

Assignment 10

Physics/ECE 176

Made available: Wednesday, April 13, 2011
Due: by 5pm, Wednesday, April 20, 2011.

Problem 1: Molecule with multiple binding sites

A certain biological molecule has two binding sites A and B , each of which can be empty or can hold one molecule. Site A can bind a single molecule of type α with binding energy $\epsilon_{A,\alpha}$ or bind a single molecule of type β with a binding energy $\epsilon_{A,\beta}$. Independently of site A , site B can bind a single molecule of type α with binding energy $\epsilon_{B,\alpha}$ or bind a single molecule of type β with a binding energy $\epsilon_{B,\beta}$. The biological molecule is in thermodynamic equilibrium with a reservoir whose constant temperature is T and that contains mobile α and β molecules such that their constant chemical potentials have the values μ_α and μ_β respectively. In terms of the temperature T , chemical potentials μ_α and μ_β , and four binding energies $\epsilon_{i,j}$, write down a mathematical expression for the probability that the biological molecule will have one α molecule and one β molecule bound to it.

Note: an olfactory receptor might have several binding sites, each of which can bind thousands of different kinds of molecules, and many mammals like mice have over a thousand different kinds of receptors. So the problem of figuring out how the olfactory sensory array of some animal reacts to a mixture of odorants is daunting indeed. A practical application might be designing a robot with an electronic nose that can smell the chemicals associated with a land mine, but despite many efforts, no artificial nose has come close to matching what even a honeybee is capable of discriminating.

Problem 2: Schroeder Problems

1. **Problem 7.6** on page 261.

When you apply your result to an ideal gas, use Eq. (6.93) on page 255, you should obtain the result $\sigma_N = (\overline{N})^{1/2}$.

2. **Problem 7.10** on page 265.

To help you get started, I'll answer part (a) for you and part of (b) and (c). For the case of bosons, all five particles can occupy the lowest energy state E_1 since there is no limit to how many bosons can occupy a given energy level. For the case of distinguishable (classical) particles, again all five particles can occupy the lowest energy state. For identical fermions, only one particle can occupy a given state (ignoring the spin of the particles) so the ground state of the system will consist of one particle in each of the lowest five energy states. These could be indicated pictorially like this:

bosons, or distinguishable particles:

5	-
4	-
3	-
2	-
1	*****

fermions:

5	*
4	*
3	*

2	*
1	*

Here the numbers 1, 2, ... indicate energy levels starting from the lowest level 1, the asterisks * denote a particle in that level, and the dash - means that there are no particles in that level.

For part (b), to add a unit of energy to the ground state of this five-particle system, one of the five particles has to be placed in a higher energy level. Let's say we consider the case of distinguishable particles. Then there are five ways to choose any one of the distinguishable particles in level 1 and promote it to level 2 like this:

5	-
4	-
3	-
2	*
1	****

so the first excited state has a degeneracy of 5.

For part (c), let's consider the case of fermions when we have added two units of energy to a system with five identical fermions. There is only one way to add one unit of energy, which is to raise the highest energy fermion one energy level, like this:

6	*
5	-
4	*
3	*
2	*
1	*

If we try to raise any fermion below the 5th level, it would take two or more energy units, e.g., it takes two energy units to raise the fermion in level 4 to the first free level, level 6. To add another energy unit from here, there are only two choices: raise the fermion in energy level 6 by one level, to level 7, or raise the fermion in level 4 to level 5, so the two choices look like this for a 5-fermion state with two energy units above the ground state

7	*	-
6	-	*
5	-	*
4	*	-
3	*	*
2	*	*
1	*	*

The degeneracy of each state is 1 since the particles are identical. You should be able to take it from here.

3. **Problem 7.12** on page 269.

4. **Problem 7.18** on page 271.

5. **Problem 7.20** on page 276

Hint: calculate the Fermi temperature T_F and see if the temperature of the Sun's core is large or small compared to T_F .

6. **Problem 7.21** on page 276.

