Detecting New Physics through Precision Measurements

- Willis Lamb (Nobel Prize 1955) measured the difference between energies of $^2S_{1/2}$ and $^2P_{1/2}$ states of hydrogen atom
  - 4 micro electron volts difference compared to few electron volts binding energy
  - States should be degenerate in energy according to tree-level calculation

- Harbinger of vacuum fluctuations to be calculated by Feynman diagrams containing quantum loops
  - Modern quantum field theory of electrodynamics followed (Nobel Prize 1965 for Schwinger, Feynman, Tomonaga)
Parameters of Electro-Weak Interactions

- Gauge symmetries related to the electromagnetic and weak forces in the standard model, extension of QED
  - U(1) gauge group with gauge coupling $g$
  - SU(2) gauge group with gauge coupling $g'$

- And gauge symmetry-breaking via vacuum expectation value of Higgs field $\nu \neq 0$

- Another interesting phenomenon in nature: the U(1) generator and the neutral generator of SU(2) get mixed (linear combination) to yield the observed gauge bosons
  - Photon for electromagnetism
  - Z boson as one of the three gauge bosons of weak interaction
- Linear combination is given by Weinberg mixing angle $\Theta_W$
Parameters of Electro-Weak Interactions

At tree level, all of the observables can be expressed in terms of three parameters of the SM Lagrangian: \( v, g, g' \) or, equivalently, \( v, e, s \equiv \sin \theta_W \) (also \( c \equiv \cos \theta_W \))

\[
\begin{align*}
\alpha &= \frac{e^2}{4\pi}, \\
G_F &= \frac{1}{2\sqrt{2}v^2}, \\
m_Z &= \frac{ev}{\sqrt{2}sc}, \\
m_W &= \frac{ev}{\sqrt{2}s}, \\
s_{\text{eff}}^2 &= s^2
\end{align*}
\]

Radiative corrections to the relations between physical observables and Lagrangian params:

\[
\begin{align*}
m_Z^2 &= \frac{e^2v^2}{2s^2c^2} + \Pi_{ZZ}(m_Z^2) \\
m_W^2 &= \frac{e^2v^2}{2s^2} + \Pi_{WW}(m_W^2)
\end{align*}
\]

\[
G_F = \frac{1}{2\sqrt{2}v^2} \left[ 1 - \frac{\Pi_{WW}(0)}{m_W^2} + \delta_{\text{VB}} \right]
\]

\[
\begin{align*}
\mu &\rightarrow \nu_\mu \nonumber \\
W &\rightarrow e + \bar{\nu}_e \\
\mu &\rightarrow \nu_\mu \nonumber \\
\Pi_{WW} &\rightarrow e + \bar{\nu}_e + \ldots
\end{align*}
\]
Radiative Corrections to Electromagnetic Coupling

\[
\alpha = \frac{e^2}{4\pi} \left[ 1 + \lim_{q^2 \to 0} \frac{\Pi_{\gamma\gamma}(q^2)}{q^2} \right]
\]

This one is tricky: the hadronic contribution to \( \Pi'_{\gamma\gamma}(0) \) cannot be computed perturbatively.

We can however trade it for another experimental observable:

\[
R_{\text{had}}(q^2) = \frac{\sigma_{\text{had}}(q^2)}{\sigma_{\ell^+\ell^-}(q^2)}
\]

\[
\alpha(m_Z) = \frac{e^2}{4\pi} \left[ 1 + \frac{\Pi_{\gamma\gamma}(m_Z)}{m_Z^2} \right] = \frac{\alpha}{1 - \Delta\alpha(m_Z)}
\]

\[
\Delta\alpha(m_Z) = \Delta\alpha_\ell(m_Z) + \Delta\alpha_{\text{top}}(m_Z) + \Delta\alpha^{(5)}_{\text{had}}(m_Z)
\]

Calculable

\[
\Delta\alpha^{(5)}_{\text{had}}(m_Z) = -\frac{m_Z^2}{3\pi} \int_{4m_Z^2}^{\infty} \frac{R_{\text{had}}(q^2) dq^2}{q^2 (q^2 - m_Z^2)} = 0.02758 \pm 0.00035
\]

(This hadronic contribution is one of the biggest sources of uncertainty in EW studies)
Radiative Corrections to W Boson Mass

All these corrections can be combined into relations among physical observables, e.g.:

\[ m_W^2 = m_Z^2 \left[ \frac{1}{2} + \frac{1}{2} \sqrt{1 - \frac{2\sqrt{2} \pi \alpha}{G_F m_Z^2}} (1 + \Delta r) \right] \]

\( \Delta r \) can be parametrized in terms of two universal corrections and a remainder:

\[ \Delta r = \Delta \alpha(m_Z) - \frac{c^2}{s^2} \Delta \rho + \Delta r_{\text{rem}} \]

The leading corrections depend quadratically on \( m_t \) but only logarithmically on \( m_H \):

\[ \Delta \rho = \frac{\Pi_{ZZ}(0)}{m_Z^2} - \frac{\Pi_{WW}(0)}{m_W^2} \approx \frac{3 \alpha}{16 \pi c^2} \left( \frac{m_t^2}{s^2 m_Z^2} + \log \frac{m_H^2}{m_W^2} + \ldots \right) \]

\[ \frac{\delta m_W^2}{m_W^2} \approx \frac{c^2}{c^2 - s^2} \Delta \rho , \quad \delta \sin^2 \theta_{\text{eff}} \approx -\frac{c^2 s^2}{c^2 - s^2} \Delta \rho \]
Motivation for Precision Measurements

- The electroweak gauge sector of the standard model is constrained by three precisely known parameters
  - $\alpha_{EM} (M_Z) = 1 / 127.918(18)$
  - $G_F = 1.16637 (1) \times 10^{-5}$ GeV$^{-2}$
  - $M_Z = 91.1876 (21)$ GeV

- At tree-level, these parameters are related to other electroweak observables, e.g. $M_W$
  - $M_W^2 = \pi \alpha_{EM} / \sqrt{2} G_F \sin^2 \theta_W$
    - Where $\theta_W$ is the Weinberg mixing angle, defined by $\cos \theta_W = M_W/M_Z$
Motivation for Precision Measurements

- Radiative corrections due to heavy quark and Higgs loops and exotica

Motivate the introduction of the $\rho$ parameter:

$$M_W^2 = \rho [M_W(\text{tree})]^2$$

with the predictions $\Delta \rho = (\rho-1) \sim M_{\text{top}}^2$ and $\Delta \rho \sim \ln M_H$

