Physics 212: Lecture 19

Monday, April 8, 1999
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1 Announcements

No homework until next week, please put the time instead in your course projects.

Times to work out:

1. When to hold the oral presentations: last week of April? April 26 (Monday), April 29 (Thursday), plus one more session.
2. Meet next week to discuss using LaTeX and emacs for putting slides and paper together?
3. We have one more tour scheduled for Professor Simon's lab. Interest in any other tours?

2 Reading

Reading Section 5.3 of Griffiths on two models of solid state matter.

A classic reference for the quantum mechanical theory of solids is *Solid State Physics* by Neil Ashcroft and N. David Mermin (Holt, Rinehart, and Winston, 1976), which is what most physics graduate students in the US use to learn this subject. The book has a good discussion of why one can often treat electrons as non-interacting entities, a most remarkable fact. The deeper reason for this comes from something called "Landau theory", that the low-lying excitations of a many-fermion system are "quasiparticles" that act like independent objects.

You may also enjoy browsing through *States of Matter* by David L. Goodstein, which gives a nice discussion of superfluidity.

3 Course Projects

Please put several hours into your course project every week for the next few weeks. I will be decreasing the amount of homework per week during the rest of the semester but I expect you to put the equivalent amount of time into preparing your talk and paper.

Please don't hesitate to meet with me if you have questions about your project, either the physics that you will be discussing or how to prepare it. I will be organizing one or two evening lectures for us to discuss the preparation of lecture slides and a technical paper.

4 Comments on the Second Midterm

We will discuss briefly the answers, and strategies for getting the answers, for the recent take-home exam.

5 No More Principles

With our discussion of the symmetrization principle, the assumption that multi-particle wave functions for identical particles have to be either totally symmetric (bosons) or antisymmetric (fermions) under particle exchange, we have covered all the principles of nonrelativistic

6 Free Electron Gas Model of a Solid

We will discuss the material in Section 5.3 of Griffiths, two elementary models of the solid state in which one makes the crazy assumption that the electrons are non-interacting.

The first idea is called the "free electron gas" and is one of those extreme simplifications that shouldn't work but works quite well. The idea is to take a metal made up of atoms containing one or two valence electrons each. The periodic lattice of atom nuclei is then approximated as a uniform positive charge and the electrons are assumed to be completely free and noninteracting over the entire volume of the metal. We can then try treating the metal as a three-dimensional infinite well ($V = 0$ inside, $V = \infty$ outside) with macroscopic dimensions L_x, L_y, L_z, and we can immediately write down the wave functions for states characterized by the number of nodes in each direction (n_x, n_y, n_z) since this is a direct generalization of our one-dimensional infinite well discussion. When the lengths L_i are large, the levels basically form a continuum. We apply the Pauli exclusion principle and fill up each of the levels with two electrons at a time until we have used up all the electrons.

This model makes some remarkable predictions, several of which are nontrivial and significant. First, even at absolute zero, there are electrons at the top of the levels (at the so-called Fermi surface) that are moving at high speed, a few percent of the speed of light, so one can think of a metal as "hot" all the time. Second, even at absolute zero temperature and without considering any interactions between the electrons, the electron gas exerts a substantial pressure on its container, one could call this a "exclusion pressure" since it is a direct consequence of the Pauli exclusion principle. This is the pressure in white dwarf stars and in neutron stars that prevents gravity from collapsing the star completely. Third, only electrons very close to the Fermi surface are capable of colliding with one another since "deep" electrons can't jump to a nearby free energy level. This means that only a very small fraction of the electrons in a metal can contribute to the specific heat (roughly the ratio T/T_F where $T_F = E_F/k$ is the Fermi temperature), explaining one of the great experimental conundrums at the turn of this century, why the specific heats of metals were so much smaller than what it should be.

7 White Dwarfs and the Chandrasekhar Limit

8 Band Structure of Solids: Electrons in a Periodic Potential

We next discuss Section 5.3.2 of Griffiths, an elementary model of the "band structure" of a solid in which we replace the homogeneous positive background by a periodic lattice of positive charges. Most of the interesting mathematics can be obtained by the extreme measure of replacing the Coulomb attraction around each atom by a δ-function attractive potential, so the band theory becomes solving the many but independent particle Schrodinger equation:

$$-\frac{\hbar^2}{2m}\partial_x^2\psi + V(x)\psi = E\psi, \tag{1}$$

where the periodic potential V is given by a "Dirac comb":

$$V(x) = -\alpha\sum_{j=0}^{N-1}\delta(x - ja), \tag{2}$$

consisting of a periodic regular spacing of attractive Dirac δ-functions.

The key new mathematical insight for electrons in a periodic lattice is "Bloch's theorem", that the single-particle wave functions have a special form of a plane wave with wavenumber k times a function $f(x)$ with the periodicity of the lattice:

$$\psi_k(x) = e^{ikx} f(x), \tag{3}$$

where

$$f(x + ja) = f(x), \tag{4}$$

if a is the lattice spacing in one-space dimension. Felix Bloch of Stanford rediscovered what was known well before him by mathematicians as "Floquet theory", the linear stability of solutions to odes with periodic coefficients.

I won't reproduce the mathematics discussed in Griffiths but just mention the highlights: for a macroscopic solid of length L huge compared to the lattice spacing and de Broglie wavelength, one gets effectively a *continuum* of energy levels in the sense of infinitely many levels that are arbitrarily close to one another. However, the continuum is broken up into intervals (ranges of energy) with gaps between the intervals. When one applies the Pauli exclusion principle and fills up all the levels to the Fermi energy E_F, one finds two distinct cases: the Fermi energy sits in the middle of a group of levels, giving a metal since if one applies an electric field which increases slightly the energy of an electron, there are nearby states of higher energy for the electron to "jump to". The other case is that the Fermi energy is just at the top of group of levels that is separated by a gap from the next group of levels. Now if one applies an electric field, the electrons can't move since they would have to jump a finite amount of energy to get to the next highest available energy state. This is the case of an insulator (non-conducting) material.

By the way, it is a big thing whether a macroscopic amount of electrons or anything has a gap in the energy levels or not. Superconductivity, the flow of electricity with zero resistance, occurs because as the temperature is decreased for many metals (and other substances), a finite gap opens up between the ground state and the next available many-particle state. Crudely speaking, the vibrations of a crystal lattice (phonons) cause a weak *attractive* force between electrons near the Fermi energy. The electrons can bind into a single state called a "Cooper pair" and such a pair acts as a bosonic particle. The Cooper pairs, as bosons, can all condense into the lowest energy level and then it turns out to be an energy gap that separates the ground state from the next available state and one gets zero resistance. The development of a theory to explain the existence of this energy gap (the Bardeen-Cooper-Schrieffer or BCS theory) was one of the great intellectual advances this century and the ideas behind the argument remain the basis for much theoretical work in the immense field of condensed matter physics.