Problems That Require Writing

1. A scientist wants to study the properties of a certain equilibrium solid substance for various values of the temperature $T$, volume $V$, and chemical potential $\mu$ by placing the substance in a large reservoir for which these three parameters are held constant during a given experiment.

   (a) (5 points) Find an appropriate thermodynamic potential $K(T, \mu, V)$ for this experimental setup.

   (b) (5 points) Show how to calculate the entropy $S$, pressure $P$, and the number of particles $N$ for this system in terms of $K$.

2. (6 points) A single helium atom with mass $m_{\text{He}}$ is placed inside an ideal gas of neon atoms, each with mass $m_{\text{Ne}} > m_{\text{He}}$. If the gas is in thermodynamic equilibrium with temperature $T$, write down in terms of $m_{\text{He}}$, $m_{\text{Ne}}$, and $T$ a mathematical expression for the probability for the He atom to have a speed between 100 m/s and 400 m/s.

   Note: you do not have to evaluate your expression, just write it down.

3. (10 points) A physical system consists of $N$ identical, non-interacting, and distinguishable objects, each of which has two non-degenerate energy levels, $E_1 = 0$ and $E_2 = \epsilon > 0$. If this system is in equilibrium with a reservoir that has a constant temperature $T$, derive an expression for the entropy $S(T)$ of this system as a function of temperature $T$, evaluate the infinite-temperature limit of your expression, and discuss whether your infinite-temperature limit of $S$ makes physical sense.

   Hint: relate $S$ to $F$.

4. (6 points) Consider a three-dimensional gas of identical molecules that is in thermodynamic equilibrium with temperature $T$. Find the probability density $D(\theta)$ for observing a molecule to have a velocity vector $v$ that makes an angle $\theta$ with the $z$ axis for angles $\theta$ that lie in the range $[\theta, \theta + d\theta]$. 
5. **(6 points)** What is the most likely kinetic energy \( E = \frac{1}{2}mv^2 \) for a gas of molecules whose speed distribution \( D(v) \) is constant over the range \( 0 \leq v \leq v_{\text{max}} \) and zero for \( v > v_{\text{max}} \), as shown in the figure below?

![Diagram of speed distribution](image)

The following equations may be useful:

\[
dU = TdS - PdV + \mu dN, \quad F = U - TS, \quad F = -kT \ln(Z). \tag{1}
\]

\[
\beta = \frac{1}{kT}, \quad Z = \sum_s e^{-\beta U(s)} = \sum_n d_n e^{-\beta E_n}, \quad p_s = \frac{e^{-\beta E_s}}{Z}. \tag{2}
\]

\[
Z_N = Z_N^1 \quad \text{for distinguishable independent identical particles.} \tag{3}
\]

\[
\langle X \rangle = \sum_s p_s X_s, \quad \langle X \rangle = \frac{\int e^{-\beta E(q)} X(q) dq}{\int e^{-\beta E(q)} dq}, \quad \langle E \rangle = \sum_s p_s E_s = -\frac{1}{Z} \frac{\partial Z}{\partial \beta}. \tag{4}
\]

\[
D(v) = \left( \frac{m}{2\pi kT} \right)^{3/2} 4\pi v^2 e^{-\beta mv^2/2}, \quad 1 = \int_{-\infty}^{\infty} dv_x \int_{-\infty}^{\infty} dv_y \int_{-\infty}^{\infty} dv_z \left( \frac{m}{2\pi kT} \right)^{3/2} e^{-\beta mv^2/2}. \tag{5}
\]

\[
v^2 = v_x^2 + v_y^2 + v_z^2, \quad dv_x dv_y dv_z = v^2 \sin(\theta) dv d\theta d\phi. \tag{6}
\]