- In conjunction with $M_{\text{top}}$, the $W$ boson mass constrains the mass of the Higgs boson, and possibly new particles beyond the standard model.
Contributions from Supersymmetric Particles

- Radiative correction depends on mass splitting ($\Delta m^2$) between squarks in SU(2) doublet

- After folding in limits on SUSY particles from direct searches, SUSY loops can contribute $\sim$100 MeV to $M_W$
Uncertainty from $\alpha_{\text{EM}}(M_Z)$

- $\delta \alpha_{\text{EM}}$ dominated by uncertainty from non-perturbative contributions: hadronic loops in photon propagator at low $Q^2$
- equivalent $\delta M_W \approx 4$ MeV for the same Higgs mass constraint
  - Was equivalent $\delta M_W \approx 15$ MeV a decade ago!
Progress on $M_{\text{top}}$ at the Tevatron

- From the Tevatron, $\Delta M_{\text{top}} = 0.9$ GeV $\Rightarrow \Delta M_H / M_H = 8\%$
- equivalent $\Delta M_W = 6$ MeV for the same Higgs mass constraint
- Current world average $\Delta M_W = 15$ MeV
  - progress on $\Delta M_W$ has the biggest impact on Higgs constraint
1998 Status of $M_W$ vs $M_{\text{top}}$

Experimental errors 68% CL:
- LEP2/Tevatron (1998)

- SM
- MSSM
- light SUSY
- heavy SUSY

$M_W = 114$ GeV
$M_H = 400$ GeV

Heinemeyer, Hollik, Stockinger, Weber, Weiglein
2012 Status of $M_W$ vs $M_{\text{top}}$
Motivation

- Generic parameterization of new physics contributing to W and Z boson self-energies through radiative corrections in propagators


![Graph showing $\Pi_{WW}(q^2)$ and $\Pi_{ZZ}(q^2)$ with $S+U \sim$ slope and $S \sim$ slope]
Motivation

- Generic parameterization of new physics contributing to W and Z boson self-energies: \( S, T, U \) parameters (Peskin & Takeuchi)

Additionally, \( M_W \) is the only measurement which constrains \( U \)

\[ M_H \sim 120 \text{ GeV} \]
\[ M_H > 600 \text{ GeV} \]

\( M_W \) and Asymmetries are the most powerful observables in this parameterization

(from P. Langacker, 2012)
Asymmetries definable in electron-positron scattering sensitive to Weinberg mixing angle $\vartheta_W$

- Higgs and Supersymmetry also contribute radiative corrections to $\vartheta_W$ via quantum loops

$A_{FB}$ is the angular (forward – backward) asymmetry of the final state

$A_{LR}$ is the asymmetry in the total scattering probability for different polarizations of the initial state
**W Boson Production at the Tevatron**

Quark-antiquark annihilation dominates (80%)

Lepton $p_T$ carries most of $W$ mass information, can be measured precisely (achieved 0.01%)

Initial state QCD radiation is $O(10 \text{ GeV})$, measure as soft 'hadronic recoil' in calorimeter (calibrated to ~0.5%)

Pollutes $W$ mass information, fortunately $p_T(W) \ll M_W$
W Boson Production at the Tevatron

Quark → Gluons → Lepton
Antiquark

Quark-antiquark annihilation dominates (80%)

Lepton $p_T$ carries most of $W$ mass information, can be measured precisely (achieved 0.01%)

Initial state QCD radiation is $O(10 \text{ GeV})$, measure as soft 'hadronic recoil' in calorimeter (calibrated to $\sim 0.5\%$)
Pollutes $W$ mass information, fortunately $p_T(W) \ll M_W$
D0 Detector at Fermilab

Electromagnetic Calorimeter measures electron energy
Hadronic calorimeters measure recoil particles

Scintillator fiber tracker provides lepton track direction

Electromagnetic Calorimeter measures electron energy
Hadronic calorimeters measure recoil particles
Quadrant of Collider Detector at Fermilab (CDF)

Select W and Z bosons with central ($|\eta| < 1$) leptons

EM calorimeter provides precise electron energy measurement

Drift chamber provides precise lepton track momentum measurement

Calorimeters measure hadronic recoil particles
Collider Detector at Fermilab (CDF)

- Muon detector
- Central hadronic calorimeter
- Drift chamber tracker (COT)
W mass measurement – decay kinematics

- Main complication: invariant mass cannot be reconstructed from 2-body leptonic decay mode
  - Because neutrino is not detectable directly
- Exploit the “Jacobian edge” in lepton transverse momentum spectrum

\[
\frac{d\sigma}{d\cos\hat{\theta}} = \sigma_0(\hat{s}) \left[ \frac{1}{2} (1 + \cos\hat{\theta})^2 + \frac{1}{2} (1 - \cos\hat{\theta})^2 \right]
\]

\[= \sigma_0(\hat{s})(1 + \cos^2\hat{\theta})\]

W boson rest frame
W mass measurement – decay kinematics

- Main complication: invariant mass cannot be reconstructed from 2-body leptonic decay mode
  - Because neutrino is not detectable directly
- Exploit the “Jacobian edge” in lepton transverse momentum spectrum

\[
\frac{d\sigma}{dp_T} = \frac{d\sigma}{d((m_W/2) \sin \hat{\theta})} \\
= \frac{2}{m_W} \frac{d\sigma}{d\sin \hat{\theta}} \\
= \frac{2}{m_W} \frac{d\sigma}{d\cos \hat{\theta}} \left| \frac{d\cos \hat{\theta}}{d\sin \hat{\theta}} \right| \\
= \frac{2}{m_W} \sigma_0(\hat{s})(1 + \cos^2 \theta) |\tan \hat{\theta}| \\
= \sigma_0(\hat{s}) \frac{4p_T}{m_W^2} (2 - 4p_T^2/m_W^2) \left( \frac{1}{\sqrt{1 - 4p_T^2/m_W^2}} \right)
\]
W mass measurement – decay kinematics

- Main complication: invariant mass cannot be reconstructed from 2-body leptonic decay mode
  - Because neutrino is not detectable directly
- Exploit the “Jacobian edge” in lepton transverse momentum spectrum

We can transfer $\frac{d\sigma}{dp_T}$ to $\frac{d\sigma}{dm_T}$ by using $m_T = 2p_T$:

$$\frac{d\sigma}{dm_T} = \frac{1}{2} \frac{d\sigma}{dp_T}$$

$$= \sigma_0(\hat{s}) \frac{m_T}{m_W} (2 - \frac{m_T^2}{m_W^2}) \left( \frac{1}{\sqrt{1 - m_T^2/m_W^2}} \right)$$
W mass measurement – decay kinematics

