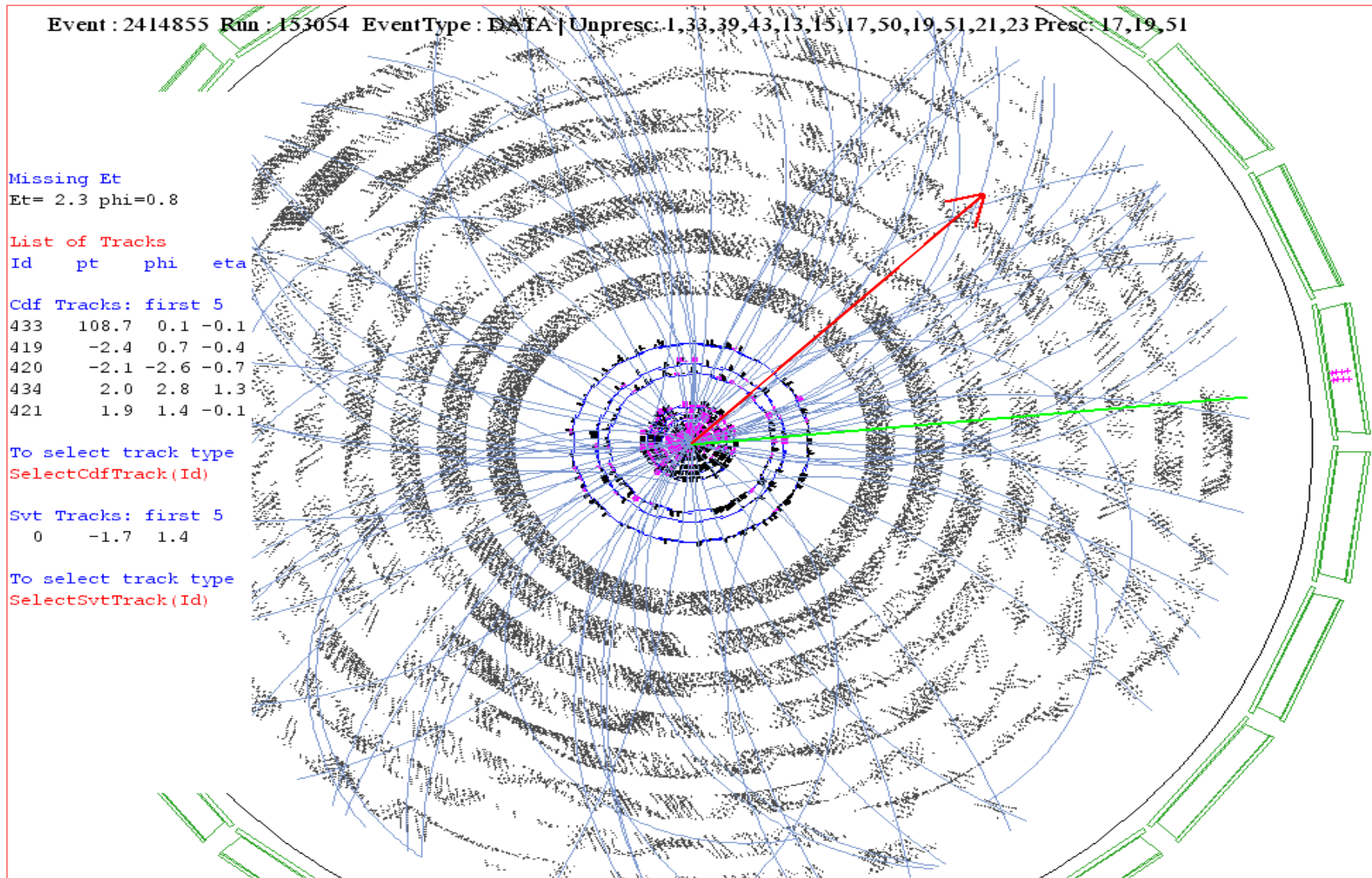


CDF Track Reconstruction

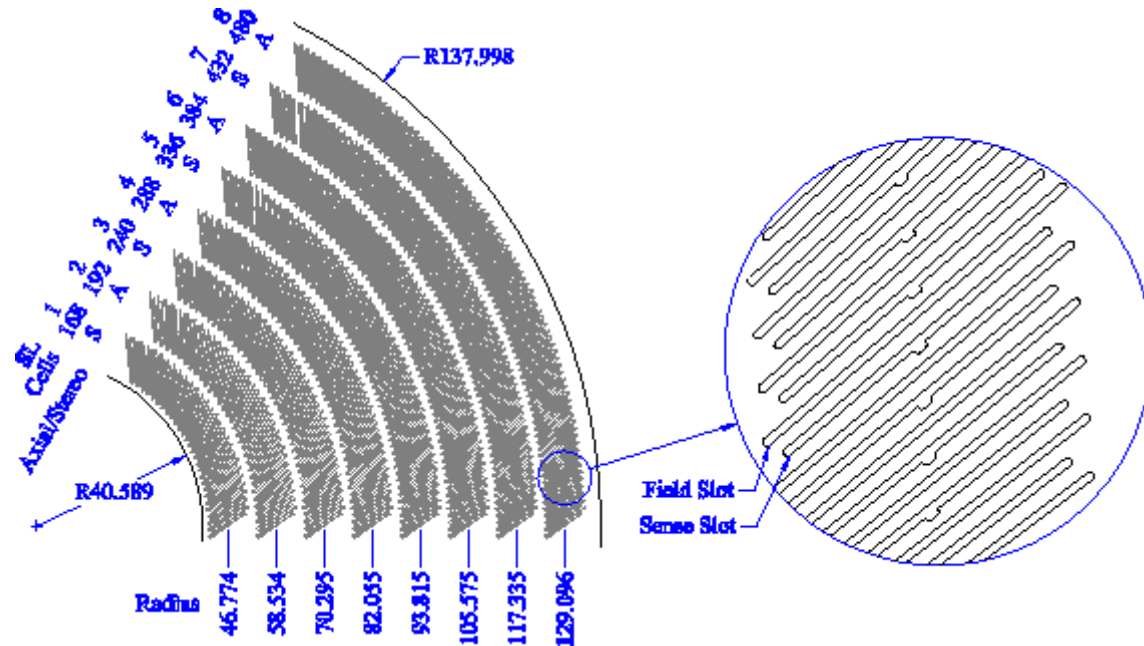
(part 1)



Chris Hays, Duke University
CDF Detector Lecture Series

The Detectors

COT

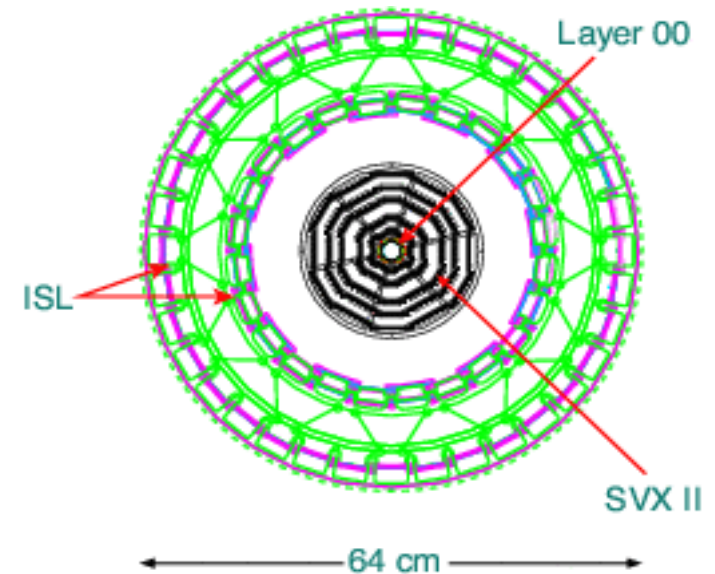


Open-cell drift chamber

*wires strung under tension
between two endplates*

8 “superlayers” of 12 wires each
maximum drift distance ~ 1 cm
alternate r - ϕ only, 2° stereo

