Name:

Signature for Honor Code:____

"I have neither received nor given aid in completion of this examination."

Second Midterm for Physics 212

Professor Henry Greenside

Handed out: 2:15 pm, Monday, April 5, 1999
Due by: 3:30 pm, Tuesday, April 6, 1999

The following is a 24-hour take-home exam which is due in my office, Physics 047, by 3:30 pm on Tuesday, April 6. If I am not in my office when you come by, please slip the exam under my door.

You are bound by the Duke Honor code to complete the exam entirely on your own, without consultation with anyone other than myself. You may use as references the two course textbooks (Griffiths and French and Taylor), the class lecture notes and your own course notes but no other material. If you have any questions whatsoever, please send me e-mail, give me a call at 660-2548, or arrange to meet me at my office.

While you do not need to write in complete sentences, your answers must be sufficiently detailed and readable for me to understand what you are doing and why. In questions of the form "True or False and Why", make sure that you indicate true or false and make sure that you justify your answer as quantitatively and thoroughly as possible.

If you use extra pages to complete a given question, please put your name on each extra page, label the extra page by the problem number, and staple all the extra pages to your exam.

Total Score (out of 60): ______

Second Midterm Grade: ______

1. **(4 points)** Two hydrogen atoms in excited states with principal quantum numbers n and n' are one centimeter apart. True or false and why: when calculating the energy of this system of two atoms, one does *not* need to take into account the exchange energy between the electrons.

2. **(4 points)** You are given a container of gas which is a mixture of orthohelium and parahelium atoms in their ground states. Explain experimentally how to separate these atoms, yielding one container of orthohelium and another container of parahelium.

3. **(4 points)** A scientist makes the following claim:

> "The magnitude-squared $|Y_l^m(\theta, \phi)|^2$ of a spherical harmonic is independent of the angle ϕ. Therefore if I prepare a beam of hydrogen atoms that are all in identical quantum states, a measurement of some property of the hydrogen atoms can not depend on the angle ϕ."

Explain why this scientist is wrong, that generally one would expect measurements to depend on the angle ϕ of observation. Note: To be specific, think of the spherical coordinate system as being centered at the point where the beam intersects some laboratory apparatus, with the $\theta = 0$ axis aligned along the beam direction.

4. **(6 points)** An alpha particle in its ground state is a spinless particle consisting of two neutrons and two protons, none of which have any orbital angular momentum. Let's make the bold assumption that the protons and neutrons move independently around in some potential well which we crudely take to be an infinite spherical well, with potential $V = 0$ for $r < a$ and $V = \infty$ for $r > a$ where the nuclear radius $a \approx 10^{-13}\,\text{cm}$. Assuming that each energy level of this well can hold two protons and two neutrons (i.e., the two particles are equivalent under the strong interaction and we ignore the relatively small Coulomb forces between protons), write down explicitly the normalized wave function for the ground state of the alpha particle in terms of the space and spin states of the four nucleons.

5. **(4 points)** In quantum mechanics, a classically forbidden region is that region of space where the kinetic energy of a particle would be negative. For a hydrogen atom in its ground state, what is the classically forbidden region and what is the probability of observing the electron in that region?

6. **(6 points)** Suppose that you have three particles and you also have three distinct single-particle states $\psi_a(x)$, $\psi_b(x)$, and $\psi_c(x)$ with energies $E_a < E_b < E_c$. For each of the following three cases:

   (a) the particles are distinguishable,

   (b) the particles are identical bosons,

   (c) the particles are identical fermions,

   give a list of the distinct energy values (in increasing order) that can be obtained for all the possible three-particle states and also give their corresponding degeneracies. Note: the particles need not be in different states, e.g., $\psi_a(x_1)\psi_a(x_2)\psi_c(x_3)$ with energy $2E_a + E_c$ would be a possibility for the case of distinguishable particles.

7. **(8 points)** For hydrogen-like atoms with a single s valence electron (e.g., the alkali atoms lithium, sodium, and potassium), the screening effect of the closed-shell electrons can be accounted for by considering the electrons to move in a modified spherical potential:

$$V(r) = -\frac{e^2}{r}\left(1 + \frac{\alpha}{r}\right), \qquad \text{CGS units} \tag{1}$$

where α is a constant. Find the energy levels for this potential and discuss how they differ from those of a hydrogen atom.