- Lepton transverse momentum not invariant under transverse boost
- But measurement resolution on leptons is good

Black curve: truth level, no $p_T(W)$

Blue points: detector-level with lepton resolution and selection, But no $p_T(W)$

Shaded histogram: with $p_T(W)$
**W mass measurement – decay kinematics**

- Define “transverse mass” → approximately invariant under transverse boost
- But measurement resolution of “neutrino” is not as good due to recoil

\[
m_T = \sqrt{\left( E_T^l + E_T^\nu \right)^2 - \left( \vec{p}_T^l + \vec{p}_T^\nu \right)^2} = \sqrt{2 p_T^l p_T^\nu (1 - \cos \Delta \phi)}
\]
CDF Event Selection

- **Goal:** Select events with high $p_T$ leptons and small hadronic recoil activity
  - to maximize $W$ mass information content and minimize backgrounds
- **Inclusive lepton triggers:** loose lepton track and muon stub / calorimeter cluster requirements, with lepton $p_T > 18$ GeV
  - Kinematic efficiency of trigger $\sim 100\%$ for offline selection
- **Offline selection requirements:**
  - Electron cluster $E_T > 30$ GeV, track $p_T > 18$ GeV
  - Muon track $p_T > 30$ GeV
  - Loose identification requirements to minimize selection bias
- **$W$ boson event selection:** one selected lepton, $|u| < 15$ GeV & $p_T(\nu) > 30$ GeV
  - $Z$ boson event selection: two selected leptons
CDF W & Z Data Samples

<table>
<thead>
<tr>
<th>Sample</th>
<th>Candidates</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W \rightarrow e\nu$</td>
<td>470126</td>
</tr>
<tr>
<td>$W \rightarrow \mu\nu$</td>
<td>624708</td>
</tr>
<tr>
<td>$Z \rightarrow e^+e^-$</td>
<td>16134</td>
</tr>
<tr>
<td>$Z \rightarrow \mu^+\mu^-$</td>
<td>59738</td>
</tr>
</tbody>
</table>

- **Integrated Luminosity** (collected between February 2002 – August 2007):
  - Electron and muon channels: $L = 2.2 \text{ fb}^{-1}$
  - Identical running conditions for both channels, guarantees cross-calibration
- **Event selection gives fairly clean samples**
  - Mis-identification backgrounds $\sim 0.5\%$
Analysis Strategy
Maximize the number of internal constraints and cross-checks

Driven by two goals:

1) Robustness: constrain the same parameters in as many different ways as possible

2) Precision: combine independent measurements after showing consistency
Outline of Analysis

Energy scale measurements drive the $W$ mass measurement

- **Tracker Calibration**
  - alignment of the COT ($\sim$2400 cells) using cosmic rays
  - COT momentum scale and tracker non-linearity constrained using $J/\psi \rightarrow \mu\mu$ and $\Upsilon \rightarrow \mu\mu$ mass fits
  - Confirmed using $Z \rightarrow \mu\mu$ mass fit

- **EM Calorimeter Calibration**
  - COT momentum scale transferred to EM calorimeter using a fit to the peak of the $E/p$ spectrum, around $E/p \sim 1$
  - Calorimeter energy scale confirmed using $Z \rightarrow ee$ mass fit

- **Tracker and EM Calorimeter resolutions**

- **Hadronic recoil modelling**
  - Characterized using $p_T$-balance in $Z \rightarrow ll$ events
Drift Chamber (COT) Alignment

COT endplate geometry
Reconstruction of particle trajectories, calibration to \(\sim 2 \, \mu\text{m} \) accuracy:


C. Hays et al, NIM A538, 249 (2005)
Internal Alignment of COT

- Use a clean sample of $\sim400k$ cosmic rays for cell-by-cell internal alignment

- Fit COT hits on both sides simultaneously to a single helix (AK, H. Gerberich and C. Hays, NIMA 506, 110 (2003))
  - Time of incidence is a floated parameter in this 'dicosmic fit'
Residuals of COT cells after alignment

Final relative alignment of cells ~2 μm (initial alignment ~50 μm)
Cross-check of COT alignment

- Cosmic ray alignment removes most deformation degrees of freedom, but “weakly constrained modes” remain.
- Final cross-check and correction to beam-constrained track curvature based on difference of $<E/p>$ for positrons vs electrons.
- Smooth ad-hoc curvature corrections as a function of polar and azimuthal angle: statistical errors $\Rightarrow \Delta M_W = 2 \text{ MeV}$.
Signal Simulation and Fitting
Signal Simulation and Template Fitting

- All signals simulated using a Custom Monte Carlo
  - Generate finely-spaced templates as a function of the fit variable
  - Perform binned maximum-likelihood fits to the data
- Custom fast Monte Carlo makes smooth, high statistics templates
  - And provides analysis control over key components of the simulation

\[ L = \prod_{i=1}^{N} \frac{e^{-m_i} m_i^{n_i}}{n_i!} \]

- We will extract the W mass from six kinematic distributions: Transverse mass, charged lepton \( p_T \) and missing \( E_T \) using both electron and muon channels

\[ M_W = 80 \text{ GeV} \]

\[ M_W = 81 \text{ GeV} \]
Generator-level Signal Simulation

- Generator-level input for W & Z simulation provided by RESBOS (C. Balazs & C.-P. Yuan, PRD56, 5558 (1997) and references therein), which
  - Calculates triple-differential production cross section, and $p_T$-dependent double-differential decay angular distribution
  - Calculates boson $p_T$ spectrum reliably over the relevant $p_T$ range: includes tunable parameters in the non-perturbative regime at low $p_T$

- Multiple radiative photons generated according to PHOTOS (P. Golonka and Z. Was, Eur. J. Phys. C 45, 97 (2006) and references therein)
Constraining Boson $p_T$ Spectrum

- Fit the non-perturbative parameter $g_2$ and QCD coupling $\alpha_s$ in RESBOS to $p_T(ll)$ spectra:

$$\Delta M_W = 5 \, \text{MeV}$$

Position of peak in boson $p_T$ spectrum depends on $g_2$

Tail to peak ratio depends on $\alpha_s$
Outline of Analysis

Energy scale measurements drive the $W$ mass measurement

- **Tracker Calibration**
  - alignment of the COT ($\sim 2400$ cells) using cosmic rays
  - COT momentum scale and tracker non-linearity constrained using $J/\psi \rightarrow \mu\mu$ and $Y \rightarrow \mu\mu$ mass fits
    - Confirmed using $Z \rightarrow \mu\mu$ mass fit

