}}) \times 10^{-5}$$

Setting $S_E$ to 1 using E/p calibration from combined $W \rightarrow \nu$ and $Z \rightarrow ee$ samples

$$\Delta M_W = 13 \text{ MeV}$$

Tail of E/p spectrum used for tuning model of radiative material
Z→ee Mass Cross-check and Combination

- Performed “blind” measurement of Z mass using E/p-based calibration
  - Consistent with PDG value (91188 MeV) within 1.4σ (statistical)
  - \( M_Z = 91230 \pm 30_{\text{stat}} \pm 10_{\text{calorimeter}} \pm 8_{\text{momentum}} \pm 5_{\text{QED}} \pm 2_{\text{alignment}} \) MeV

- Combine E/p-based calibration with Z→ee mass for maximum precision

\[ \Delta M_W = 10 \text{ MeV} \]
Constraining Boson $p_T$ Spectrum

- Fit the non-perturbative parameter $g_2$ and QCD coupling $\alpha_s$ in RESBOS to $p_T(\ell\ell)$ spectra:

$$\Delta M_W = 5 \text{ MeV}$$

Position of peak in boson $p_T$ spectrum depends on $g_2$

Tail to peak ratio depends on $\alpha_s$

\[
\int L \, dt = 2.2 \text{ fb}^{-1}
\]

\[
\begin{array}{c|c|c}
\text{MC} & \text{data} \\
\hline
\mu = 8.907 \text{ GeV} & \mu = 8.891 \pm 0.027 \text{ GeV} \\
\sigma = 6.676 \text{ GeV} & \sigma = 6.699 \pm 0.019 \text{ GeV}
\end{array}
\]

\[
\chi^2 / \text{DoF} = 31.9 / 29
\]

\[
\begin{array}{c|c|c}
\text{MC} & \text{data} \\
\hline
\mu = 8.921 \text{ GeV} & \mu = 8.856 \pm 0.052 \text{ GeV} \\
\sigma = 6.693 \text{ GeV} & \sigma = 6.698 \pm 0.037 \text{ GeV}
\end{array}
\]

\[
\chi^2 / \text{DoF} = 18.7 / 29
\]
$W$ Transverse Mass Fit

CDF II

$\int L \, dt \approx 2.2 \text{ fb}^{-1}$

Muons

$M_W = (80379 \pm 16_{\text{stat}}) \text{ MeV}$

$\chi^2/\text{dof} = 58 / 48$
$W$ Mass Fit using Lepton $p_T$

CDF II

$\int L \, dt \approx 2.2 \, fb^{-1}$

$M_W = (80393 \pm 21_{\text{stat}}) \, \text{MeV}$

$\chi^2/\text{dof} = 60/62$

Electrons

- Data
- Simulation
## Summary of $W$ Mass Fits

<table>
<thead>
<tr>
<th>Charged Lepton</th>
<th>Kinematic Distribution</th>
<th>Fit Result (MeV)</th>
<th>$\chi^2$/DoF</th>
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<tbody>
<tr>
<td>Electron</td>
<td>Transverse mass</td>
<td>80408 ± 19</td>
<td>52/48</td>
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<tr>
<td>Electron</td>
<td>Charged lepton $p_T$</td>
<td>80393 ± 21</td>
<td>60/62</td>
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<tr>
<td>Electron</td>
<td>Neutrino $p_T$</td>
<td>80431 ± 25</td>
<td>71/62</td>
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<tr>
<td>Muon</td>
<td>Transverse mass</td>
<td>80379 ± 16</td>
<td>57/48</td>
</tr>
<tr>
<td>Muon</td>
<td>Charged lepton $p_T$</td>
<td>80348 ± 18</td>
<td>58/62</td>
</tr>
<tr>
<td>Muon</td>
<td>Neutrino $p_T$</td>
<td>80406 ± 22</td>
<td>82/62</td>
</tr>
</tbody>
</table>

