Print your name clearly:____

Signature:____

"I agree to neither give nor receive aid during this quiz."

Quiz 5 for Physics 176

Professor Greenside
Thursday, April 7, 2011

This 15-minute quiz is closed book and no electronic devices are allowed on your table. Please write all answers on the supplementary blank pages so that you will have plenty of space, and write your name and problem number on each supplementary page. If your answer is not reasonably readable, you will lose credit. Unless stated otherwise, you need to explain or justify your reasoning to get full credit.

Problems That Require Writing

1. A scientist wants to study the properties of a certain equilibrium solid substance for various values of the temperature T, volume V, and chemical potential μ by placing the substance in a large reservoir for which these three parameters are held constant during a given experiment.

   (a) **(5 points)** Find an appropriate thermodynamic potential $K(T, \mu, V)$ for this experimental setup.

   (b) **(5 points)** Show how to calculate the entropy S, pressure P, and the number of particles N for this system in terms of K.

2. **(6 points)** A single helium atom with mass m_{He} is placed inside an ideal gas of neon atoms, each with mass $m_{\mathrm{Ne}} > m_{\mathrm{He}}$. If the gas is in thermodynamic equilibrium with temperature T, write down in terms of m_{He}, m_{Ne}, and T a mathematical expression for the probability for the He atom to have a speed between $100\,\mathrm{m/s}$ and $400\,\mathrm{m/s}$.

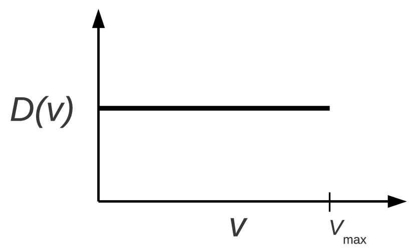
   Note: you do not have to evaluate your expression, just write it down.

3. **(10 points)** A physical system consists of N identical, non-interacting, and distinguishable objects, each of which has two non-degenerate energy levels, $E_1 = 0$ and $E_2 = \epsilon > 0$. If this system is in equilibrium with a reservoir that has a constant temperature T, derive an expression for the entropy $S(T)$ of this system as a function of temperature T, evaluate the infinite-temperature limit of your expression, and discuss whether your infinite-temperature limit of S makes physical sense.

   Hint: relate S to F.

4. **(6 points)** Consider a three-dimensional gas of identical molecules that is in thermodynamic equilibrium with temperature T. Find the probability density $\mathcal{D}(\theta)$ for observing a molecule to have a velocity vector $\mathbf{v}$ that makes an angle θ with the z axis for angles θ that lie in the range $[\theta, \theta + d\theta]$.

5. **(6 points)** What is the most likely kinetic energy $E = (1/2)mv^2$ for a gas of molecules whose speed distribution $\mathcal{D}(v)$ is constant over the range $0 \le v \le v_{\max}$ and zero for $v > v_{\max}$, as shown in the figure below?



The following equations may be useful:

$$dU = T dS - P dV + \mu\, dN, \qquad F = U - TS, \qquad F = -kT\ln(Z). \tag{1}$$

$$\beta = \frac{1}{kT}, \qquad Z = \sum_s e^{-\beta U(s)} = \sum_n d_n e^{-\beta E_n}, \qquad p_s = \frac{e^{-\beta E_s}}{Z}. \tag{2}$$

$$Z_N = Z_1^N \quad \text{for distinguishable independent identical particles.} \tag{3}$$

$$\langle X \rangle = \sum_s p_s X_s, \qquad \langle X \rangle = \frac{\int e^{-\beta E(q)} X(q)\, dq}{\int e^{-\beta E(q)}\, dq}, \qquad \langle E \rangle = \sum_s p_s E_s = -\frac{1}{Z}\frac{\partial Z}{\partial \beta}. \tag{4}$$

$$\mathcal{D}(v) = \left(\frac{m}{2\pi kT}\right)^{3/2} 4\pi v^2\, e^{-\beta m v^2/2}, \qquad 1 = \int_{-\infty}^{\infty} dv_x \int_{-\infty}^{\infty} dv_y \int_{-\infty}^{\infty} dv_z \, \left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\beta m v^2/2}. \tag{5}$$

$$v^2 = v_x^2 + v_y^2 + v_z^2, \qquad dv_x\, dv_y\, dv_z = v^2 \sin(\theta)\, dv\, d\theta\, d\phi. \tag{6}$$