Lecture 1: Physics 212

Thursday, January 14, 1999
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1 Reading

1. Chapters 6 and 7 of French and Taylor.
2. Please review the concept of polarized light (linear, circular, elliptical), which you have hopefully seen in your electrodynamics course and also in a high school or freshman physics courses when discussing elementary optics. A particularly good and accessible discussion of polarization together with many nice home experiments can be found in the book *Waves* by Frank Crawford, Jr.
3. You can find a more detailed and mathematical discussion of the Chapter 7 material in Chapter 1 of Gordon Baym's book "Lectures on Quantum Mechanics".

2 Questions

1. We discussed in lecture how to write down the mathematical expression for the electric field $\mathbf{E}(t,z)$ of a linearly polarized light beam of wavenumber k and angular frequency $\omega = ck$ propagating in the z-direction with x-polarization, namely:

$$\mathbf{E}(t,z) = \hat{\mathbf{x}}\, E_0 \cos(kz - \omega t + \phi), \tag{1}$$

where E_0 is some constant magnitude, $\hat{\mathbf{x}} = (1,0,0)$ is the unit vector in the x-direction, and ϕ is some arbitrary constant phase (some constant in the interval $[0, 2\pi]$). The Maxwell equations show that the magnetic field $\mathbf{B}(t,z)$ is simply given by $(1/c)\hat{\mathbf{z}} \times \mathbf{E}$ and so is known once the electric field is known. (Please show this yourself since it is elementary.)

Given this, what is the mathematical expressions for the electric field $\mathbf{E}(t,z)$ of a left-circularly polarized monochromatic light beam of intensity I_0, propagating in the z-direction, again with wavenumber $k = \omega/c$ and angular frequency ω? What is the expression for a right-circularly polarized beam?

2. The mathematical expression for a z-beam of monochromatic light (e.g., generated from the sun or from an incandescent lamp by passing the light through a prism or high quality color filter) would be a superposition of linearly polarized plane waves with all possible angles θ of polarization:

$$\mathbf{E}(t,z) = \int_0^{2\pi} d\theta\, \hat{\mathbf{n}}(\theta)\, E_0(\theta) \cos\big(kz - \omega t + \phi(\theta)\big), \tag{2}$$

where the unit vector $\hat{n}$ is always normal to the z direction and can be written:

$$\hat{\mathbf{n}} = \big(\cos(\theta), \sin(\theta), 0\big). \tag{3}$$

Both the amplitude $E_0(\theta)$ and phase $\phi(\theta)$ can vary with the angle of polarization θ.

The expression for white light from the sun or from a light bulb would involve one more integral over all possible frequencies:

$$\mathbf{E}(t,z) = \int_0^{\infty} d\omega \int_0^{2\pi} d\theta\, \hat{\mathbf{n}}(\theta)\, E_0(\theta,\omega) \cos\big(kz - \omega t + \phi(\theta,\omega)\big), \tag{4}$$

where now the amplitude and phase can depend on both the polarization angle θ and light frequency ω, and the wavenumber is given in terms of ω by using the dispersion relation for light in a vacuum, $k = \omega/c$.

Given these expressions, right down expressions for monochromatic beam of light and for general white light as superpositions of right- and left-circularized monochromatic beams.

3. Show directly that Eq. (1) and $\mathbf{B} = (1/c)\hat{z} \times \mathbf{E}$ are solutions of the Maxwell equations in vacuum (here given in CGS units)

$$\nabla \cdot \mathbf{B} = 0, \quad \nabla \cdot \mathbf{E} = 0, \quad \nabla \times \mathbf{B} = (1/c)\partial_t \mathbf{E}, \quad \nabla \times \mathbf{E} = -(1/c)\partial_t \mathbf{B}. \tag{5}$$

4. Can a beam of linearly polarized light transmit angular momentum to an object that it strikes? Can a beam of circularly polarized light transmit momentum to an object that it strikes? If the light has angular frequency ω and intensity I, how much momentum and angular momentum is transferred in both cases if the object is a perfect mirror and the beam strikes at normal incidence?

5. Explain why sound waves in air can not be polarized while sound waves in a crystal can be polarized.

6. What is pleochroism?

7. TV stations transmit VHF signals using a long horizontal wire (at least in the US, I understand that England uses a vertical wire for VHF transmission). Explain why the VHF TV signal is linearly polarized and in what direction. In what direction should you orient a VHF antenna in your home to best pick up the signal?

8. Why is light from a solid-state laser (used in hand-held pointers) linearly polarized? What determines the axis of polarization? (I don't know the answer to this one but am curious.)

9. What is the axis of linear polarization for sunglasses and why is it chosen that way? Are sunglasses effective for reflections from all media: water, glass, asphalt, metal, snow?

10. In your electrodynamics course or earlier physics course, you learned about Brewster's angle: an unpolarized light beam reflecting off of a dielectric medium (e.g., window glass or water surface, something non-metallic) such that the angle of reflection is perpendicular to the angle of refraction leads to a reflected beam that is linear polarized with an axis parallel to the surface. (Please review your understanding of why this is the case.)

    What happens if an unpolarized beam of light reflects off of a linear polarizing sheet, e.g., one of the green pieces of polaroid that we passed around in class? Will the reflected beam still be polarized? Does rotating the axis of polarization of the sheet make any difference to the reflected beam? Find two linear polarizers and a laser light pointer and test your thinking.

11. Is it meaningful to talk about a wavefunction $\psi(t, x)$ for a linearly polarized beam of light?

12. A beam of light that passes through a linear polarizer can be described by a particular ket, e.g., $|y\rangle$ for a y-polarized beam. What is the Hamiltonian operator $\hat{H}$ that one should insert into the Schrodinger equation $i\hbar\partial_t|\Psi\rangle = \hat{H}|\Psi\rangle$ to describe the evolution of this ket in time, once the photon has emerged from a polarizing filter?

13. By thinking about the quantum interpretation of linearly polarized light passing through a linear polarizer rotated an angle θ with respect to the axis of the polarized light, give a criterion for when you can expect the Maxwell equations to give an accurate description of polarization. Does your criterion depend on the frequency and intensity of the light beam?

14. Explain the statement: an individual photon does not have a meaningful color (energy) or electric field.

    Why, then, is it meaningful to talk about individual photons?

15. Circularly polarized light can be made by gluing a linear polarizer to a so-called quarter-wave plate with the axis of linear polarization at forty-five degrees compared to the orthogonal fast and slow axes of the quarter-wave plate. Light that passes through the linear polarizer and then through the quarter-wave plate will be circularly polarized. What kind of polarized light do you get if you shine light though this sandwich in the opposite direction, quarter-wave plate first then the linear polarizer?

    You are given two squares of plastic, one of which is a linear polarizer and the other a right-circular polarizer. Without having access to any other polarizer pieces, explain how to tell which is which.