8. **(12 points)** Octatetraene is an organic molecule consisting of a linear chain of eight carbon atoms bonded one to another in succession, with alternating single and double bonds. (The carbon bonds not in the chain connect to hydrogen atoms.) This molecule is known experimentally to absorb light strongly at the wavelength

$$\lambda = 3020\,\text{Å}. \tag{2}$$

You now know enough quantum mechanics to understand this fact and to predict similar absorption peaks for other molecules like chlorophyll that have a one-dimensional chain of alternating bonds. To see this, let's make the following assumptions:

(a) Each carbon atom in such a chain contributes one non-bonded valence electron so that there are 8 non-bonded electrons in this molecule.

(b) Because of the alternating single and double bonds, each non-bonded valence electron can wander freely over the entire chain but obviously can't get off the chain. So let's try approximating the molecule by a one-dimensional infinite potential well whose length L is experimentally seven bond lengths, $7 \times 1.39\,\text{Å} \approx 10\,\text{Å}$.

(c) Let's assume that the electrons are non-interacting.

Here is then what you need to do:

(a) Based on the above assumptions, draw the five lowest energy levels of this molecule and give the energies of each level.

(b) Using the fact that electrons are spin 1/2 particles that obey the Pauli exclusion principle, fill up the successive energy levels of this one-dimensional well with the eight free electrons.

(c) To one significant digit, estimate the wavelength of a photon that provides the minimum energy needed to raise an electron to the next highest unfilled state. This possibly is the absorption peak wavelength and should be compared with the experimental value of 3020 Å.

(d) True or false and why: a photon with wavelength Eq. (2) is also the minimum energy needed to excite an electron in a singly ionized octatetraene ion (i.e., a molecule that is missing one electron.)

Some helpful information:

$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}, \qquad \hbar c \approx 2000\,\text{eV-Å}, \qquad \mathrm{m_e c^2} \approx 500\,\text{keV}, \qquad \mathrm{c} = \frac{\omega}{\mathrm{k}}, \qquad \mathrm{k} = \frac{2\pi}{\lambda}. \tag{3}$$

9. **(12 points)** As a simple model of phonons (quantized sound waves that act as spinless boson particles) in a crystal lattice, consider a system of three coupled harmonic oscillators described by the Hamiltonian:

$$H = \sum_{i=1}^{3} \frac{p_i^2}{2m} + \sum_{i=1}^{3} \frac{1}{2} m\omega^2 x_i^2 + \frac{1}{2}k(x_1 - x_2)^2 + \frac{1}{2}k(x_1 - x_3)^2 + \frac{1}{2}k(x_2 - x_3)^2, \tag{4}$$

where m is the mass of each particle, ω is the oscillator frequency, and k is the coupling constant between oscillators.

(a) By making a transformation to center-of-mass-like coordinates,

$$X = \frac{1}{3}(x_1 + x_2 + x_3), \qquad x = x_1 - x_2, \qquad y = x_3 - \frac{x_1 + x_2}{2}, \tag{5}$$

and

$$P = p_1 + p_2 + p_3, \qquad p_x = \frac{1}{2}(p_1 - p_2), \qquad p_y = \frac{2}{3}\left(p_3 - \frac{p_1 + p_2}{2}\right) \tag{6}$$

show that the Hamiltonian Eq. (4) transforms into a separable Hamiltonian and give the new Hamiltonian explicitly.

(b) Using the separability of the new Hamiltonian, determine and give the energies E_{N,n_x,n_y} and wave functions $\psi_{N,n_x,n_y}(x_1, x_2, x_3)$ for this three-particle system. Eq. (4) is one of the few cases for which one can find analytical solutions for interacting quantum particles.

(c) Sketch the energy spectrum in the weak and strong interaction limits ($k \ll 1$ and $k \gg 1$) *without regard to symmetrization.* In both limits, give the degeneracies of the first few states.

(d) Finally, which of the states are physically realizable for spinless bosons? I.e., for which states can one form completely symmetric wave functions?