- **EM Calorimeter Calibration**
  - COT momentum scale transferred to EM calorimeter using a fit to the peak of the $E/p$ spectrum, around $E/p \sim 1$
    - Calorimeter energy scale confirmed using $Z \rightarrow ee$ mass fit

- **Tracker and EM Calorimeter resolutions**

- **Hadronic recoil modelling**
  - Characterized using $p_T$-balance in $Z \rightarrow ll$ events
Custom Monte Carlo Detector Simulation

- A complete detector simulation of all quantities measured in the data
- First-principles simulation of tracking
  - Tracks and photons propagated through a high-resolution 3-D lookup table of material properties for silicon detector and COT
  - At each material interaction, calculate
    - Ionization energy loss according to detailed formulae and Landau distribution
    - Generate bremsstrahlung photons down to 0.4 MeV, using detailed cross section and spectrum calculations
    - Simulate photon conversion and Compton scattering
    - Propagate bremsstrahlung photons and conversion electrons
    - Simulate multiple Coulomb scattering, including non-Gaussian tail
  - Deposit and smear hits on COT wires, perform full helix fit including optional beam-constraint
Custom Monte Carlo Detector Simulation

- A complete detector simulation of all quantities measured in the data
- First-principles simulation of tracking
  - Tracks and photons propagated through a high-resolution 3-D lookup table of material properties for silicon detector and COT
3-D Material Map in Simulation

- Built from detailed construction-level knowledge of inner tracker: silicon ladders, bulkheads, port-cards etc.

- Tuned based on studies of inclusive photon conversions

- Radiation lengths vs \((\phi, z)\) at different radii shows localized nature of material distribution

- Include dependence on type of material via Landau-Pomeranchuk-Migdal suppression of soft bremsstrahlung
Tracking Momentum Scale
Tracking Momentum Scale

Set using $J/\psi \rightarrow \mu\mu$ and $\Upsilon \rightarrow \mu\mu$ resonance and $Z \rightarrow \mu\mu$ masses

- Extracted by fitting $J/\psi$ mass in bins of $1/p_T(\mu)$, and extrapolating momentum scale to zero curvature
- $J/\psi \rightarrow \mu\mu$ mass independent of $p_T(\mu)$ after 4% tuning of energy loss

\[
\int L \, dt = 2.2 \, \text{fb}^{-1}
\]

Scale correction = \((\mathbf{-1.299 \pm 0.022}) \times 10^{-3}\)

Slope = \((\mathbf{0.8 \pm 6.4}) \times 10^{-5} \, \text{GeV}\)

Default energy loss * 1.04

\[
\Delta p/p = (-1.284 \pm 0.024_{\text{stat}}) \times 10^{-3}
\]

\[
\chi^2/\text{dof} = 95/86
\]

$J/\psi \rightarrow \mu\mu$ mass fit (bin 5)
Tracking Momentum Scale

$\gamma \rightarrow \mu \mu$ resonance provides

- Momentum scale measurement at higher $p_T$

$$\int L \, dt = 2.2 \text{ fb}^{-1}$$

$\Delta p/p = (-1.335 \pm 0.025_{\text{stat}}) \times 10^{-3}$

$\chi^2/\text{dof} = 59/48$

$\gamma \rightarrow \mu \mu$

mass fit
Using the $J/\psi$ and $\Upsilon$ momentum scale, performed “blinded” measurement of $Z$ mass

- $Z$ mass consistent with PDG value (91188 MeV) (0.7$\sigma$ statistical)

\[ M_Z = 91180 \pm 12_{\text{stat}} \pm 9_{\text{momentum}} \pm 5_{\text{QED}} \pm 2_{\text{alignment}} \text{ MeV} \]

\[ \int L \, dt = 2.2 \text{ fb}^{-1} \]

\[ M_{\mu\mu} = (91180 \pm 12_{\text{stat}}) \text{ MeV} \]

\[ \chi^2/\text{dof} = 30 / 30 \]
Tracker Linearity Cross-check & Combination

- Final calibration using the $J/\psi$, $\Upsilon$ and $Z$ bosons for calibration

- Combined momentum scale correction:

\[
\frac{\Delta p}{p} = \left( -1.29 \pm 0.07_{\text{independent}} \pm 0.05_{\text{QED}} \pm 0.02_{\text{align}} \right) \times 10^{-3}
\]

\[
\int L \, dt = 2.2 \, \text{fb}^{-1}
\]

\[
\Delta M_W = 7 \, \text{MeV}
\]
EM Calorimeter Response
Calorimeter Simulation for Electrons and Photons

- Distributions of lost energy calculated using detailed GEANT4 simulation of calorimeter
  - Leakage into hadronic calorimeter
  - Absorption in the coil
  - Dependence on incident angle and $E_T$

- Energy-dependent gain (non-linearity) parameterized and fit from data

- Energy resolution parameterized as fixed sampling term and tunable constant term
  - Constant terms are fit from the width of $E/p$ peak and $Z \rightarrow ee$ mass peak
EM Calorimeter Scale

- E/p peak from $W \rightarrow \nu \tau$ decays provides measurements of EM calorimeter scale and its ($E_T$-dependent) non-linearity

$\Delta S_E = (9_{\text{stat}} \pm 5_{\text{non-linearity}} \pm 5_{\text{x0}} \pm 9_{\text{Tracker}}) \times 10^{-5}$

Setting $S_E$ to 1 using E/p calibration from combined $W \rightarrow \nu \tau$ and $Z \rightarrow \nu \tau$ samples

$\int L \, dt = 2.2 \text{ fb}^{-1}$

Tail of E/p spectrum used for tuning model of radiative material

$\Delta M_W = 13 \text{ MeV}$
Measurement of EM Calorimeter Non-linearity

- Perform $E/p$ fit-based calibration in bins of electron $E_T$
- GEANT-motivated parameterization of non-linear response:
  \[ S_E = 1 + \beta \log(E_T / 39 \text{ GeV}) \]
- Tune on W and Z data: $\beta = (5.2 \pm 0.7_{\text{stat}}) \times 10^{-3}$

\[ \Rightarrow \Delta M_W = 4 \text{ MeV} \]
Z→ee Mass Cross-check and Combination

- Performed “blind” measurement of Z mass using E/p-based calibration
  - Consistent with PDG value (91188 MeV) within 1.4σ (statistical)
  - $M_Z = 91230 \pm 30_{\text{stat}} \pm 10_{\text{calorimeter}} \pm 8_{\text{momentum}} \pm 5_{\text{QED}} \pm 2_{\text{alignment}}$ MeV