CDF III: $\int L \, dt = 2.2 \, fb^{-1}$
Combined Results

- Combined electrons (3 fits): $M_W = 80406 \pm 25$ MeV, $P(\chi^2) = 49\%$

- Combined muons (3 fits): $M_W = 80374 \pm 22$ MeV, $P(\chi^2) = 12\%$

- All combined (6 fits): $M_W = 80387 \pm 19$ MeV, $P(\chi^2) = 25\%$
New CDF Result (2.2 fb\(^{-1}\))
Transverse Mass Fit Uncertainties (MeV)

<table>
<thead>
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<th>electrons</th>
<th>muons</th>
<th>common</th>
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<td>Total systematic</td>
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<td>15</td>
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<tr>
<td>Total</td>
<td>26</td>
<td>23</td>
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</table>

Systematic uncertainties shown in green: statistics-limited by control data samples
Combined W Mass Result, Error Scaling
Previous $M_W$ vs $M_{\text{top}}$

- LEPEWWG (2011) 68% CL (excluding $M_w$, $m_{\text{top}}$ & direct Higgs exclusion)
- 68% CL (by area) $M_w$ (2009), $m_{\text{top}}$

Higgs mass constraints:
- $115 < m_H < 127$
- $600 < m_H < 1000$
Updated $M_W$ vs $M_{\text{top}}$
W Boson Mass Measurements from Different Experiments

Previous world average = 80399 ± 23 MeV

New CDF result is significantly more precise than other measurements.

**World Average**

- D0 I: 80483 ± 84
- CDF I: 80433 ± 79
- DELPHI: 80336 ± 67
- L3: 80270 ± 55
- OPAL: 80416 ± 53
- ALEPH: 80440 ± 51
- D0 II (PRL 108, 151804): 80375 ± 23
- CDF II (PRL 108, 151803): 80387 ± 19
- World Average: 80385 ± 15

ArXiv: 1204.0042
PDF Uncertainties – scope for improvement

- Factor of 5 bigger samples of W and Z bosons available

- Newer PDF sets, e.g. CT10W include more recent data, such as Tevatron W charge asymmetry data

- Dominant sources of W mass uncertainty are the $d_{\text{valence}}$ and $\bar{d}-\bar{u}$ degrees of freedom
  - Understand consistency of data constraining these d.o.f.
  - PDF fitters increase tolerance to accommodate inconsistent datasets

- Tevatron and LHC measurements that can further constrain PDFs:
  - Z boson rapidity distribution
  - W → ℓν lepton rapidity distribution
  - W boson charge asymmetry
Summary

- The W boson mass is a very interesting parameter to measure with increasing precision

- New CDF W mass result from 2.2 fb$^{-1}$ is the most precise in the world

  \[ M_W = 80387 \pm 12_{\text{stat}} \pm 15_{\text{syst}} \text{ MeV} \]
  \[ = 80387 \pm 19 \text{ MeV} \]

- New indirect limit $M_H < 152$ GeV @ 95% CL

  - SM Higgs prediction is pinned in the low-mass range => confront mass from direct search results

- Looking forward to $\Delta M_W < 10$ MeV from 10 fb$^{-1}$ of CDF data
Standard model confirmed at high precision if light (~125 GeV) Higgs boson

Standard model inconsistent at > 5σ if Higgs mass > 600 GeV
Backup
Measurement of EM Calorimeter Non-linearity

- Perform E/p fit-based calibration in bins of electron $E_T$
- GEANT-motivated parameterization of non-linear response:
  \[ S_E = 1 + \beta \log(\frac{E_T}{39 \text{ GeV}}) \]
- Tune on W and Z data: $\beta = (5.2 \pm 0.7_{\text{stat}}) \times 10^{-3}$
  \[ \Rightarrow \Delta M_W = 4 \text{ MeV} \]
Tuning Recoil Resolution Model with $Z$ events

At low $p_T(Z)$, $p_T$-balance constrains hadronic resolution due to underlying event.

At high $p_T(Z)$, $p_T$-balance constrains jet resolution.

