HW 2

PROBLEM 1  (10 PT)

IN TIME T:  \( M = \frac{T}{\tau_A} \) STEPS

\( N = \frac{T}{\tau_B} \) STEPS

\( \Delta \vec{X}^{(t)} = \sum_{i=1}^{M} \Delta \vec{X}_i \),  \( |\Delta \vec{X}_i| = v_A \tau_A \)

\( \Delta \vec{Y}^{(t)} = \sum_{i=1}^{N} \Delta \vec{Y}_i \),  \( |\Delta \vec{Y}_i| = v_B \tau_B \)

\( \langle |X - \bar{X}|^2 \rangle = \langle \vec{X}^2 \rangle + \langle \vec{Y}^2 \rangle - 2 \langle \vec{X} \cdot \vec{Y} \rangle \)

BUT  \( \langle \vec{X}^2 \rangle = M d_x^2 \)

\( \langle \vec{Y}^2 \rangle = N d_y^2 \)
\[ \langle \vec{x} \cdot \vec{y} \rangle = \sum_{i=1}^{M} \sum_{j=1}^{N} \langle \Delta \vec{x}_i \cdot \Delta \vec{y}_j \rangle \]

\[ = \sum_{i=0}^{M} \sum_{j=0}^{N} \ d_i d_j \langle \cos \Theta_{ij} \rangle \]

But \( \langle \cos \Theta_{ij} \rangle \) is zero, since we are taking the average.

\[ \Rightarrow \langle |\vec{x} - \vec{y}|^2 \rangle = \prod d_x^2 + N d_y^2 \]

\[ = (\sigma_A^2 \ z_A + \sigma_B^2 \ z_B) \ \delta \]

\[ = D \ \delta \]
**Problem 2**

(1) 4 pt

\[ \Delta E = \int \Delta \theta A \cdot N \cdot \frac{1}{V} \cdot \frac{1}{2} m u^2 \]

Since \( \frac{1}{2} m u^2 \) is energy lost by each particle

\[ \frac{N}{V} \text{ density of particle} \]

\[ \int \Delta \theta A = \Delta V \text{ volume} \]

\[ \frac{1}{6} = \text{fraction hitting the surface} \]
\[
\frac{dE}{dt} = U A \frac{N}{V} \frac{1}{6} u^2
\]

And
\[
N = \frac{P}{kT} \quad \Rightarrow \quad U = \sqrt{\frac{3kT}{m}}
\]

Since
\[
\frac{1}{2} \langle m u^2 \rangle = \frac{3}{2} kT
\]

\[
\Rightarrow \quad \frac{dE}{dt} = \frac{1}{12} A \frac{P}{kT} m \left( \frac{3kT}{m} \right)^{3/2}
\]

\[
= \frac{1}{h} A P \left( \frac{3kT}{m} \right)^{3/2}
\]
(2) 4 pt.

Now

\[ v \cos \theta \Delta t \Delta A = \text{volume} \]

\[ \Rightarrow \Delta E(\theta, \varphi) = \left[ (v \cos \theta) \Delta t \Delta A \right] \frac{N}{V} \frac{\sin \theta \, d\theta \, d\varphi}{\mu I} \frac{1}{\mu_0} \mu_0 \]

Again

\[ \omega = \sqrt{\frac{3kT}{m}} \]

\[ \frac{dE}{dt} = \iint_{0}^{\frac{5}{2}} \int_{0}^{\frac{5}{2}} \frac{A}{8\pi} \frac{m}{(3kT/m)^{3/2}} \frac{P}{kT} \cos \theta \sin \theta \, d\theta \, d\varphi \]

\[ = \frac{3AP}{8\pi} \left( \frac{3kT}{m} \right)^{1/2} \int_{0}^{\pi/2} \cos \theta \sin \theta \, d\theta \int_{0}^{2\pi} \, d\varphi \]

\[ = \frac{3}{8} AP \left( \frac{3kT}{m} \right)^{1/2} \]
(3) 4 PT.

\[ \frac{dE}{dt} = \frac{3}{8} \times 10^{-6} \text{ m} \cdot 10^3 \frac{N}{m^2} \left[ \frac{2(8 \frac{3m}{K\text{mol}})(3 \cdot 10^8 \text{ k})}{3 \cdot 10^{-2} \text{ K}^2/\text{mol}} \right]^{7/2} \]

\[ = 20 \frac{\text{J}}{\text{s}} \]

(4) 4 PT.

\[ \frac{1}{2} m \langle u^2 \rangle = \frac{3}{2} k T \]

\[ \Rightarrow T = \frac{m \langle u^2 \rangle}{3k} = \text{const.} \]

\[ \frac{dN}{dt} = \int_{0}^{\pi} \int_{0}^{2\pi} u \cos \theta \frac{A}{V} \frac{N}{2} \frac{\sin \theta \, d\theta \, d\psi}{4\pi} \]

\[ = \frac{1}{4} \frac{A}{V} \left( \frac{3kT}{m} \right)^{3/2} N \]
\[ N(t) = N_0 e^{-\frac{t}{\tau}} \]