Silicon



Double-sided wafers

r - ϕ only, small-angle stereo,
stereo only (90° z)

1 layer on beam pipe (Layer 00)
5 surrounding layers (SVXII)
1-2 outer layers (ISL)

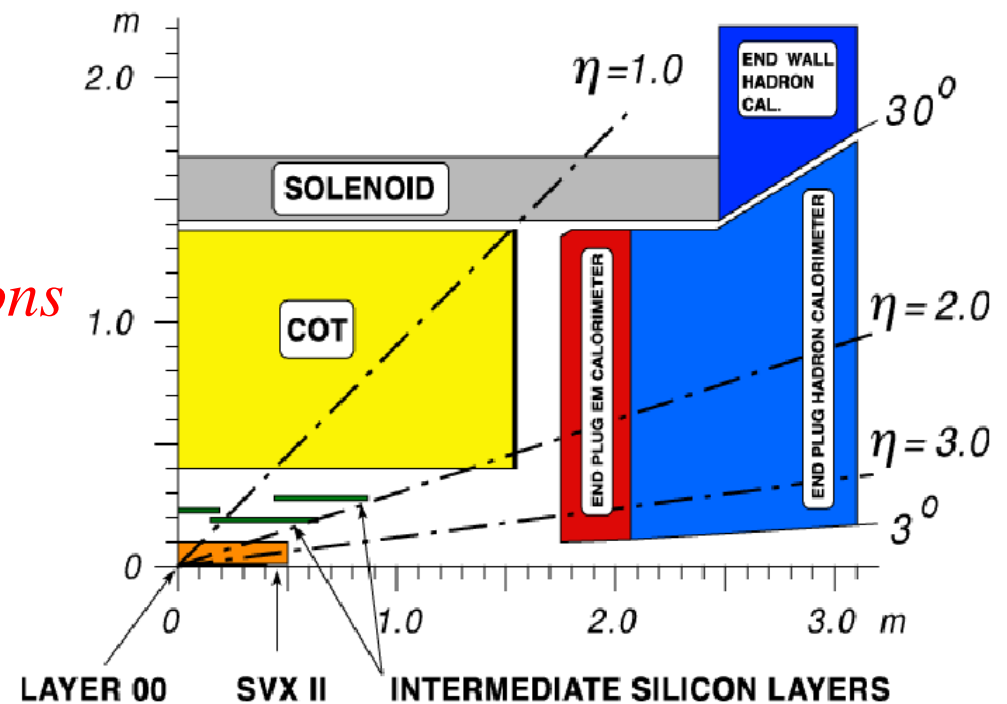
The Challenge

Integrate all tracker information into a reconstructed track collection

Accurately measure fiducial charged particles'
momentum, trajectory, origin, charge
within CPU constraints

Correct hit-track association crucial:
maintain gaussian parameter resolutions
reduce track duplication
prevent biases

Utilize all hit information:
maximize reconstruction efficiency
achieve best resolutions
reduce track duplication (split tracks)



The Algorithm

Staged algorithm to reconstruct fully fiducial tracks first, working from the outside in

- * ***COT-only pattern recognition:***

- reconstruct tracks that pass through at least half the COT (radially)

- * ***COT-seeded silicon pattern recognition (“OI”):***

- use COT tracks to search for silicon hits

- * ***z_0 - t_0 vertexing:***

- use COT-only tracks, silicon 90° hits as vertex seeds

- * ***Silicon-only pattern recognition (“SiSA”):***

- reconstruct silicon tracks with remaining hits

- * ***Silicon-seeded COT pattern recognition (“IO”):***

- attach COT hits at edge of COT fiduciality

- * ***Final track collection:***

- perform best fits, remove duplicate tracks, create quality classes

The People

COT pattern recognition and calibration:

Peter Tamburello, Kevin Burkett, Aseet Mukherjee, Peter Wittich, Paolo Gatti, Patrizia Azzi, Avi Yagil, Chris Hays, Ashutosh Kotwal, Yimei Huang, Mircea Coca

COT-seeded silicon pattern recognition and calibration:

Kurt Rinnert, Stephanie Menzemer, Weiming Yao, Ken Bloom, Rick Snider, Jason Nielsen, Matt Herndon, Aaron Dominguez, Lester Miller, Joe Boudreau

z_0 - t_0 vertexing:

Beate Heinemann, Peter Tamburello, Claudia Lecci, Chris Hays, Matt Herndon, Pasha Murat

Silicon-only pattern recognition:

Stephanie Menzemer, Thorsten Scheidle, Avi Yagil

Silicon-seeded COT pattern recognition:

Chris Hays, Yimei Huang, Ashutosh Kotwal, Heather Gerberich, Kurt Rinnert, Claudia Lecci, Stephanie Menzemer, Matt Herndon, Rick Snider

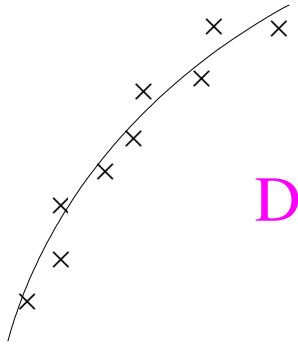
Final track collection:

Jason Nielsen, Song Ming Wang, Avi Yagil, Yimei Huang, Chris Hays, Matt Herndon

Tracking Basics

Charged particles follow helical trajectory in uniform magnetic field

* Radius of curvature determined by transverse momentum



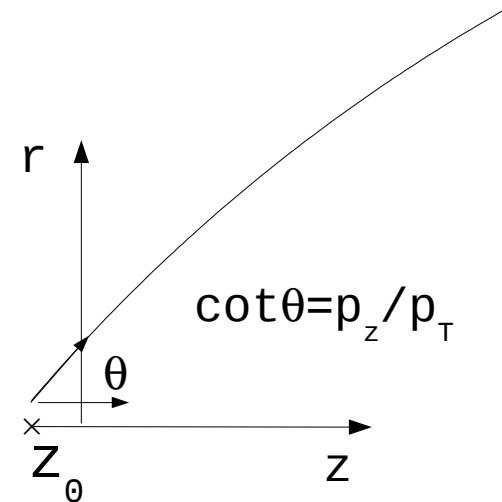
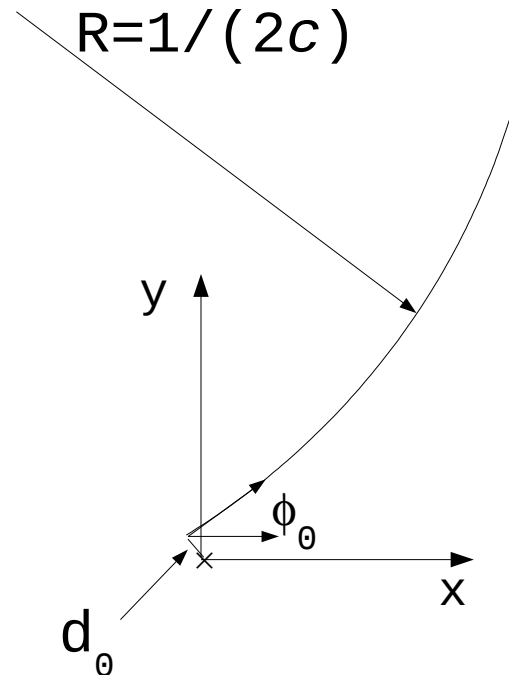
$$\mathbf{F} = q\mathbf{v}/c \times \mathbf{B} \longrightarrow p_T = q/c \times BR$$

Define curvature as $c = 1/(2R)$: $p_T = 0.002116/c \text{ GeV}$

Axial parameters (r- ϕ)

