Assignment 6 Physics/ECE 176

Made available:Saturday, February 19, 2011Due:Tuesday, March 1, 2011, at beginning of class

Problem 1: How does the entropy of water vary with temperature?

The relation

$$dS = \frac{Q}{T} = \frac{C_V \, dT}{T},\tag{1}$$

can be written in the form

$$\frac{dS}{dT} = \frac{C_V}{T},\tag{2}$$

i.e., if you plot the entropy S(T) versus T, its slope at a given temperature T is known in terms of C_V and T.

Given the data in Figure 1.14 on page 30 of Schroeder (which gives the general behavior of C_V for any solid), given the assumption that C_V has an approximately constant value in the liquid phase and has an approximately different constant value in the gas phase, and given that C_V for water is somewhat larger than the values for ice and steam (as you saw in the last problem of Quiz 2), use Eq. (2) to draw carefully a qualitative plot of S(T) vs T for water starting from absolute zero T = 0, that spans the three water phases of solid, liquid, and gas.

Problem 2: Entropy change upon connecting two containers of different temperature

Two containers of volume V_1 and V_2 each contain N atoms of the same ideal gas at the same pressure P but with temperatures of T_1 and T_2 respectively. In terms of the heat capacity at constant pressure C_P (which you can assume does not vary with temperature) and of the temperatures T_1 and T_2 , calculate the total change of entropy ΔS_{total} that arises when the two containers are connected by a tube of negligible volume so that thermodynamic equilibrium is attained.

Discuss whether your expression for ΔS_{total} is always positive if $T_1 \neq T_2$; do you always get an irreversible process upon connecting the two containers?

While any physical process must lead to a non-negative change in total entropy, this may not be the case for mathematical approximations of physical processes, so it is useful to check this to see if the math makes scientific sense.

Problem 3: Schroeder Problems

- 1. Problem 2.27 on page 74.
- 2. Problem 2.36 on page 79.
- 3. Problem 2.37 on page 81.

4. Problem 3.10 on page 97. Note that the latent heat L for melting ice is 3.33×10^5 J/kg and the specific heat c_V from freezing to boiling is about 4.2×10^3 J/(kg K).

For part (a) of this problem, also estimate approximately the ratio of the multiplicity of the water molecules in the water phase to the multiplicity of the same water molecules in the ice phase. Does the multiplicity increase by a lot even though the volume has changed just by a little?

Answers to one digit (but you need to provide two digits): (a) $\Delta S \approx 40 \text{ J/K}$; (b) $\Delta S \approx 10 \text{ J/K}$; (c) $\Delta S \approx -40 \text{ J/K}$; (d) $\Delta S_{\text{total}} \approx 4 \text{ J/K}$.

- 5. Problem 3.13 on page 97. This short simple problem is insightful since it explains why the formation of order by biological growth does not contradict the second law of thermodynamics, a widespread and unfortunate misunderstanding. You can use Schroeder problem 2.36 above to help you with your estimate, e.g., how many kilograms and therefore moles of grass do you think you can grow in a square meter of soil?
- 6. Problem 3.23 on page 107. Please also plot S(T) versus T in the form S(y)/(Nk) versus $y = kT/(\mu B)$ and include this plot with your assignment. Your plot should verify that this mathematical expression for S(T) gives the qualitative behavior we derived in class without using any mathematics, that S(T) starts at zero, increases monotonically, and then saturates with a constant plateau.

You may find it useful first to do Problem 3.19 (which you are not required to do), i.e., make sure you know how to derive Eqs. (3.30), (3.31), and (3.33) on pages 104-105 of Schroeder. In deriving the answer, rather than memorizing various trig-like identities for the hyperbolic functions, just use the basic definitions such as $tanh(x) = (e^x - e^{-x})/(e^x + e^{-x})$, then simplify the algebra to see if you can identify a related hyperbolic function such as cosh or sinh through their definitions.

Please also answer these two questions:

- (a) On your plot of S(T) vs T, draw qualitatively how the curve changes if the magnetic field B is increased to a constant larger value and briefly justify your answer.
- (b) From Eq. (3.28) on page 103 of Schroeder and from Eq. (3.25) on page 99, show that the entropy S(U) can be written in the explicit mathematical form:

$$S(x)/(Nk) = \left[\ln(2) - \frac{1}{2}\ln\left(1 - x^2\right) + \frac{x}{2}\ln\left(\frac{1 - x}{1 + x}\right)\right],\tag{3}$$

where $x = U/(N\mu B)$ is the dimensionless energy measured in the natural unit of $N\mu B$. Plot Eq. (3) versus x and verify that this reproduces Fig. 3.8 on page 101 which means that you now understand where this figure comes from. (You don't need to hand in this plot, but do make the plot.)

- 7. Problem 3.20 on page 107. The goal of this problem is to give you some feeling for numerical magnitudes for a realistic paramagnet. One way to get the entropy is to use the equation you derived in in Problem 3.23 above. Another way is to use Eq. (3.28) on page 103 of Schroeder with Eqs. (3.25) and (3.31).
- 8. Problem 3.27 A short simple problem, just to see if you understand the idea of using the "thermodynamic identity" as a way to derive various thermodynamic relations.
- 9. Problem 3.34 on pages 114-115 of Schroeder. This is an interesting and important problem, especially for those of you interested in biophysics and material science, since it shows that the elastic properties of rubber arise from entropy, not from energy. This problem also explains why rubber bands contract upon heating.

Note: For part (d), stretching this particular rubber band does not change the energy so consider your thermodynamic identify with dU = 0. After some algebra, you should get the answer:

$$F = \frac{kT}{2l} \ln\left(\frac{1+L/(Nl)}{1-L/(Nl)}\right). \tag{4}$$

- 10. Problem 3.36 of Schroeder. A straightforward but important problem to give you some practice thinking about the physical meaning chemical potential μ of some equilibrium system.
- 11. Problem 3.39 of Schroeder. The two-dimensional version of the Sackur-Tetrode equation is:

$$S = Nk \left[2 + \ln \left(\frac{A}{N} \frac{2\pi m}{h^2} \frac{U}{N} \right) \right]$$
(5)

Once you have your answer for the chemical potential μ_{2d} of a two-dimensional ideal gas, assume that the two-dimensional surface gas is in thermodynamic equilibrium with a surrounding three-dimensional ideal gas of the same atoms and deduce a mathematical relation between the surface density $s = N_s/A = s(P)$ (where N_s is the number of atoms on the surface) and the pressure P of the 3D gas and discuss whether to what extent your answer makes sense. For example, what happens as one increases the pressure of the 3D gas? What happens as one increases the temperature of the system for fixed pressure?

Problem 4: Time to Finish This Homework Assignment

Please tell me the approximate time in hours that it took you to complete this homework assignment.