WHERE \[ \tau = \frac{4 \gamma}{A} \left( \frac{3 k T}{m} \right)^{3/2} \]

AND FROM \[ PV = N k T \]

\[ P(t) = \frac{k T}{V} N_0 e^{-\frac{t}{\tau}} = P_0 e^{-\frac{t}{\tau}} \]
- PROBLEM 3 (10 pts.)

\[ \frac{1}{2} M_{\text{star}} \langle U_{\text{star}}^2 \rangle = \frac{1}{2} M_{\text{sphere}} \langle U_{\text{sphere}}^2 \rangle \]

Since both are equal to \( \frac{3}{2} k T \)

\[ u_{\text{sphere}} = \left( \frac{M_{\text{sphere}}}{M_{\text{star}}} \right)^{1/2} u_{\text{star}} \]

\[ \approx 10^{-18} \text{ m/s} \quad \Rightarrow \quad c = 3 \cdot 10^8 \text{ m/s} \]
Problem 1.21. The hailstones strike the window at intervals of $1/30$ second (on average). During this time, the average force exerted by the window on the hailstone must be
\[ F_x = m \frac{\Delta v_x}{\Delta t}, \]
where $\Delta v_x$ is the change in the component of the hailstone's velocity perpendicular to the window. Assuming elastic collisions and a velocity of 15 m/s at 45°, this change in velocity is $2v \cos 45° = 21$ m/s. The average pressure is just the average force divided by the surface area:
\[ P = \frac{F}{A} = \frac{m \Delta v_x}{A \Delta t} = \frac{(0.002 \text{ kg})(21 \text{ m/s})}{(0.5 \text{ m}^2)(0.033 \text{ s})} = 2.5 \text{ N/m}^2. \]
This is less than atmospheric pressure by a factor of about 40,000. (However, the instantaneous pressure during a collision is much higher, and the force of each hailstone is localized, not distributed over the whole window.)

Problem 1.24. Each lead atom has six degrees of freedom: three from kinetic energy and three from potential, corresponding to vibrations in the three orthogonal directions. The atomic mass of lead is about 207, so a gram of lead contains $1/207$ moles. Therefore the total thermal energy is approximately
\[ U_{\text{thermal}} = \frac{1}{2} k_n R T = \frac{6}{2} \frac{1}{207}(8.31 \text{ J/K})(300 \text{ K}) = 36 \text{ J}. \]

Problem 1.26. The flow of energy from the battery to the resistor is work, not heat. Even though the resistor gets warm, there is no spontaneous flow of energy from the battery to the resistor that is caused by a difference in their temperatures. In fact, the battery could very well be cooler than the resistor, yet keep providing energy to it. The flow of energy from the resistor to the water, however, is a spontaneous one caused by the resistor being hotter than the water. This energy flow is therefore classified as heat, not work.
Problem 1.27. Temperature increase with no heat added: The resistor in the previous problem provides an example; it gets hot as the battery supplies energy in the form of work (not heat). Other examples would be "heating" a cup of tea in the microwave, or compressing air to pump up a bicycle tire, or simply rubbing your hands together. Heat input with no increase in temperature: I can think of two types of examples. The first is a phase change, like boiling a pot of water on the stove. Heat is constantly flowing in, but the temperature of the water remains at 100°C (or at whatever the boiling temperature is at your altitude). The second type of example is when the system does work on its surroundings to compensate for the energy put in as heat. For instance, you could have a gas in a cylinder with a flame under it, while letting the piston out fast enough that the gas actually cools.

Problem 1.31. (A helium expansion example.)

(a)

(b) The work done is minus the area under the graph (shaded). The easiest way to compute this area is to note that the average pressure during the process is 2 atm, so

\[ W = -P \Delta V = -(2 \text{ atm})(2 \text{ liters}) \approx -(2 \times 10^5 \text{ Pa})(2 \times 10^{-3} \text{ m}^3) = -400 \text{ J}. \]

The minus sign indicates that 400 J of work is done by the gas on its surroundings.

(c) Each helium atom has three degrees of freedom, so at any point the thermal energy of the helium is \( U = \frac{3}{2} N k T = \frac{3}{2} PV \). The change in energy during this process is

\[ \Delta U = \frac{3}{2} [P_f V_f - P_i V_i] = \frac{3}{2} [(3 \text{ atm})(3 \text{ liters}) - (1 \text{ atm})(1 \text{ liter})] \]

\[ = 12 \text{ liter} \cdot \text{ atm} = 1200 \text{ J}. \]

(d) By the first law,

\[ Q = \Delta U - W = 1200 \text{ J} - (-400 \text{ J}) = 1600 \text{ J}. \]

This amount of heat enters the gas.

(e) To cause such an increase in pressure (and temperature) as the gas expands, you must provide heat, for instance, by holding a flame under the cylinder and letting the piston out slowly enough to allow the pressure to rise as desired.