Stereo parameters (r-z)

x beam line



COT-only Pattern Recognition

C. Hays et al., CDF Note 6992

Hits → **Segments** → **Tracks**

- * *Hit corrections (calibrations)*

- * *Segment finding*

- * *Segment linking*

- * *Histogram linking*

- * *Segment linking*

- * *z_0 vertexing*

- * *Hit linking*

- * *Hit dropping*

- * *Hit adding*

- * *t_0 vertexing*

- * *t_0 constraint*

Axial track construction
two parallel algorithms

Stereo track construction
two algorithms in series

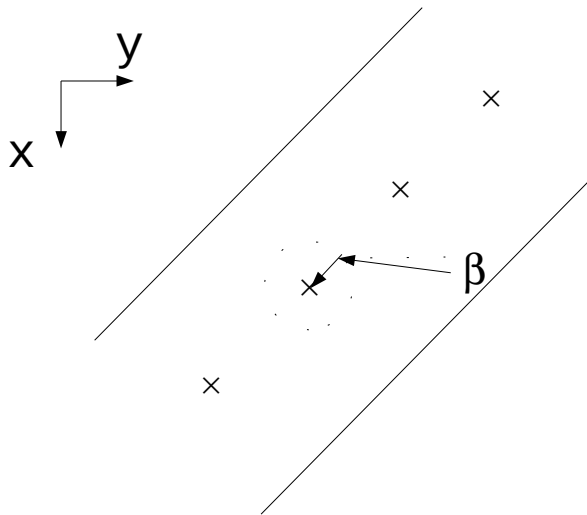
Final fit
includes track pruning and hit recovery

COT Hit Calibrations

Goal: Translate measured hit time and width into hit position

* Requires knowledge of collision time and model of charge drift

Drift model based on Garfield simulation



Constant drift velocity $v \sim 53 \mu\text{m/ns}$
beyond a radius $\rho \sim 2 \text{ mm}$ from the wire
at an angle $\beta \sim 0.1$ with respect to the horizontal

Radial drift inside a radius ρ
and an increased drift speed $v_n \sim 3 \text{ mm/ns}$
within a distance $d_n \sim 500 \mu\text{m}$

Additional “slewing” correction due to electronics rise time

More charge $\rightarrow$ faster rise, reaches threshold sooner $t_{\text{slew}} \sim 280 (1 - (1 - 1/w)^{1/2})$

Collision time, drift velocity fit each run, others assumed constant over time

COT Segment Finding

Build segments in each superlayer

- * Start with 3-hit seeds
- * Extend to full superlayer segment

Seed construction

start with inner hits in superlayer
search for hits in neighboring wires and cells

5 possible seed hit combinations

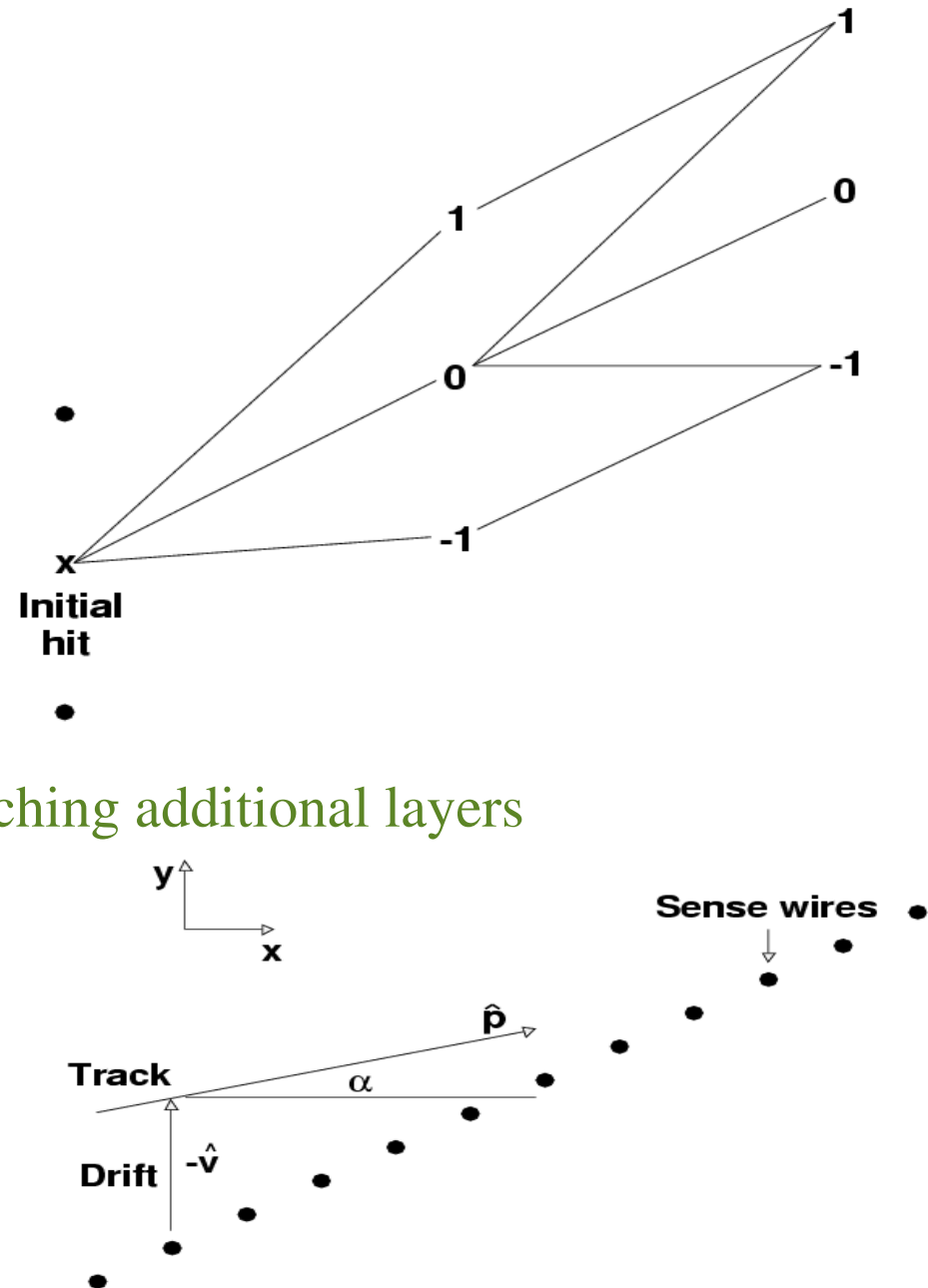
apply quality requirements to seeds before searching additional layers

Aspect angle:

angle between track direction and
direction perpendicular to drift

$$\sin \alpha = \hat{v} \cdot \hat{p}$$

radial tracks have zero aspect angle



COT Segment Finding

Aspect angle calculated from 1st, 3rd seed hits:

$$\tan \alpha = \frac{t_3 + |(\Delta t)_{\alpha=0}| - t_1}{|(\Delta t)_{\alpha=0}| / \tan \tau}$$

$\tau = 35^\circ$ (cell tilt angle), $|(\Delta t)_{\alpha=0}| = 2d \sin \tau / v$,

d : sense wire spacing (7.112 mm)