- Combine E/p-based calibration with Z→ee mass for maximum precision

$\Delta M_W = 10$ MeV
Hadronic Recoil Model
Constraining the Hadronic Recoil Model

Exploit similarity in production and decay of $W$ and $Z$ bosons

Detector response model for hadronic recoil tuned using $p_T$-balance in $Z \rightarrow ll$ events

Transverse momentum of Hadronic recoil ($u$) calculated as 2-vector-sum over calorimeter towers
Hadronic Recoil Simulation

Recoil momentum 2-vector \( \mathbf{u} \) has

- a soft 'spectator interaction' component, randomly oriented
  - Modelled using minimum-bias data with tunable magnitude

- A hard 'jet' component, directed opposite the boson \( p_T \)
  - \( p_T \)-dependent response and resolution parameterizations
  - Hadronic response \( R = \frac{\mathbf{u}_{\text{reconstructed}}}{\mathbf{u}_{\text{true}}} \) parameterized as a logarithmically increasing function of boson \( p_T \) motivated by \( Z \) boson data

\[
\int L \, dt \sim 2.2 \text{ fb}^{-1}
\]
Tuning Recoil Response Model with $Z$ events

Project the vector sum of $p_T(\ell\ell)$ and $u$ on a set of orthogonal axes defined by boson $p_T$

Mean and rms of projections as a function of $p_T(\ell\ell)$ provide information on hadronic model parameters

$$\int L \, dt = 2.2 \text{ fb}^{-1}$$

$$\chi^2 / \text{DoF} = 8 / 9$$

Hadronic model parameters tuned by minimizing $\chi^2$ between data and simulation

$$\Delta M_W = 4 \text{ MeV}$$
Tuning Recoil Resolution Model with \( Z \) events

At low \( p_T(Z) \), \( p_T \)-balance constrains hadronic resolution due to underlying event.

\[ \Delta M_W = 4 \text{ MeV} \]

At high \( p_T(Z) \), \( p_T \)-balance constrains jet resolution.

\[ \chi^2 / \text{DoF} = 8.9 / 9 \]

\[ \int L \, dt \approx 2.2 \text{ fb}^{-1} \]
Testing Hadronic Recoil Model with $W$ events

Compare recoil distributions between simulation and data

Recoil projection (GeV) on lepton direction

Recoil projection (GeV) perpendicular to lepton direction
Testing Hadronic Recoil Model with $W$ events

Recoil model validation plots confirm the consistency of the model.

CDF II

Data

Simulation

mu = 5.912 ± 0.005 GeV

sigma = 3.522 ± 0.004 GeV

CDF II

Data

Simulation

p_T(W), electron channel

mu = 5.918 GeV

sigma = 3.522 GeV

CDF II

Data

Simulation

p_T(W), muon channel

u (recoil)
Parton Distribution Functions

- Affect $W$ kinematic lineshapes through acceptance cuts
- In the rest frame, $p_T = m \sin \theta^* / 2$
- Longitudinal cuts on lepton in the lab frame sculpt the distribution of $\theta^*$, hence biases the distribution of lepton $p_T$
  - Relationship between lab frame and rest frame depends on the boost of the $W$ boson along the beam axis
- Parton distribution functions control the longitudinal boost
- Uncertainty due to parton distribution functions evaluated by fitting pseudo-experiments (simulated samples with the same statistics and selection as data) with varied parton distribution functions
  - Current uncertainty 10 MeV
  - Largest source of systematic uncertainty
  - Expected to reduce with lepton and boson rapidity measurements at Tevatron and LHC
W Mass Fits
Blind Analysis Technique

- All W and Z mass fit results were blinded with a random [-75,75] MeV offset hidden in the likelihood fitter.
- Blinding offset removed after the analysis was declared frozen.
- Technique allows to study all aspects of data while keeping Z mass and W mass result unknown within 75 MeV.
$W$ Transverse Mass Fit

CDF II

\[ \int L \, dt \approx 2.2 \, \text{fb}^{-1} \]

Muons

Data

Simulation

$M_W = (80379 \pm 16_{\text{stat}}) \, \text{MeV}$

$\chi^2/\text{dof} = 58/48$
$W$ Mass Fit using Lepton $p_T$

CDF II

$\int L \, dt = 2.2 \text{ fb}^{-1}$

$M_W = (80393 \pm 21_{\text{stat}}) \text{ MeV}$

$\chi^2/\text{dof} = 60 / 62$

Events / 0.25 GeV

Electrons

Data

Simulation
## Summary of $W$ Mass Fits

<table>
<thead>
<tr>
<th>Charged Lepton</th>
<th>Kinematic Distribution</th>
<th>Fit Result (MeV)</th>
<th>$\chi^2$/DoF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electron</td>
<td>Transverse mass</td>
<td>80408 ± 19</td>
<td>52/48</td>
</tr>
<tr>
<td>Electron</td>
<td>Charged lepton $p_T$</td>
<td>80393 ± 21</td>
<td>60/62</td>
</tr>
<tr>
<td>Electron</td>
<td>Neutrino $p_T$</td>
<td>80431 ± 25</td>
<td>71/62</td>
</tr>
<tr>
<td>Muon</td>
<td>Transverse mass</td>
<td>80379 ± 16</td>
<td>57/48</td>
</tr>
<tr>
<td>Muon</td>
<td>Charged lepton $p_T$</td>
<td>80348 ± 18</td>
<td>58/62</td>
</tr>
<tr>
<td>Muon</td>
<td>Neutrino $p_T$</td>
<td>80406 ± 22</td>
<td>82/62</td>
</tr>
</tbody>
</table>

CDF II \( \int L \, dt = 2.2 \, \text{fb}^{-1} \)

- Muons: $p_T^\gamma$  $\quad 80406 \pm 22$
- Muons: $p_T^l$  $\quad 80348 \pm 18$
- Muons: $m_T$  $\quad 80379 \pm 16$
- Electrons: $p_T^\gamma$  $\quad 80431 \pm 25$
- Electrons: $p_T^l$  $\quad 80393 \pm 21$
- Electrons: $m_T$  $\quad 80408 \pm 19$

W boson mass (MeV/$c^2$): 80100 to 80600
CDF Result (2.2 fb\(^{-1}\))
Transverse Mass Fit Uncertainties (MeV)