$\Delta M_W = 4$ MeV
Testing Hadronic Recoil Model with $W$ events

Compare recoil distributions between simulation and data

Recoil projection (GeV) on lepton direction
# Backgrounds in the W sample

## Muons

<table>
<thead>
<tr>
<th>Background</th>
<th>% of $W \rightarrow \mu\nu$ data</th>
<th>$\delta m_W$ (MeV)</th>
<th>$m_T$ fit</th>
<th>$p_T^\mu$ fit</th>
<th>$p_T^\nu$ fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z \rightarrow \mu\mu$</td>
<td>$7.35 \pm 0.09$</td>
<td>2</td>
<td>4</td>
<td>5</td>
<td></td>
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<tr>
<td>$W \rightarrow \tau\nu$</td>
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<td>DIF</td>
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<td>Cosmic rays</td>
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<tr>
<td><strong>Total</strong></td>
<td></td>
<td>3</td>
<td>5</td>
<td>6</td>
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## Electrons

<table>
<thead>
<tr>
<th>Background</th>
<th>% of $W \rightarrow e\nu$ data</th>
<th>$\delta m_W$ (MeV)</th>
<th>$m_T$ fit</th>
<th>$p_T^e$ fit</th>
<th>$p_T^\nu$ fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z \rightarrow ee$</td>
<td>$0.139 \pm 0.014$</td>
<td>1</td>
<td>2</td>
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<td>$W \rightarrow \tau\nu$</td>
<td>$0.93 \pm 0.01$</td>
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<tr>
<td>QCD</td>
<td>$0.39 \pm 0.14$</td>
<td>4</td>
<td>2</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td>4</td>
<td>3</td>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

Backgrounds are small (except $Z \rightarrow \mu\mu$ with a forward muon)
**Z →ee Mass Cross-check using Electron Tracks**

- Performed “blind” measurement of Z mass using electron tracks
  - Consistent with PDG value within $1.8\sigma$ (statistical)
- Checks tracking for electrons vs muons, and model of radiative energy loss

![Graph showing the comparison between data and simulation with $M_Z = (91268 \pm 47_{stat})$ MeV and $\chi^2/\text{dof} = 62/46$. The integration of the luminosity is $\int L \, dt = 2.2 \text{ fb}^{-1}$.]
$W$ Transverse Mass Fit

CDF II

$\int L \, dt \approx 2.2 \, \text{fb}^{-1}$

Electrons

$M_W = (80408 \pm 19_{\text{stat}}) \, \text{MeV}$

$\chi^2/\text{dof} = 52/48$
$W$ Lepton $p_T$ Fit

CDF II

$\int L \, dt \approx 2.2 \, fb^{-1}$

$M_W = (80348 \pm 18_{\text{stat}}) \, \text{MeV}$

$\chi^2/\text{dof} = 54/62$

- **Data**
- **Simulation**

**Muons**
$W$ Missing $E_T$ Fit

CDF II

$\int L \, dt \approx 2.2 \, \text{fb}^{-1}$

Electrons

$M_W = (80431 \pm 25_{\text{stat}}) \, \text{MeV}$

$\chi^2/\text{dof} = 71 / 62$

- Data
- Simulation

events / 0.25 GeV

$p_T^e(v) \, (\text{GeV})$
$W$ Missing $E_T$ Fit

CDF II

$\int L \, dt \approx 2.2 \text{ fb}^{-1}$

$M_W = (80406 \pm 22_{\text{stat}}) \text{ MeV}$

$\chi^2/\text{dof} = 79/62$

*Data*  
*Simulation*

**Muons**

**events / 0.25 GeV**

**$p_T^\mu (\nu)$ (GeV)**
$W$ Mass Fit Residuals, Electron Channel
$W$ Mass Fit Residuals, Muon Channel
$W$ Mass Fit Window Variation, $m_T$ Fit
W Mass Fit Window Variation, $p_T(l)$ Fit
$W$ Mass Fit Window Variation, $p_T(\nu)$ Fit