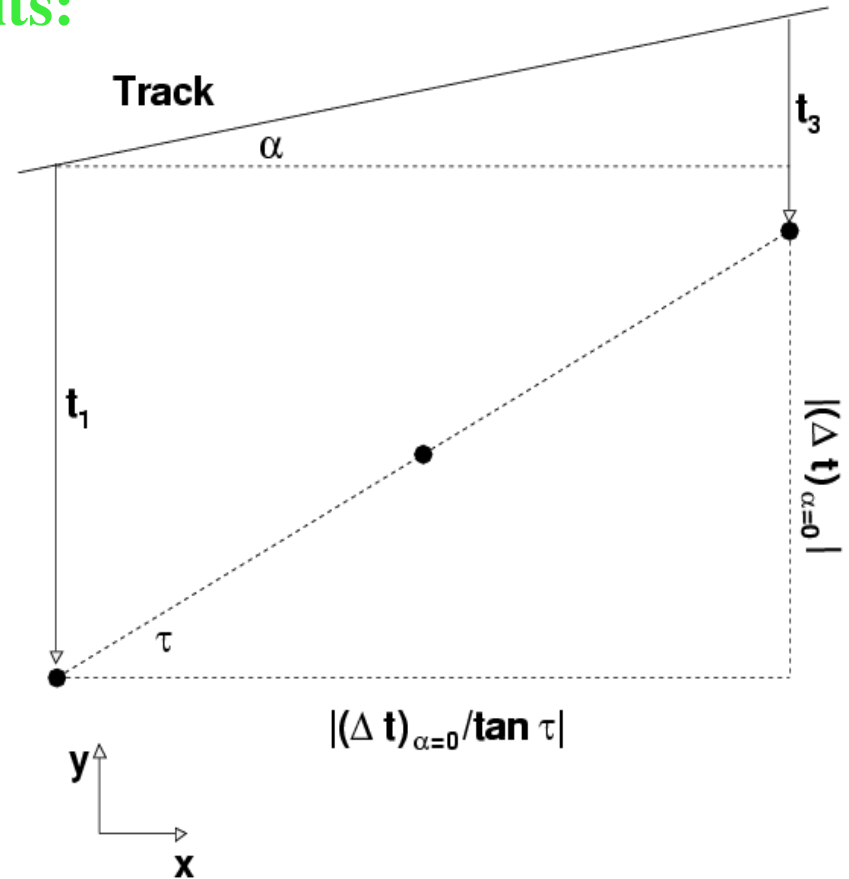
v : drift velocity

$\therefore |(\Delta t)_{\alpha=0}| \sim 150 \text{ ns}$

$\tan \alpha$ required to be $< \sim 1.3$ ($\alpha < \sim 50^\circ$)

Each hit combination has 4 possible aspect angles

* drift direction ambiguity



COT Segment Finding

Require middle hit to be consistent with a line

$$t_{\text{exp}} = |t_3 + t_1|/2 + t_0$$

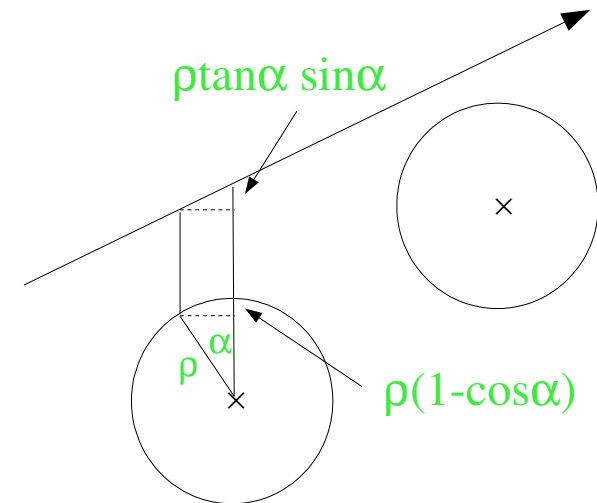
Superlayer t_0 includes:

- * calibration offset
- * average flight time
- * readout time of wires

four drift signs tested through $t_i \rightarrow -t_i$

Hits on opposite sides of wires corrected for radial drift region

$$\begin{aligned} \Delta t_{\text{exp}} &= \rho(1 - 1/\cos\alpha)/v \\ &\approx -\rho/(2v) \tan^2\alpha \end{aligned}$$



Measured t of middle hit must be:

- * within 15 ns of t_{exp} if t_1 and t_3 have same sign
- * within 25 ns otherwise (additional corrections do not cancel)

Order good seeds by increasing slope (i.e., by p_T)

COT Segment Finding

Search for hits starting from good segment seeds

Fit 3 hits to a line ($y = mx + b$) using method of least squares

Expected time on a given wire:

$$t_{\text{exp}} = \frac{mx_w + b - y_w}{v_y - mv_x}$$

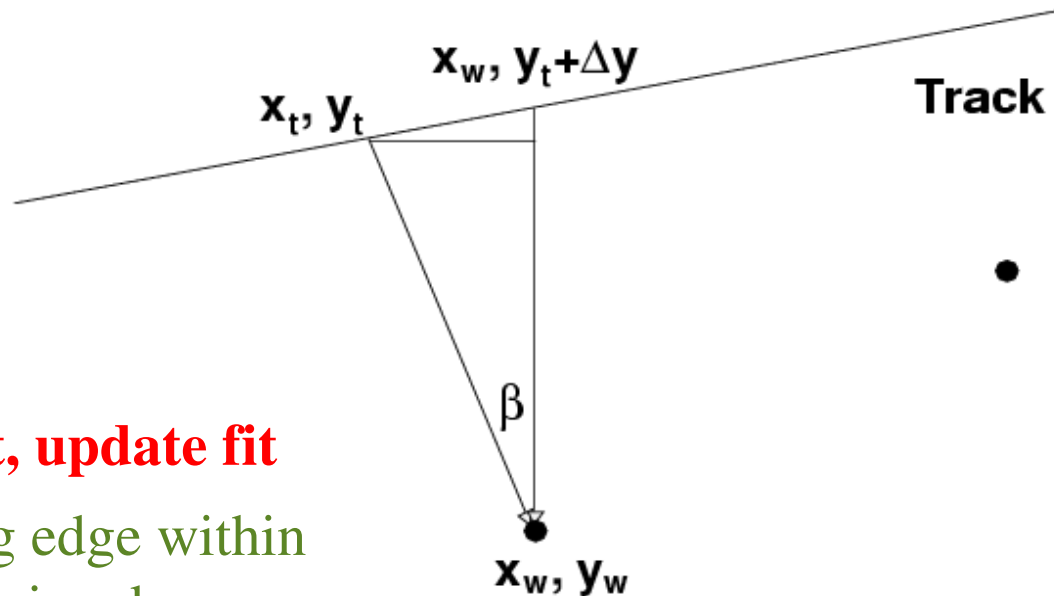
If $t_{\text{exp}} > t_{\text{max}}$, search neighboring cell

If $|t_h - t_{\text{exp}}| < 20 \text{ ns}$, add hit to segment, update fit

If no hit within 20 ns, and no hit trailing edge within 20 ns (obscured hit), mark wire as missed

Discard segment if more misses than hits at any point in the search

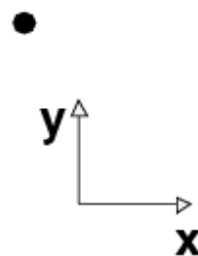
If initial wire not inner wire, search backwards through cell



$$t = (y_t - y_w) / v_y$$

$$= ((mx_w + b - \Delta y) - y_w) / v_y,$$

$$t = (x_t - x_w) / v_x = -\Delta y / (mv_x)$$



COT Segment Finding

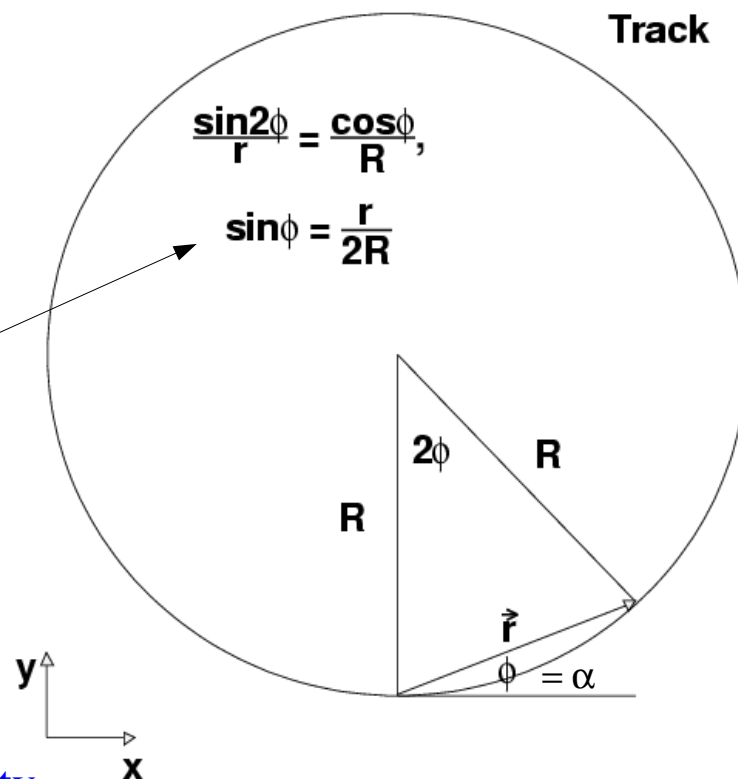
Prune wrong hits from segment

- * find residuals from best-fit line of all hits
- * remove worst hit if residual > 20 ns
- * refit and iterate until no hits have residual > 20 ns
- * perform final hit search to recover missing hits