<table>
<thead>
<tr>
<th>Source</th>
<th>electrons</th>
<th>muons</th>
<th>common</th>
</tr>
</thead>
<tbody>
<tr>
<td>W statistics</td>
<td>19</td>
<td>16</td>
<td>0</td>
</tr>
<tr>
<td>Lepton energy scale</td>
<td>10</td>
<td>7</td>
<td>5</td>
</tr>
<tr>
<td>Lepton resolution</td>
<td>4</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Recoil energy scale</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Recoil energy resolution</td>
<td>7</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Selection bias</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Lepton removal</td>
<td>3</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Backgrounds</td>
<td>4</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>pT(W) model</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Parton dist. Functions</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>QED rad. Corrections</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td><strong>Total systematic</strong></td>
<td><strong>18</strong></td>
<td><strong>16</strong></td>
<td><strong>15</strong></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>26</strong></td>
<td><strong>23</strong></td>
<td></td>
</tr>
</tbody>
</table>

Systematic uncertainties shown in green: statistics-limited by control data samples
2012 Status of $M_W$ vs $M_{\text{top}}$

![Graph showing the relationship between $M_W$ and $m_t$ with shaded regions for different models and error bands.](image)

- Experimental errors 68% CL:
  - LEP2/Tevatron: today

- MSSM and light SUSY regions
- Heavy SUSY region

- $M_H = 114$ GeV
- $M_H = 127$ GeV

Heinemeyer, Hollik, Stockinger, Weiglein, Zeune '12
W Boson Mass Measurements from Different Experiments

Previous world average = 80399 ± 23 MeV

World average computed by TeVEWWG, ArXiv: 1204.0042

- D0 I: 80483 ± 84 MeV
- CDF I: 80433 ± 79 MeV
- DELPHI: 80336 ± 67 MeV
- L3: 80270 ± 55 MeV
- OPAL: 80416 ± 53 MeV
- ALEPH: 80440 ± 51 MeV

5 fb$^{-1}$
- D0 II (PRL 108, 151804): 80375 ± 23 MeV

2.2 fb$^{-1}$
- CDF II (PRL 108, 151803): 80387 ± 19 MeV

World Average: 80385 ± 15 MeV

W boson mass (MeV/c$^2$)
Future $M_w$ Measurements at Tevatron and LHC

- Factor of 2-5 bigger samples of W and Z bosons available at Tevatron
- Huge samples at LHC
- For most of the sources of systematic uncertainties, we have demonstrated that we can find ways to constrain them with data and scale systematic uncertainties with data statistics
- Exception is the PDF uncertainty, where we have not made a dedicated effort to constrain the PDFs within the analysis

- We need to address specific PDF degrees of freedom to answer the question:
  - Can we approach total uncertainty on $M_w \sim 10$ MeV at the Tevatron?
PDF Uncertainties – scope for improvement

- Newer PDF sets, e.g. CT10W include more recent data, such as Tevatron W charge asymmetry data

- Dominant sources of W mass uncertainty are the $d_{\text{valence}}$ and $\bar{d}-\bar{u}$ degrees of freedom
  - Understand consistency of data constraining these d.o.f.
  - PDF fitters increase tolerance to accommodate inconsistent datasets

- Tevatron and LHC measurements that can further constrain PDFs:
  - $Z$ boson rapidity distribution
  - $W \rightarrow l\nu$ lepton rapidity distribution
  - $W$ boson charge asymmetry
PDF Constraint – $W$ Charge Asymmetry


\[
A(\eta) = \frac{d\sigma/d\eta(W^+ \rightarrow e^+\nu) - d\sigma/d\eta(W^- \rightarrow e^-\bar{\nu})}{d\sigma/d\eta(W^+ \rightarrow e^+\nu) + d\sigma/d\eta(W^- \rightarrow e^-\bar{\nu})}
\]

\[
\propto \frac{u(x_1)D(x_2) - U(x_1)d(x_2)}{u(x_1)D(x_2) + U(x_1)d(x_2)}
\]

where $q(x)$ [$Q(x)$] denotes the quark (antiquark) density at momentum faction $x$

\[
x_1 x_2 \propto M_W^2/s \quad \& \quad \ln(x_1/x_2) \propto \eta
\]
Missing $E_T$ in Inclusive W Boson Events (CMS)
Systematic Uncertainties in Electron Asymmetry (CMS)

Table 1: Summary of the systematic uncertainties on the asymmetry. All values are given in units of $10^{-3}$.

| $|\eta|$ bin | Signal Yield | Energy Scale & Res. | Charge MisId. | Efficiency Ratio |
|------------|-------------|---------------------|---------------|-----------------|
| 0.0 < $|\eta|$ < 0.2 | 1.8 | 0.6 | <0.1 | 4.5 |
| 0.2 < $|\eta|$ < 0.4 | 2.5 | 0.6 | <0.1 | 4.4 |
| 0.4 < $|\eta|$ < 0.6 | 2.7 | 0.3 | <0.1 | 4.4 |
| 0.6 < $|\eta|$ < 0.8 | 2.5 | 0.3 | <0.1 | 4.4 |
| 0.8 < $|\eta|$ < 1.0 | 1.9 | 0.6 | 0.1 | 4.4 |
| 1.0 < $|\eta|$ < 1.2 | 2.4 | 1.0 | 0.1 | 4.9 |
| 1.2 < $|\eta|$ < 1.4 | 2.6 | 0.8 | 0.1 | 5.4 |
| 1.6 < $|\eta|$ < 1.8 | 3.1 | 0.8 | 0.1 | 9.2 |
| 1.8 < $|\eta|$ < 2.0 | 2.0 | 1.6 | 0.2 | 8.7 |
| 2.0 < $|\eta|$ < 2.2 | 2.0 | 2.6 | 0.3 | 10.0 |
| 2.2 < $|\eta|$ < 2.4 | 2.9 | 2.4 | 0.3 | 12.5 |

Correction for backgrounds

Measured using $Z \rightarrow ee$ events
Systematic Uncertainties in Electron Asymmetry (CMS)

CMS

840 pb⁻¹ at \( \sqrt{s} = 7 \) TeV

\[ p_T(e) > 35 \text{ GeV} \]

- \( W \rightarrow e\nu \)

Electron Charge Asymmetry

MCFM:
- CT10
- HERAPDF1.5
- MSTW2008NLO
- NNPDF2.2 (NLO)

theory bands: 68% CL

Electron Pseudorapidity: \( \eta \)
Trilinear and Quartic Gauge Couplings

- Prediction of “forces” based on the idea of gauge invariance in Quantum Field Theory
  \[ \Psi \rightarrow e^{i g \xi(x)} \Psi \] (gauge transformation of fermion field)

