Assignment 2
Physics/ECE 176

Made available: Friday, January 21, 2011
Due: Thursday, January 27, 2011, by the beginning of class.

Problem 1: Distance Between Two Random Walkers

Consider two atoms A and B that undergo a random walk in a box containing some gas. Just like the situation described in lecture, atom A undergoes a random walk in which it repeatedly travels with a constant speed v_A in a randomly chosen direction until, after a collision time τ_A (which is always the same), atom A collides with another molecule and changes direction randomly. Atom B also undergoes a random walk but, because of its different properties, travels with a different constant speed v_B in a randomly chosen direction until, after a different collision time τ_B, it collides with another molecule and changes its direction randomly.

Let $\mathbf{X}(t)$ and $\mathbf{Y}(t)$ be the vector locations of atoms A and B respectively at time t with respect to some coordinate system and assume that both atoms start at the origin of the coordinate system so that $\mathbf{X}(0) = \mathbf{Y}(0) = \mathbf{0}$. Show that the average square of the distance between the two atoms itself behaves like a random walk in that it increases linearly with time t like this

$$\left\langle |\mathbf{X}(t) - \mathbf{Y}(t)|^2 \right\rangle = D\,t, \tag{1}$$

and give the diffusion coefficient D explicitly in terms of the parameters v_A, v_B, τ_A and τ_B. (The brackets $\langle \cdots \rangle$ indicate an ensemble average.) Discuss briefly whether your conclusion changes if the two atoms do not start off at the same location at time $t = 0$.

Some hints: Assume in a time t that atom A takes $M = t/\tau_A$ steps and that atom B takes $N = t/\tau_B$ steps where M and N are positive integers. Then write

$$\mathbf{X}(t) = \sum_{i=1}^{M} \Delta\mathbf{X}_i, \qquad \mathbf{Y}(t) = \sum_{j=1}^{N} \Delta\mathbf{Y}_i, \tag{2}$$

where each step $\Delta\mathbf{X}_i$ has the same length but a random direction, and where each step $\Delta\mathbf{Y}_j$ has the same length (although different than A's) but a random direction.

Problem 2: A Box With a Tiny Cold Plate

Consider a cubic box of volume V that contains an ideal gas in thermodynamic equilibrium that consists of N identical atoms, each of mass m, each moving with the same speed v. The gas is initially at temperature T with pressure P. A tiny flat metallic square of area A is attached to the inside surface of the box so that the square lies flush with the wall and such that the square is cooled to such a low temperature (by external machinery) that any gas atoms that come in contact with the square permanently stick.

1. Assuming the simplest kinetic model in which 1/6 of the atoms are moving perpendicular to and toward the flat plate, show that the amount of the energy per unit time dE/dt that must be removed from the metal square in order to keep its temperature constant and cold is given by the expression

$$\frac{dE}{dt} = \frac{1}{4} A P \left(\frac{3kT}{m} \right)^{1/2}. \tag{3}$$

2. Assuming now that the atoms in the gas all have the same speed but have isotropic velocities (this means the velocities point in all possible directions with equal likelihood), show that Eq. (3) becomes

$$\frac{dE}{dt} = \frac{3}{8} A P \left(\frac{3kT}{m} \right)^{1/2} . \tag{4}$$

   Note how the numerical coefficient changes little, 3/8 versus 1/4, when you use the presumably more realistic isotropic model; the crude kinetic model gets the job done.

   Hint: You will want to set up an expression like this:

$$\Delta E(\theta, \phi) = ([v \cos(\theta) \Delta t] A) \times \frac{N}{V} \times \frac{\sin(\theta)\, d\theta\, d\phi}{4\pi} \times \cdots , \tag{5}$$

   which represents the small contribution $\Delta E(\theta, \phi)$ to the energy from particles arriving at the plate within a short time Δt from the direction (θ, ϕ), for a spherical coordinate system centered on the plate with its z-axis pointing into the box and perpendicular to the area A. You then need to integrate Eq. (5) over some appropriate region to add up all the relevant contributions.

3. Using Eq. (4), estimate without a calculator to *one* significant digit the power (energy per unit time, with units of joules/second or J/s) that needs to be removed from a one-millimeter metal square if the square is attached to the inside of a one liter box containing diatomic nitrogen (molecular weight 28 g) at STP (standard temperature and pressure so $T \approx 290\, K$ and $P \approx 1.0 \times 10^5\, \mathrm{N/m^2}$). Also assume that the gas parameters have not yet had time to change significantly from their initial values.

   Note: When estimating expressions without a calculator, please save time by simplifying your expression algebraically as much as possible **before** substituting any numbers. That is, avoid calculating intermediate numbers that are not needed for the final answer. Also round numbers to one digit before combining them. In future quizzes and exams, you will lose credit if I see you multiplying out any numbers by hand.

4. Determine mathematically and then show by an appropriate hand drawing (qualitative sketch) how the pressure $P(t)$, temperature $T(t)$, and number of particles $N(t)$ vary with time for this gas that is in steady contact with a tiny cold metal square. Make sure to state clearly any physical assumptions that you use in your analysis.

   From your analysis, determine and give an expression for the characteristic time τ for the pressure to change substantially.

Problem 3: Long-Term Speed of an Indestructible Sphere

An indestructible sphere of mass 100 kg is placed in the interstellar space of the Milky Way galaxy, a few light years from any star. What will its speed be (as measured from Earth) after a sufficiently long time?

Some comments: you can assume that the probability for the sphere to fall into a star is zero. (This is true even for stars, which are so tiny (2 light-seconds) compared to their average separation (a light year) that they essentially never physically collide, even when two galaxies collide with one another.) The mass of a typical star is of order 10^{30} kg, the typical relative speed of one star with respect to another is of order 10 km/s, and there are about 10^{10} stars in a typical galaxy.

Problem 4: Schroeder and Related Problems

1. Problem 1.21 on page 14 of Schroeder.
2. Problem 1.24 on page 17 of Schroeder.

3. Problem 1.26 on page 19 of Schroeder.
4. Problem 1.27 on page 19 of Schroeder.
5. Problem 1.31 on page 22 of Schroeder.

   Note: this should be an easy review problem and you should be able to do it, even though I won't get around to discussing Section 1.5 until the day the homework is due.

Problem 5: Time to Finish This Homework Assignment

Please tell me the approximate time in hours that it took you to complete this homework assignment.