**Upper**

- **CDF II preliminary**
- $\int L dt = 2.2$ fb$^{-1}$

- **Graphs**
  - $W \rightarrow e\nu$
  - $W \rightarrow \mu\nu$

- **Axes**
  - $\Delta M_W$ (MeV)
  - Start of fit window (GeV)
  - End of fit window (GeV)

**Lower**

- **CDF II preliminary**
- $\int L dt = 2.2$ fb$^{-1}$

- **Graphs**
  - $W \rightarrow e\nu$
  - $W \rightarrow \mu\nu$

- **Axes**
  - $\Delta M_W$ (MeV)
  - Start of fit window (GeV)
  - End of fit window (GeV)
$W$ Mass Fit Results

- Electron and muon $m_T$ fits combined
  \[ m_W = 80390 \pm 20 \text{ MeV}, \frac{\chi^2}{\text{dof}} = 1.2/1 \ (28\%) \]

- Electron and muon $p_T$ fits combined
  \[ m_W = 80366 \pm 22 \text{ MeV}, \frac{\chi^2}{\text{dof}} = 2.3/1 \ (13\%) \]

- Electron and muon MET fits combined
  \[ m_W = 80416 \pm 25 \text{ MeV}, \frac{\chi^2}{\text{dof}} = 0.5/1 \ (49\%) \]

- All electron fits combined
  \[ m_W = 80406 \pm 25 \text{ MeV}, \frac{\chi^2}{\text{dof}} = 1.4/2 \ (49\%) \]

- All muon fits combined
  \[ m_W = 80374 \pm 22 \text{ MeV}, \frac{\chi^2}{\text{dof}} = 4/2 \ (12\%) \]

- All fits combined
  \[ m_W = 80387 \pm 19 \text{ MeV}, \frac{\chi^2}{\text{dof}} = 6.6/5 \ (25\%) \]
### \( p_T(l) \) Fit Systematic Uncertainties

<table>
<thead>
<tr>
<th>Systematic (MeV/( c^2 ))</th>
<th>Electrons</th>
<th>Muons</th>
<th>Common</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lepton Energy Scale</td>
<td>10</td>
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<td>Lepton Energy Resolution</td>
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### $p_T(\nu)$ Fit Systematic Uncertainties

<table>
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## Combined Fit Systematic Uncertainties

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<th>Source</th>
<th>Uncertainty (MeV)</th>
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## Systematic Uncertainties in QED Radiative Corrections

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<th>effects:</th>
<th>CDF0</th>
<th>CDF1a</th>
<th>CDF1b</th>
<th>CDFII 200pb⁻¹</th>
<th>CDFII 2.3fb⁻¹</th>
<th>DØ 1fb⁻¹</th>
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<th>CDF1a</th>
<th>CDF1b</th>
<th>CDFII 200pb⁻¹</th>
<th>CDFII 2.3fb⁻¹</th>
<th>DØ 1fb⁻¹</th>
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<td>Breit-Wigner</td>
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<td>EWK scheme</td>
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</tbody>
</table>
EM Calorimeter Uniformity

- Checking uniformity of energy scale in bins of electron pseudo-rapidity

\[ \int L \, dt = 2.2 \, \text{fb}^{-1} \]
Parton Distribution Functions

• Affect W kinematic lineshapes through acceptance cuts

• We use CTEQ6 as the default PDF

• Use ensemble of 'uncertainty' PDFs
  – Represent variations of eigenvectors in the PDF parameter space
  – Compute $\delta M_W$ contribution from each error PDF

• Using MSTW2008 PDF ensemble defined for 68% CL, obtain systematic uncertainty of 10 MeV

• Comparing CTEQ and MSTW at 90% CL, yield similar uncertainty (CTEQ is 10% larger)
  – Cross-check: default MSTW2008 relative to default CTEQ6 yields 6 MeV shift in W mass
Tracking Momentum Scale

$\gamma \rightarrow \mu\mu$ resonance provides

- Cross-check of non-beam-constrained (NBC) and beam-constrained (BC) fits
- Difference used to set additional systematic uncertainty

\[ \int L \, dt \approx 2.2 \, fb^{-1} \]

\[ \Delta p/p = (-1.185 \pm 0.02) \times 10^{-3} \]

\[ \chi^2/\text{dof} = 48 / 38 \]