- Introduction of a vector potential \( A_\mu \) (a.k.a. gauge field),
  \[ \partial_\mu \Psi \rightarrow D_\mu \Psi = (\partial_\mu - i g A_\mu) \Psi \]

- \( A_\mu \rightarrow A_\mu + \partial_\mu \xi \)

- Gauge-invariant Field Strength tensor \( F_{\mu\nu} \)
  \[ F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \]

- For gauge transformation in the internal space described by the (Abelian) U(1) group

- Kinetic energy associated with e.g. “electromagnetic field”
  \[ F_{\mu\nu} F^{\mu\nu} \]
Trilinear and Quartic Gauge Couplings

- For non-Abelian Gauge group, Gauge-invariant Field Strength tensor $F_{\mu\nu}$

  \[ F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + g[A_\mu, A_\nu] \]

- (gauge and Lorentz-invariant) kinetic energy term $F_{\mu\nu} F^{\mu\nu}$ yields

  - terms which are quadratic in gauge field (these yield the gauge boson propagator)

  - Cubic terms

  - Quartic terms

- Latter two types of terms yield trilinear and quartic gauge couplings
Anomalous Trilinear Gauge Coupling

- effective parametrizations for anomalous couplings:
  - $WWV$ vertex: $\Delta g_1^Z, \Delta \kappa_Z, \Delta \kappa_\gamma, \lambda_Z, \lambda_\gamma$
  - constraints from $WW, WZ, W\gamma$, and EW $Zjj$ measurements

  - $ZZV$ vertex (not in SM): $h_3^V, h_4^V, f_4^V, f_5^V$
  - constraints from $ZZ$ and $Z\gamma$ measurements

- 1- and 2-dimensional 95% confidence intervals for aTGC from 7 TeV data, e.g.
  - without and with form factors to avoid unitarity violation
  $$\mathcal{F}(s) = \frac{1}{(1 + \frac{s}{\Lambda_{FF}^2})^n}$$
  ($\Lambda_{FF}$: form factor scale)

(from A. Vest, TU-Dresden)
Spontaneous Symmetry Breaking of Gauge Symmetry

- postulate of scalar Higgs field which develops a vacuum expectation value via spontaneous symmetry breaking (SSB)

- Phase transition → vacuum state possesses non-trivial quantum numbers
  - Dynamical origin of this phase transition is not known
  - Implies vacuum is a condensed, superconductor-like state

- Radial (Higgs boson) and azimuthal (longitudinal gauge boson) excitations are related !!
Quartic Gauge Couplings

\[ V = W, Z \]

- the mechanism responsible for EWSB must regulate \( \sigma(V_L V_L \rightarrow V_L V_L) \) to restore unitarity above \( \sim 1 - 2 \text{ TeV} \)

- a light SM Higgs boson exactly cancels increase for large \( s \) (for \( HWW \) coupling)

\[
\mathcal{A}(W_L W_L \rightarrow W_L W_L) \propto \frac{g_W^2}{v^2} \left[ -s - t + \frac{s^2}{s-m_H^2} + \frac{t^2}{t-m_H^2} \right]
\]

- unitarity preservation only visible in \( VV \) scattering

\[ VV \text{ scattering is a key process to probe the SM nature of EWSB!} \]

- at the LHC: measure \( VV jj \) final states \( \rightarrow \) same-sign \( W^\pm W^\pm jj \) most promising

(from A. Vest, TU-Dresden)
Same-Sign Boson-boson Scattering

- Electroweak $W^\pm W^\pm jj$ production:
  
  $W^\pm W^\pm jj$-EW VBS: no s-channel diagrams
  
  $W^\pm W^\pm jj$-EW VBS: no s-channel diagrams

  → lowest order: $W^\pm W^\pm + 2$ jets, there is no SM inclusive $W^\pm W^\pm$ production!

  → VBS: “tagging” jets well separated in $y$ with large $m_{jj}$ (similar to EW $Zjj$ production)

- Strong $W^\pm W^\pm jj$ production:
  
  $W^\pm W^\pm jj$-strong VBS

  → no LO $gg$ or $qg$ initial state → strong $W^\pm W^\pm jj$ contributions comparably small

(from A. Vest, TU-Dresden)
**W\(^\pm\)W\(^\pm\) Scattering**

**for EW+strong measurement**
(“inclusive signal region”)

\[ m_{jj} > 500 \text{ GeV} \] (jets with largest \( p_T \))

**for EW measurement**
(“VBS signal region”)

\[ |\Delta y_{jj}| > 2.4 \]

\( |\Delta y_{jj}| \) between the 2 tagging jets

---

(from A. Vest, TU-Dresden)

---


CMS: PAS SMP-13-015
$W^±W^±$ Scattering

$μ^±μ^±jj$ Candidate Event

$m_{jj} = 2800$ GeV  $|Δy_{jj}| = 6.3$

jets: $p_T^{j1} = 271$ GeV, $p_T^{j2} = 54$ GeV, $η^{j1} = 2.9$, $η^{j2} = -3.4$

muons: $p_T^{μ1} = 180$ GeV, $p_T^{μ2} = 38$ GeV, $η^{μ1} = 1.4$, $η^{μ2} = -1.3$

$E_T^{miss} = 75$ GeV

(from A. Vest, TU-Dresden)
Summary

- The $W$ boson mass is a very interesting parameter to measure with increasing precision

- New Tevatron $W$ mass results are very precise:
  
  - $M_W = 80387 \pm 19$ MeV (CDF)
  - $= 80375 \pm 23$ MeV (D0)
  - $= 80385 \pm 15$ MeV (world average)

- New global electroweak fit $M_H = 94^{+29}_{-24}$ GeV @ 68% CL (LEPEWWG)
  
  - SM Higgs prediction is pinned in the low-mass range
  - confront directly measured mass of Higgs Boson $\sim 125$ GeV

- Looking forward to $\Delta M_W < 10$ MeV from full Tevatron dataset
goal of $\Delta M_W < 5$ MeV from LHC data
Summary

- Collider measurements can help to improve our knowledge of PDFs which are needed for making precision measurements.

