Existence and Properties of Spatiotemporal Chaos in a 2D Excitable Medium

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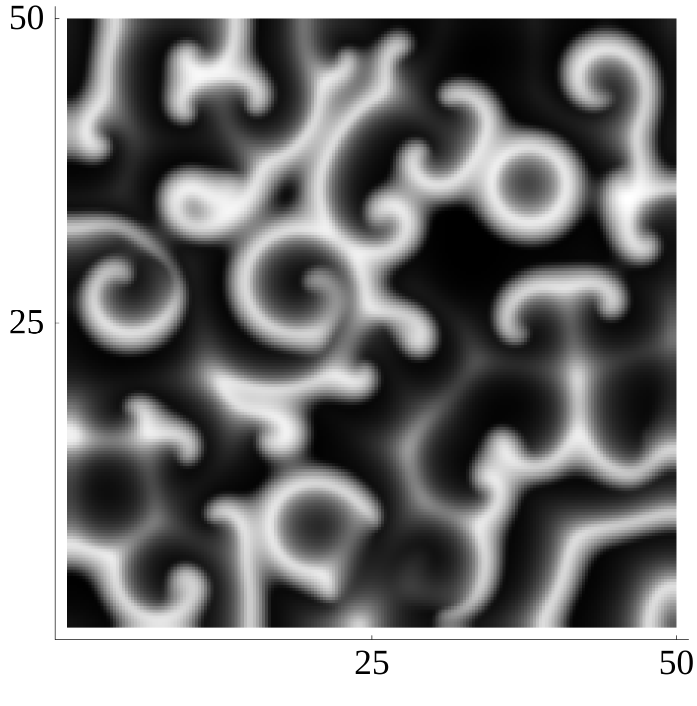
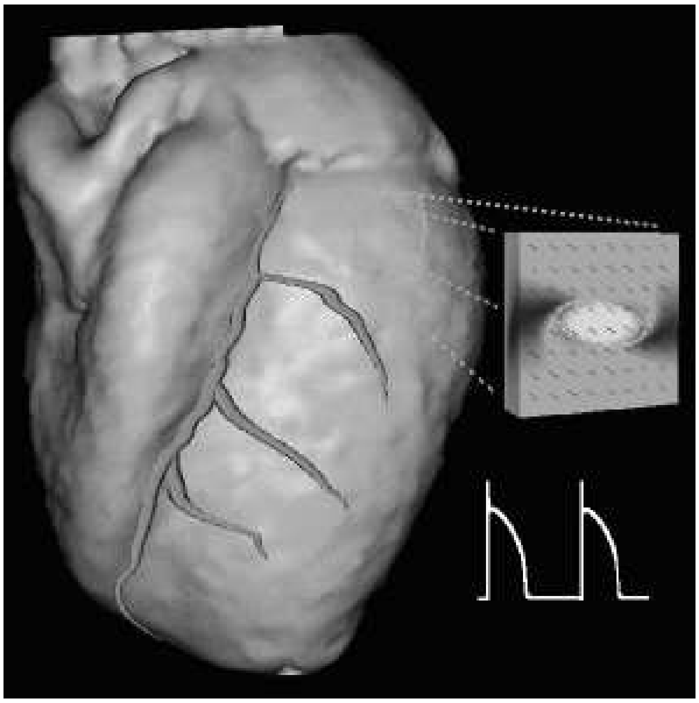


Is Fibrillation Chaotic?

Fibrillation: often fatal transition from coherent periodic beating to incoherent nonperiodic dynamics in mammalian hearts.

Question: is fibrillation chaotic?

1. If so, new approaches for prevention, control.
2. If so, new ways to characterize heart dynamics, response of heart to drugs.



Is any excitable medium chaotic?

1. Single spirals are periodic or quasiperiodic.

2. Many known qualitative examples of spatiotemporal complexity but chaotic?

3. If chaotic, relation of spiral statistics to ξ_2, λ_i, D?

4. Dependence of dynamics on size of medium?

The Bär Model

Two-dimensional model of excitable medium based on surface chemistry (Markus Bär and collaborators):

$$\partial_t u(t,x,y) = \nabla^2 u + \frac{1}{\epsilon} u(1-u)\left(u - \frac{v+b}{a}\right),$$

$$\partial_t v(t,x,y) = -v + \Theta(u - 1/3)\left[1 - 6.75\, u(u-1)^2\right].$$

Mathematical parameters: ϵ, a, b, boundary conditions, initial conditions.

Numerical parameters: $\triangle t$, $\triangle x$, T, integration algorithm.

Conclusions

Existence and properties of spatiotemporal chaos in the Bär model of an excitable medium is subtle and complicated:

1. Transient dynamics for $L \lesssim 15$ (details depend on bcs).

2. Supertransients: $\langle \tau \rangle \propto \exp(aL)$.

3. For $L \gtrsim 25$, dynamics statistically stationary, and accurately described as extensively chaotic.

4. Fractal dimension per spiral varies from 3 to 7 for $0.05 \le \epsilon \le 0.11$.

5. ξ_2 not directly related to ξ_δ.

6. $D = d\langle N \rangle + \gamma \sigma^2$

References

A Quantitative Measurement of Spatial Order in Ventricular Fibrillation
Philip V. Bayly et al, Journal of Cardiovascular Electrophysiology **4**(5):533-546 (1993).

Size-dependent transition to high-dimensional chaotic dynamics in a two-dimensional excitable medium
Matthew Strain and Henry Greenside. Phys. Rev. Lett. **80**(11):2306-2309 (1998) and to be published.

`http://www.phy.duke.edu/~strain/atwork/excite`

`http://www.phy.duke.edu/~hsg`