Note: Since each wave function can now hold four particles (two protons, two neutrons, each of opposite spin), you will have to modify Eq. (7.38) accordingly before computing your answer.

7. **Problem 7.22** on page 276.

This problem involves following the discussion on pages 274-275 rather closely, with the change that the energy of a relativistic particle in a box is given by

$$\epsilon = pc = \frac{hc}{2L} (n_x^2 + n_y^2 + n_z^2)^{1/2} = \frac{hcn}{2L}, \quad (1)$$

where n_x , n_y , and n_z can be any positive integer and n is the radius in the n -space of Figure 7.10 on page 273. For part (b), you should get the answer $U = (3/4)N\epsilon_F$.

8. **Problem 7.23** on pages 276-277.

This problem lets you follow the thinking that led to the discovery of what is called the “Chandrasekhar limit” for the maximum mass of a white dwarf star, discovered by the 19-year old Subrahmanyan Chandrasekhar (Physics Nobel 1983) during a boat ride from India to England around 1930. The Chandrasekhar limit has proved to be extremely important to modern cosmology since it implies that all white dwarf stars explode in nearly the same way, which allows such exploding stars to be used as distance markers for distances up to ten billion light years. (If you know the luminosity $\mathcal{L}$ of the exploding white dwarf and can measure the light intensity I of the dwarf at Earth, you can deduce the distance d from Earth via the relation $\mathcal{L} = 4\pi d^2 I$.)

- Skip part (a), just use the result you derived in an earlier homework problem that the gravitational potential energy is $-(3/5)GM^2/R$ for a sphere of uniform density with radius R , and mass M .
- For part (b), the assumption of “one proton and one neutron for each electron” allows you to deduce the number N of electrons from the mass M of the Sun since electrons have a mass that is about $1/2000$ of $m_p \approx m_n$ which means that their contribution to the mass of the Sun is negligible compared to the mass of the protons and neutrons.
- For part (c), you should find that the equilibrium radius is given by

$$R = 0.029 \frac{h^2}{Gm_e m_p^{5/3}} M^{-1/3}. \quad (2)$$

Use Mathematica to generate a suitable plot and include the plot (and code to generate the plot) with your assignment.

- For (d), you should get an answer between 7000 and 8000 km, which is a bit bigger than Earth’s radius (6400 km) but about 100 times smaller than the radius of the Sun. So a white dwarf is a small massive extremely dense object.
- For (e), you should get a Fermi energy of 2×10^5 eV to one significant digit, but please give your answer to two significant digits since you have access to Mathematica or a calculator.
- For (f), use your answer for Problem 7.22.
- For (g), you should get $M \approx 3 \times 10^{30}$ kg although your answer should be to two significant digits. Note that your answer will not be *the* Chandrasekhar limit because you are assuming a star with uniform density while Chandrasekhar did the harder calculation in which the density increased from the star’s surface to the star’s core.

9. **Problem 7.38** on page 293.

10. **Problem 7.39** on page 293.

This problem raises an interesting subtle issue. It is widely believed that organisms that use light for vision (eyes) or as a source of energy (plants and various bacteria) have evolved to maximize their ability to capture the photons that arrive at sea level through the atmosphere. But since the location of the peak of the equilibrium photon spectrum Eq. (7.84) on page 292 is different depending on whether you use wavelength λ or energy ϵ as a variable, which peak has life used to optimize the gathering of light? If this intrigues you, take a look at the article “Some paradoxes, errors, and resolutions concerning the spectral optimization of human vision” by B. Soffer and D. Lynch, American Journal of Physics, Volume 67, pages 946-953 (1999), which has several interesting insights.

11. **Problem 7.43** For (b), you can use the Mathematica `Plot` command. For (c), use the Mathematica function `NIntegrate` to approximate the area under the Planck spectrum between two specified integration bounds.

Problem 3: Time to Finish This Homework Assignment

Please tell me the approximate time in hours that it took you to complete this homework assignment.