- For the first time, LHC is creating the opportunity to test a key prediction of the SM:
  - The unitarization of longitudinal boson scattering at high energy

- Same-sign WW scattering signal observed
  - Ongoing searches for other channels: WZ and Wγ in vector boson scattering mode
  - Opposite-sign WW scattering has largest signal yield, but overwhelmed by top-antitop production background

- High-Luminosity LHC will provide opportunity to test composite Higgs models.
$W$ Transverse Mass Fit

CDF II

$\int L \, dt \approx 2.2 \, fb^{-1}$

Electrons

$M_W = (80408 \pm 19_{\text{stat}}) \, MeV$

$\chi^2/\text{dof} = 52 / 48$
$W$ Lepton $p_T$ Fit

CDF II

$\int L\, dt \approx 2.2\, fb^{-1}$

$M_W = (80348 \pm 18_{\text{stat}})\, \text{MeV}$

$\chi^2/\text{dof} = 54 / 62$

Muons

Data

Simulation

events / 0.25 GeV

$p_T(\mu)\, (\text{GeV})$

30 40 50
$W$ Missing $E_T$ Fit

CDF II

$\int L \, dt \approx 2.2 \text{ fb}^{-1}$

Electrons

$M_W = (80431\pm25_{\text{stat}}) \text{ MeV}$

$\chi^2/\text{dof} = 71/62$
$W$ Missing $E_T$ Fit

CDF II

$\int L \, dt \approx 2.2 \text{ fb}^{-1}$

Muons

$M_W = (80406 \pm 22_{\text{stat}}) \text{ MeV}$

$\chi^2/\text{dof} = 79/62$

- Blue: Data
- Red: Simulation

events / 0.25 GeV

$p_T^{\mu (v)}$ (GeV)
Lepton Resolutions

- Tracking resolution parameterized in the custom simulation by
  - Radius-dependent drift chamber hit resolution $\sigma_h \sim (150 \pm 1_{\text{stat}}) \mu m$
  - Beamspot size $\sigma_b = (35 \pm 1_{\text{stat}}) \mu m$
  - Tuned on the widths of the $Z \rightarrow \mu\mu$ (beam-constrained) and $\gamma \rightarrow \mu\mu$ (both beam constrained and non-beam constrained) mass peaks
    \[ \Rightarrow \Delta M_W = 1 \text{ MeV (muons)} \]

- Electron cluster resolution parameterized in the custom simulation by
  - $12.6\% / \sqrt{E_T}$ (sampling term)
  - Primary constant term $\kappa = (0.68 \pm 0.05_{\text{stat}}) \%$
  - Secondary photon resolution $\kappa_\gamma = (7.4 \pm 1.8_{\text{stat}}) \%$
  - Tuned on the widths of the $E/p$ peak and the $Z \rightarrow ee$ peak (selecting radiative electrons)
    \[ \Rightarrow \Delta M_W = 4 \text{ MeV (electrons)} \]
We remove the calorimeter towers containing lepton energy from the hadronic recoil calculation.

- Lost underlying event energy is measured in $\phi$-rotated windows

$$\Delta M_W = 2 \text{ MeV}$$

**Electron channel W data**

**Muon channel W data**
# Backgrounds in the W sample

## Muons

<table>
<thead>
<tr>
<th>Background</th>
<th>% of $W \rightarrow \mu \nu$ data</th>
<th>$\delta m_W$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z \rightarrow \mu\mu$</td>
<td>$7.35 \pm 0.09$</td>
<td>2 fit $p_T^\mu$ fit $p_T^\nu$ fit</td>
</tr>
<tr>
<td>$W \rightarrow \tau\nu$</td>
<td>$0.880 \pm 0.004$</td>
<td>0 fit 0 fit 0 fit</td>
</tr>
<tr>
<td>QCD</td>
<td>$0.035 \pm 0.025$</td>
<td>1 fit 1 fit 1 fit</td>
</tr>
<tr>
<td>DIF</td>
<td>$0.24 \pm 0.08$</td>
<td>1 fit 3 fit 1 fit</td>
</tr>
<tr>
<td>Cosmic rays</td>
<td>$0.02 \pm 0.02$</td>
<td>1 fit 1 fit 1 fit</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td>3 fit 5 fit 6 fit</td>
</tr>
</tbody>
</table>

## Electrons

<table>
<thead>
<tr>
<th>Background</th>
<th>% of $W \rightarrow e\nu$ data</th>
<th>$\delta m_W$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z \rightarrow ee$</td>
<td>$0.139 \pm 0.014$</td>
<td>1 fit 2 fit 1 fit</td>
</tr>
<tr>
<td>$W \rightarrow \tau\nu$</td>
<td>$0.93 \pm 0.01$</td>
<td>1 fit 1 fit 1 fit</td>
</tr>
<tr>
<td>QCD</td>
<td>$0.39 \pm 0.14$</td>
<td>4 fit 2 fit 4 fit</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td>4 fit 3 fit 4 fit</td>
</tr>
</tbody>
</table>

Backgrounds are small (except $Z \rightarrow \mu\mu$ with a forward muon)
$W$ Mass Fit Results

- Electron and muon $m_T$ fits combined
  \[ m_W = 80390 \pm 20 \text{ MeV}, \chi^2/\text{dof} = 1.2/1 \ (28\%) \]
- Electron and muon $p_T$ fits combined
  \[ m_W = 80366 \pm 22 \text{ MeV}, \chi^2/\text{dof} = 2.3/1 \ (13\%) \]
- Electron and muon MET fits combined
  \[ m_W = 80416 \pm 25 \text{ MeV}, \chi^2/\text{dof} = 0.5/1 \ (49\%) \]
- All electron fits combined
  \[ m_W = 80406 \pm 25 \text{ MeV}, \chi^2/\text{dof} = 1.4/2 \ (49\%) \]
- All muon fits combined
  \[ m_W = 80374 \pm 22 \text{ MeV}, \chi^2/\text{dof} = 4/2 \ (12\%) \]
- All fits combined
  \[ m_W = 80387 \pm 19 \text{ MeV}, \chi^2/\text{dof} = 6.6/5 \ (25\%) \]
### $p_T(l)$ Fit Systematic Uncertainties

<table>
<thead>
<tr>
<th>Systematic (MeV/c$^2$)</th>
<th>Electrons</th>
<th>Muons</th>
<th>Common</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lepton Energy Scale</td>
<td>10</td>
<td>7</td>
<td>5</td>
</tr>
<tr>
<td>Lepton Energy Resolution</td>
<td>4</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Recoil Energy Scale</td>
<td>6</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>Recoil Energy Resolution</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>$u_{\parallel}$ efficiency</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Lepton Removal</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Backgrounds</td>
<td>3</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>$p_T(W)$ model</td>
<td>9</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>Parton Distributions</td>
<td>9</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>QED radiation</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Total</td>
<td>19</td>
<td>18</td>
<td>16</td>
</tr>
</tbody>
</table>