BC $\gamma \rightarrow \mu\mu$ mass fit
Lepton Resolutions

- Tracking resolution parameterized in the custom simulation by
  - Radius-dependent drift chamber hit resolution $\sigma_h \sim (150 \pm 1_{\text{stat}}) \mu m$
  - Beamspot size $\sigma_b = (35 \pm 1_{\text{stat}}) \mu m$
  - Tuned on the widths of the $Z \rightarrow \mu\mu$ (beam-constrained) and $\Upsilon \rightarrow \mu\mu$ (both beam constrained and non-beam constrained) mass peaks
    
    \[ \Rightarrow \Delta M_W = 1 \text{ MeV (muons)} \]

- Electron cluster resolution parameterized in the custom simulation by
  - $12.6\% / \sqrt{E_T}$ (sampling term)
  - Primary constant term $\kappa = (0.68 \pm 0.05_{\text{stat}}) \%$
  - Secondary photon resolution $\kappa_\gamma = (7.4 \pm 1.8_{\text{stat}}) \%$
  - Tuned on the widths of the $E/p$ peak and the $Z \rightarrow ee$ peak (selecting radiative electrons)
    
    \[ \Rightarrow \Delta M_W = 4 \text{ MeV (electrons)} \]
Consistency of Radiative Material Model

- Excellent description of E/p spectrum tail

- Radiative material tune factor: $S_{X_0} = 1.026 \pm 0.003_{\text{stat}} \pm 0.002_{\text{background}}$ achieves consistency with E/p spectrum tail.
Generator-level Signal Simulation

- Generator-level input for W & Z simulation provided by RESBOS (C. Balazs & C.-P. Yuan, PRD56, 5558 (1997) and references therein), which
  - Calculates triple-differential production cross section, and $p_T$-dependent double-differential decay angular distribution
  - Calculates boson $p_T$ spectrum reliably over the relevant $p_T$ range: includes tunable parameters in the non-perturbative regime at low $p_T$

- Multiple radiative photons generated according to PHOTOS (P. Golonka and Z. Was, Eur. J. Phys. C 45, 97 (2006) and references therein)
Tracking Momentum Scale Systematics

Systematic uncertainties on momentum scale

<table>
<thead>
<tr>
<th>Source</th>
<th>$J/\psi \cdot 10^{-3}$</th>
<th>NBC-$\Upsilon \cdot 10^{-3}$</th>
<th>common $\cdot 10^{-3}$</th>
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<tbody>
<tr>
<td>QED</td>
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<td>0.045</td>
<td>0.045</td>
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<tr>
<td>B field non-uniformity</td>
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<td>0.034</td>
<td>0.032</td>
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<tr>
<td>Ionizing material</td>
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<td>0.014</td>
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<tr>
<td>Resolution</td>
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<td>0.005</td>
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<tr>
<td>Backgrounds</td>
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<tr>
<td>Misalignment</td>
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<tr>
<td>Trigger efficiency</td>
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<tr>
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<td>0.004</td>
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<tr>
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<tr>
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<tr>
<td>Total systematic</td>
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<td>0.058</td>
</tr>
<tr>
<td>Statistical</td>
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<tr>
<td>Total</td>
<td>0.092</td>
<td>0.072</td>
<td>0.058</td>
</tr>
</tbody>
</table>

$\Delta M_{W,Z} = 6 \text{ MeV}$

Uncertainty dominated by QED radiative corrections and magnetic field non-uniformity
Cross-check of COT alignment

- Cosmic ray alignment removes most deformation degrees of freedom, but “weakly constrained modes” remain
- Final cross-check and correction to beam-constrained track curvature based on difference of $<E/p>$ for positrons vs electrons
- Smooth ad-hoc curvature corrections as a function of polar and azimuthal angle: statistical errors => $\Delta M_W = 2 \text{ MeV}$