Keep best segment

- * best segment: more hits than any other segment from the initial hit
- * mark segment's hits as used so they do not seed other segments
- * fit segment's ϕ_0 , c assuming d_0 , $\beta = 0$

$$\sin \alpha = cr$$



COT Segment Finding

Use segment position and line fit to determine curvature:

$$cr^2 = \frac{mx - y}{(1 + m^2)^{1/2}} \quad (\text{x passes through center of cell})$$

Determine ϕ_0 from

- * cell position
- * relative position of segment in cell
- * rotation from track's curvature

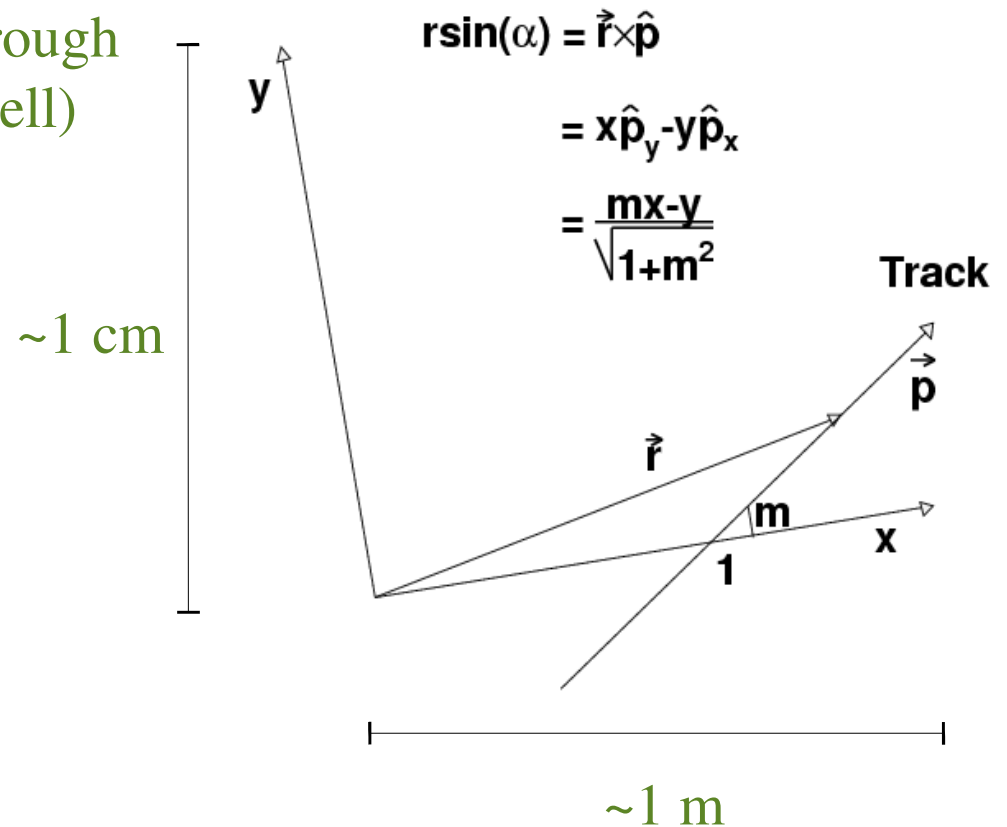
$$\phi_0 = \phi_{\text{cell}} + \tan^{-1}(y/x) - \sin^{-1}(cr)$$

Segments ordered by decreasing # hits

- * Hit dropped if used by prior segment (no shared hits)

Add “ghost” segments

- * All hits in a single cell: drift sign ambiguity



COT-only Pattern Recognition

C. Hays et al., CDF Note 6992

Hits → **Segments** → **Tracks**

- * *Hit corrections (calibrations)*

- * *Segment finding*

- * *Segment linking*

- * *Histogram linking*

- * *Segment linking*

- * z_0 vertexing

- * *Hit linking*

- * *Hit dropping*

- * *Hit adding*

- * t_0 vertexing

- * t_0 constraint

Axial track construction
two parallel algorithms

Stereo track construction
two algorithms in series

Final fit
includes track pruning and hit recovery

COT Axial Segment Linking

Link from the outer superlayer (SL8) in:

- * link to segments in SL6
- * fit all linked segments as single tracks
- * link inner SL (2,4) segments to tracks or unlinked outer segments

For each SL8 segment, calculate expected position, direction of track in SL6

start with position ($\vec{r}$) and direction ($\hat{p}$) of SL8 segment

- * obtain direction from $y = r \sin\phi = cr^2$

$$dy = 2c (x dx + y dy) \longrightarrow \frac{dy}{dx} = \frac{2cx}{1 - 2cy}$$

y-direction
x-direction

Find SL6 segment with best match in:

- * position: $l = |\vec{r}_{\text{meas}} - \vec{r}_{\text{exp}}| < l_{\text{max}}$
- * direction: $\sin^2(\Delta\theta) < \Delta_{\text{max}}^2$

COT Axial Segment Linking

position requirement:

$$|\vec{r}_{\text{meas}} \times \vec{\rho}| < 1: l < l_{\text{max}}$$

$$l_{\text{max}} = 1 \text{ cm}$$

* remove $\varepsilon \pm \pi$ ambiguity

$$\vec{r}_{\text{meas}} \cdot \vec{\rho} > 0: \cos \varepsilon > 0$$

angular requirement:

$$\sin^2(\Delta\theta) < \Delta_{\text{max}}^2$$

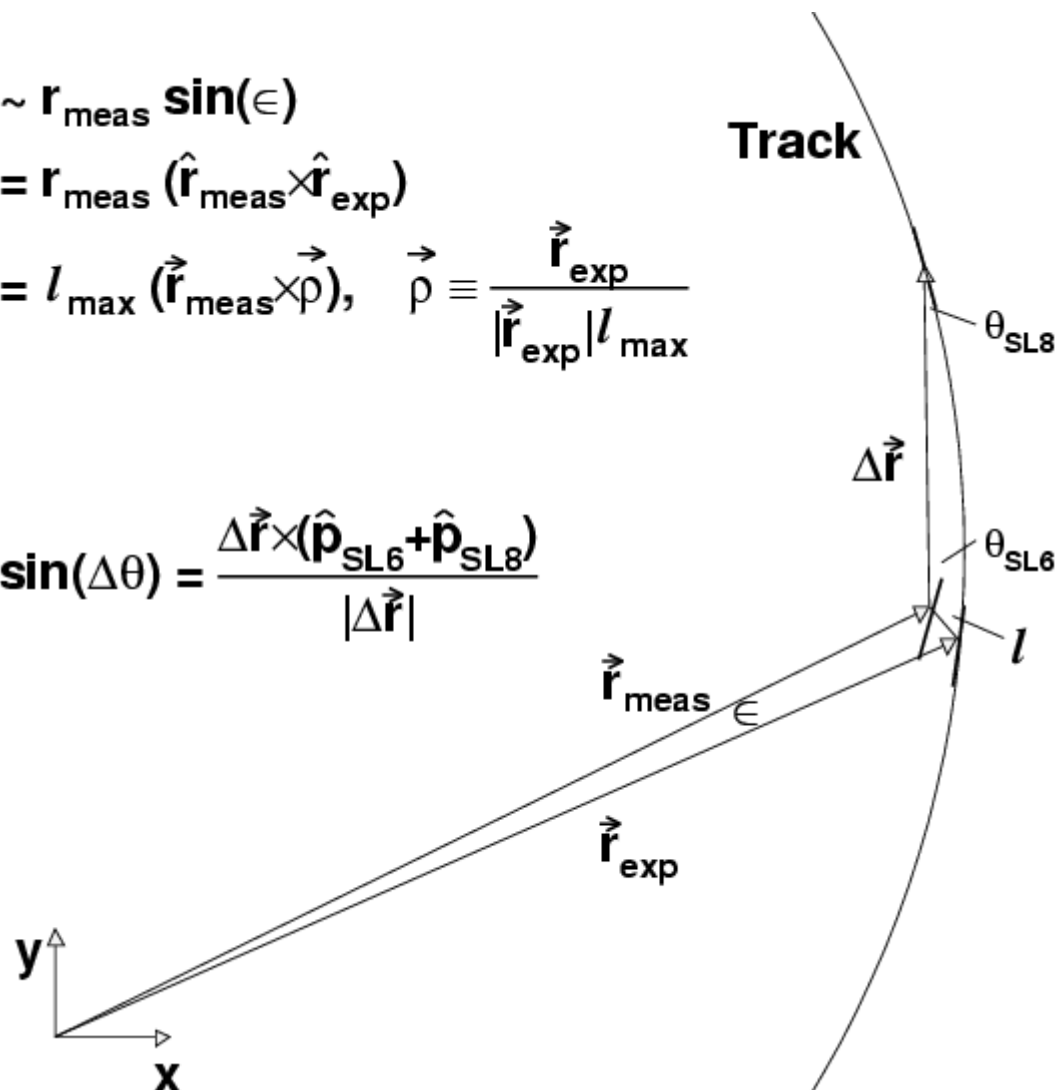
$$\Delta_{\text{max}} = 0.05$$

$$l \sim r_{\text{meas}} \sin(\varepsilon)$$

$$= r_{\text{meas}} (\hat{r}_{\text{meas}} \times \hat{r}_{\text{exp}})$$

$$= l_{\text{max}} (\vec{r}_{\text{meas}} \times \vec{\rho}), \quad \vec{\rho} \equiv \frac{\vec{r}_{\text{exp}}}{|\vec{r}_{\text{exp}}| l_{\text{max}}}$$

$$\sin(\Delta\theta) = \frac{\Delta\vec{r} \times (\hat{p}_{\text{SL6}} + \hat{p}_{\text{SL8}})}{|\Delta\vec{r}|}$$



* if multiple matches: attach segment with smallest $\sin^2(\Delta\theta)$

COT Axial Track Fit

Fit linked segments with axial parameters c, ϕ_0, d_0

* “Correct” χ^2 :

$$\chi^2 = \sum_{\text{hit}} r_{\text{hit}}^2 (\phi_{\text{hit}}^{\text{exp}} - \phi_{\text{hit}}^{\text{meas}})^2 \cos^2 \alpha / \sigma_0^2$$

Resolution varies as $\sigma_0 / \cos \alpha$

* variation assumes radial drift region dominates resolution

* “Reduced” χ^2 :

$$\chi^2 = \sum_{\text{hit}} (y_{\text{hit}}^{\text{exp}} - y_{\text{hit}}^{\text{meas}})^2$$

$$\begin{aligned} y^{\text{exp}} - y^{\text{meas}} &= r(\sin \phi^{\text{exp}} - \sin \phi^{\text{meas}}) \\ &= 2r \sin[(\phi^{\text{exp}} - \phi^{\text{meas}})/2] \\ &\quad \cos[(\phi^{\text{exp}} + \phi^{\text{meas}})/2] \\ &\approx r(\phi^{\text{exp}} - \phi^{\text{meas}}) \cos \phi \end{aligned}$$

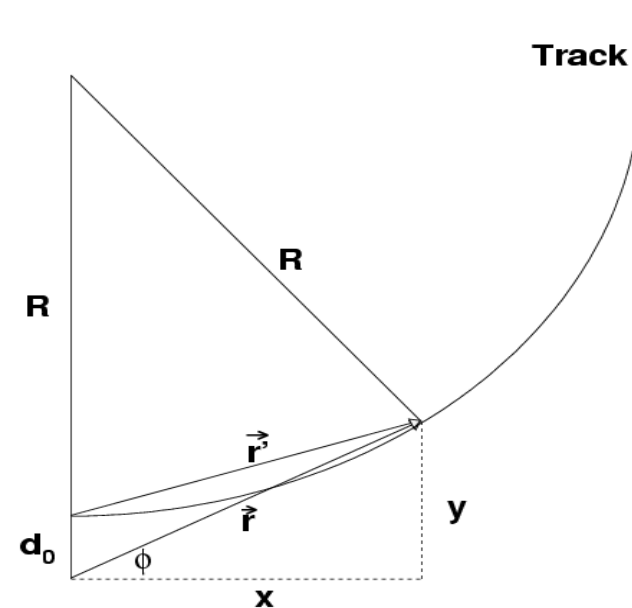
* approximates correct χ^2 for small $\Delta \phi, d_0$
(neglecting σ_0)

Geometry of non-zero d_0 :

$$r'^2 = r^2 + d_0^2 - 2yd_0 \quad (\text{Pythagoras})$$

$$R^2 = x^2 + (y - R - d_0)^2 \quad (\text{circle equation})$$

$$\longrightarrow r \sin \phi = \frac{cr^2 + d_0(1 + cd_0)}{1 + 2cd_0}$$



COT Axial Track Fit

Define parameters a and b:

$$a = \frac{c}{1 + 2cd_0}$$

$$b = d_0 - ad_0^2$$

Expected y position ($\phi_0 = 0$): $y^{\text{exp}} = ar^2 + b$

Measured hit positions:

$$x^{\text{meas}} = x'\cos\phi_0 + y'\sin\phi_0$$

$$y^{\text{meas}} = y'\cos\phi_0 - x'\sin\phi_0$$

CDF coordinates

Linearize difference between true ϕ_0 and initial estimate (ϕ_{00}): $\phi_0 = \phi_{00} + \Delta\phi$

$$x^{\text{meas}} = x^0 + y^0\Delta\phi$$

$$y^{\text{meas}} = y^0 - x^0\Delta\phi$$

3-parameter fit: $a, b, \Delta\phi$

$$\begin{pmatrix} x^0 = x'\cos\phi_{00} + y'\sin\phi_{00} \\ y^0 = y'\cos\phi_{00} - x'\sin\phi_{00} \end{pmatrix}$$

COT Axial Track Fit

Set $\vec{\nabla}\chi^2 = 0$ to obtain linear equation for fit parameters ($\mathbf{M}\vec{\mathbf{p}} = \vec{\mathbf{m}}$)

$$\begin{pmatrix} \Sigma r^4 & \Sigma r^2 x^0 & \Sigma r^2 \\ \Sigma r^2 x^0 & \Sigma (x^0)^2 & \Sigma x^0 \\ \Sigma r^2 & \Sigma x^0 & N_{\text{hits}} \end{pmatrix} \begin{pmatrix} a \\ \Delta\phi \\ b \end{pmatrix} = \begin{pmatrix} \Sigma r^2 y^0 \\ \Sigma y^0 x^0 \\ \Sigma y^0 \end{pmatrix}$$

Solve by inverting M, iterate to obtain ϕ_0

* Up to 5 iterations until $r_{\text{max}} \Delta\phi < \sigma_0$ (250 μm)

radius of wire of furthest hit

Successful fit: $4ab > 1$

* solution with d_0 closest to origin

* failed fit: set $d_0 = 0$, fit for c , ϕ_0

save covariant matrix: $\mathbf{V} = \sigma_0^2 \mathbf{M}^{-1}/2$

COT Axial Segment Linking

Link inner superlayer (SL2, SL4) segments to r - ϕ tracks

* Use covariance matrix to define an extrapolation road

l_{max} : 4σ road

σ : segment position resolution (500 μm)

$\oplus$ extrapolation uncertainty

$$\delta l_{\text{ext}} = \sum \mathbf{r}_{\text{seg}}^{(2-i)} \mathbf{r}_{\text{seg}}^{(2-j)} \mathbf{V}_{ij}$$


* Include impact parameter information when determining $\vec{p}$ direction

$$y = a r_{\text{trk}}^2 + b$$

$$(r_{\text{trk}} = (r_{\text{max}} - r_{\text{min}})/2)$$

If matching segment: update track fit and remove segment from consideration

After reaching inner SL, search outwards

* search any unattached superlayers beyond outer hit

* only search superlayers that the track passes through completely

COT Axial Histogram Linking

Hit search seeded with segment and beam positions

Histogram hits as function of curvature

- * hits on track populate same bin

Seed segments:

- * ≥ 8 hits, $c > 0.008$ ($p_T > 265$ MeV)

Reference radius for histogram

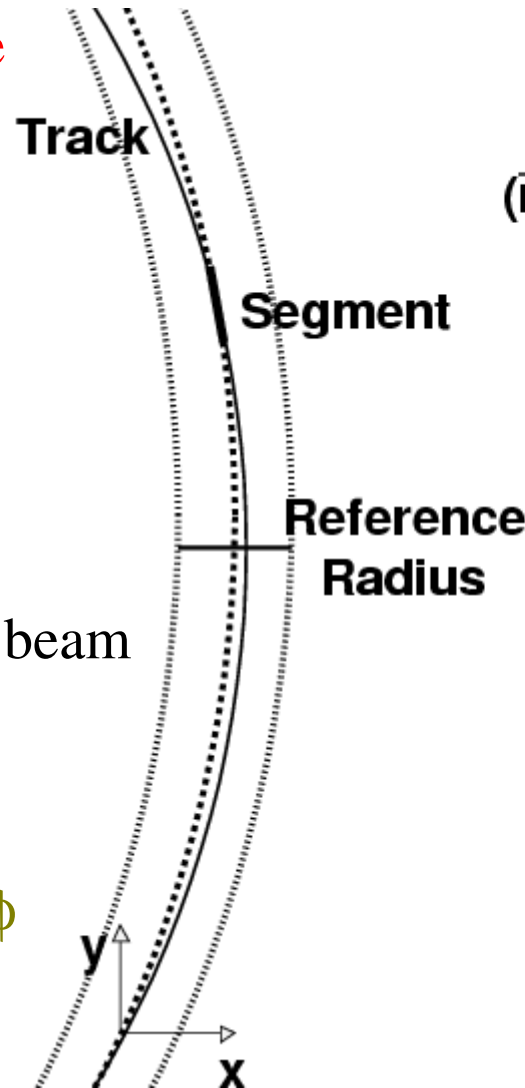
- * $r_{\text{seg}} - 20$ cm

Beam position

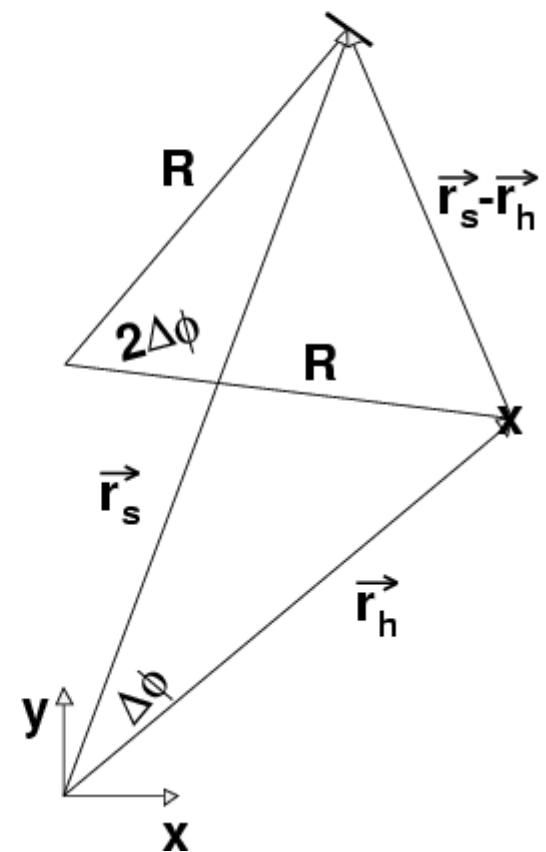
- * shift hits, reference and segment for beam

Histogram each layer's hits

- * range: $(-1, 1)$ cm, $200 \mu\text{m}$ bin size
- * center at extrapolated seed segment ϕ
- * *each hit defines a curvature:*
 - use c_h to extrapolate hit to reference radius; add to histogram



$$c = \frac{\sin(\Delta\phi)}{|\vec{r}_s - \vec{r}_h|}$$
$$(\vec{r}_s - \vec{r}_h)^2 = r_s^2 + r_h^2 - 2r_s r_h \cos(\Delta\phi)$$



COT Axial Histogram Linking

Sum histogram bins from all layers

- * bin with most hits defines track at reference radius

- * ≥ 10 hits: make track

- add seed segment hits to track (if not used by another HL track)
- add hits within $\sim 760 \mu\text{m}$ of track (defined by histogram)

Perform axial track fit

- * seed segment provides initial ϕ_0

- * keep track if successful fit and ≥ 20 hits (≥ 15 hits if seed is in SL4)

COT-only Pattern Recognition

C. Hays et al., CDF Note 6992

Hits → **Segments** → **Tracks**

- * *Hit corrections (calibrations)*

- * *Segment finding*

- * *Segment linking*

- * *Histogram linking*

- * *Segment linking*

- * z_0 vertexing

- * *Hit linking*

- * *Hit dropping*

- * *Hit adding*

- * t_0 vertexing

- * t_0 constraint

Axial track construction
two parallel algorithms

Stereo track construction
two algorithms in series

Final fit
includes track pruning and hit recovery

COT Stereo Segment Linking

Link segments to tracks starting from outer stereo superlayer (SL7)

* search superlayers within track's last axial hit and $\alpha < 45^\circ$

* search in $3 \frac{1}{2}$ cell ϕ window

Fit segments in r-z

* initial z_0 , $\cot\theta$ set to 0, iterate up to 10 times until $\Delta\cot\theta < 0.005$

* fit axes: z, path length in r- ϕ plane (L_{2D})

$$z = z_0 + L_{2D} \cot\theta$$

$$L_{2D} = 2R\phi = \sin^{-1}(cr')/c$$

$$r'^2 = \frac{r^2 - d_0^2}{1 + 2cd_0}$$

Each hit's drift time converted to a z position

* correct for flight time: $t_{\text{flight}} = [L_{2D}(1 + \cot^2\theta)^{1/2}]/c_f$

t to z conversion:

- Stereo cell rotation (6 cells endplate to endplate) produces coordinate

z-dependence: $x(z) = x(0) - y(0)\tan\Delta\phi\left(\frac{z_w}{z_{\text{max}}}\right)$ *angle subtended by 3 cells*

$y(z) = x(0)\tan\Delta\phi\left(\frac{z_w}{z_{\text{max}}}\right) + y(0)$ *wire half-length*

COT Stereo Segment Linking

Drift velocity direction changes (magnitude constant)

* write $\vec{r}(z) = \mathbf{Z}\vec{r}(0)$, then:

$$\vec{r}(z) = \mathbf{Z} \left(\vec{r}(0) + \frac{t}{|\mathbf{Z}|} \vec{v}(0) \right)$$

Linearize z position as a function of drift time: $z = z_w + \left(\frac{dz}{dt} \right)_{t=0} t + \dots$

intersection of wire with track

Defining $s \equiv \tan(\Delta\phi)/z_{\max}$, $y = ar^2 + b$ gives: $as^2r_0^2z_w^2 - sx_0z_w + ar_0^2 + b - y_0 = 0$

Since coordinate system is at $\phi_0 = 0$, $|ar_0^2 + b - y_0| \ll x_0^2/(ar_0^2)$

$$\therefore z_w \approx \frac{ar_0^2 + b - y_0}{sx_0}$$

Can now solve for dz/dt :

$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial t} = \frac{2v_{0x}sz_w + v_{0y}(1-s^2z_w^2)}{sx_0|\mathbf{Z}(z_w)|}$$

COT Stereo Segment Linking

Keeping only first order terms in sz ($sz < 0.1$):

$$z_i \approx z_w - \frac{2v_{0x}sz_w + v_{0y}t}{sx_0}$$

Iterate to improve z position estimate:

- * correct time for signal propagation
 - * determine $\vec{r}(z_i)$
- Use $|\mathbf{Z}|^{-1} = [1 - (sz_i)^2]^{-1/2} \approx 1 - (sz_i)^2/2$

- * obtain Δz in terms of x_i, y_i : $\vec{r}(z) = \mathbf{Z}(z_i + \Delta z)\mathbf{Z}^{-1}(z_i)\vec{r}(z_i) \approx \mathbf{Z}(\Delta z)\vec{r}(z_i)$

$$\Delta z \approx \frac{a r_i^2 + b - y_i}{s x_i}$$

$$z_h = z_i + \Delta z$$

L_{2D} updated with z position and hit incorporated into segment fit

COT Stereo Segment Linking

Fit segments in outer stereo SL seed segment search in next-to-outer SL

- * require outer segment to have $|z_0| < 175$ cm
- * search for segments within $\Delta\phi < 0.01$
- * keep match with best χ^2 if $\chi^2 < 100$ and $|z_0| < 175$ cm
 - z resolution: radial hit resolution (140 μm) scaled to the z resolution with dz/dt and drift speed
- * search remaining inner SL's for matches
 - match successful if $\chi^2 < 100$ and new fit has $|z_0| < 200$ cm

COT Stereo Hit Linking

Scan $\cot\theta$ for hits assuming a vertex z_0

- * histogram z_0 vertices using segment-linked tracks
- * use highest-multiplicity bin to search for hits as function of $\cot\theta$

Vertex finding algorithm:

- * define histogram within ± 160 cm, 1 cm bins
- * use prompt, quality tracks: $|d_0| < 4$ cm, $\sigma_{z0} < 3$ cm
- * determine track z position at closest approach to beam (0,0):

$$z = z_0 - d_0 \cot\theta$$

← Fill z bin, neighboring bins;
save non-zero bins as z vertices

$\cot\theta$ scan:

- * histogram z positions of all hits in SL within 8 cells of track
 - 4 bits of information for each wire in each bin
 - bit 0:* hit with leading or trailing edge within 5 cm of bin
 - bit 1:* hit with leading or trailing edge within 1 cm of bin
 - bit 2:* hit with leading edge within 1 cm of bin
 - bit 3:* hit with leading edge within 2 cm of bin

COT Stereo Hit Linking

Use hit histogram to search for best $\cot\theta$ match

- * zero bins with contributions from <6 wires in SL
- * set z_0 to highest track multiplicity vertex
- * scan $\cot\theta$ in steps of $1/L_{2D}$ (1 cm z bins)
- * search range: $z_0 - L_{2D}^{st}/L_{2D}^{ax} (155 \text{ cm} + z_0) < z < z_0 + L_{2D}^{st}/L_{2D}^{ax} (155 \text{ cm} - z_0)$

Two-dimensional path length to
center of last stereo superlayer

Two-dimensional path
length to last axial hit

$\cot\theta$ determined by number of hits within 2 cm road (bit 3)

- break any degeneracy with 1 cm road (bit 2, bit 1)
- if no hits within 2 cm road, repeat with z_0 of second highest multiplicity vertex

Use z_0 , $\cot\theta$ to search all stereo wires

- * add closest hit on each wire (within 2 cm of the expected z position)
- * fit found hits to determine best z_0 , $\cot\theta$

COT Track Fit

Perform χ^2 minimization fit on all hits attached to each track

* fit only tracks with ≥ 12 axial and ≥ 2 stereo hits

* use all relevant hit corrections:

- helix correction for magnetic field nonuniformities
- full drift model corrections

* assumed hit resolution: **140 μm**

* Taylor-expand χ^2 : $\chi^2(\vec{\mathbf{p}}) = \chi^2(\vec{\mathbf{p}}_0) + \vec{\nabla}\chi^2(\vec{\mathbf{p}}_0)\Delta\vec{\mathbf{p}} + \frac{1}{2}\Delta\vec{\mathbf{p}}\mathbf{W}(\vec{\mathbf{p}}_0)\Delta\vec{\mathbf{p}} + \dots$

initial parameters

difference between initial
and true parameters

second derivative matrix

Set gradient of χ^2 to 0: $\Delta\vec{\mathbf{p}} = -\mathbf{W}^{-1}(\vec{\mathbf{p}}_0)\vec{\nabla}\chi^2(\vec{\mathbf{p}}_0)$ ← iterate fit up to 5 times
(until $\Delta\chi^2 < 1$)

$$\chi^2 = \sum \frac{(\mathbf{D}_{\text{hit}}^{\text{exp}} - \mathbf{D}_{\text{hit}}^{\text{meas}})^2}{\sigma_{\text{hit}}^2}$$

$$\vec{\nabla}\chi^2 = 2 \sum \frac{\mathbf{D}_{\text{hit}}^{\text{exp}} - \mathbf{D}_{\text{hit}}^{\text{meas}}}{\sigma_{\text{hit}}^2} \vec{\nabla}\mathbf{D}_{\text{hit}}^{\text{exp}}$$

need derivatives of hit
distance w.r.t. parameters

COT Track Fit

Axial derivatives:

$$\frac{\partial D}{\partial \phi_0} = r, \quad \frac{\partial D}{\partial d_0} = r \frac{\partial \phi_0}{\partial d_0} = \frac{r}{x} (1 - 2ay),$$

assume azimuthal drift

$$\frac{\partial D}{\partial c} = r \frac{\partial D}{\partial c} \approx \frac{r^3}{x(1 + 2cd_0)}$$

Stereo derivatives:

$$\frac{\partial D}{\partial z_0} = -sr, \quad \frac{\partial D}{\partial \cot\theta} = -srL_{2D}$$

wire position approximations:

$$r_w = \text{const}, \quad d\phi/dz = s$$

Weight matrix:

$$\mathbf{W}_{ij} = 2 \sum \frac{\partial D}{\partial p_i} \frac{\partial D}{\partial p_j} + 2 \sum \frac{D^{\text{exp}} - D^{\text{meas}}}{\sigma_0} \frac{\partial^2 D}{\partial p_i \partial p_j}$$

expected covariance matrix
(given track parameters)

contribution from actual hits
(dropped in fit)

Use derivatives, weight matrix to iterate fit

* fit fails if parameters diverge:

$ c > 0.1$	$ z_0 > 300 \text{ cm}$
$ d_0 > 150 \text{ cm}$	$ \cot\theta > 4$

Hit Pruning and Recovery

Hit pruning:

- * intrinsic hit resolution: ~ 140 microns
- * initial window for adding hit: ~ 1 mm (20 ns)

Initial track can have hits $\sim 7\sigma$ from reconstructed track

- * likely come from other charged particles in event
- * clean up track by dropping all hits with residuals > 600 microns
 - Improves track parameter resolution and tails
 - Removes negligible number of track hits from true particle

For track $p_T \sim 1$ GeV, multiple scattering in COT increases hit resolution to ~ 200 microns

Hit recovery:

- * segment linking has inefficiencies

- wrong hit can pull segment fit, result in linking failure
- recover segments in final fit

Use best fit to search for hits in missing SL's

- * add all hits within 400 microns if find 5 or more

COT-only Pattern Recognition

C. Hays et al., CDF Note 6992

Hits → **Segments** → **Tracks**

- * *Hit corrections (calibrations)*

- * *Segment finding*

- * *Segment linking*

- * *Histogram linking*

- * *Segment linking*

- * *z_0 vertexing*

- * *Hit linking*

- * *Hit dropping*

- * *Hit adding*

- * *t_0 vertexing*

- * *t_0 constraint*

Axial track construction
two parallel algorithms

Stereo track construction
two algorithms in series

Final fit
includes track pruning and hit recovery

t_0 vertexing, silicon tracking:

→ Offline tracking part 2...

Summary

CDF has a robust COT tracking algorithm

Very high efficiency over a wide range of momenta

>99.5% for high momentum (>10 GeV),

>95% for low momentum (~ 1 GeV)

Fits within CPU constraints, <1 s/event

Correct hit-track association

segments, tracks pruned to remove stray hits

All hit information utilized:

add missing hits at the end of track reconstruction

track t_0 information incorporated (see future talk...)