

Print your name clearly:_____

Signature:_____

“I agree to neither give nor receive aid during this quiz.”

Quiz 2 for Physics 176

Professor Greenside
Tuesday, February 8, 2011

This 15-minute quiz is closed book and no electronic devices are allowed on your table. There are equations listed at the end of this quiz that may be helpful for some of the problems.

All answers should be written on supplementary blank pages so that you will have plenty of space. **Please write your name and problem number on each page.** If your answer is not reasonably readable, you will lose credit. Unless stated otherwise, **you need to explain or justify your reasoning to get full credit.**

Problems That Require Writing

1. **(4 points)** It is possible for an equilibrium system to have a negative temperature T . Using the fact that the entropy of an isolated nonequilibrium system must increase over time, $dS/dt > 0$, show that a negative temperature subsystem is hotter than any positive temperature subsystem in that heat will always transfer from the negative temperature subsystem to the positive temperature subsystem.
2. **(4 points)** The entropy $S(U)$ of a certain system depends on the energy U of the system according to the relation $S = a + b \ln(U)$ where a and b are positive constants. Calculate the heat capacity $C_V(T)$ at constant volume of this system in terms of a and b .
3. **(6 points)** Consider a system consisting of two distinguishable tetrahedral dice with sides numbered from 1 to 4:



Define a macrostate of this system to be the sum s of the digits of the two bottom faces (the faces resting on the ground), e.g. one macrostate would be $2 + 3 = 5$. List all the macrostates of this system, list all the microstates of this system, and then calculate the multiplicity $\Omega(s)$ —the number of microstates consistent with a given macrostate—for each of the macrostates.

(Turn to the next page)

4. **(8 points)** An ideal gas with initial volume V_0 and initial pressure P_0 is cooled at constant pressure until its volume is $V_0/2$ and then the gas is expanded adiabatically back to its original volume. Determine for this two-step process whether the following quantities are positive, negative, or zero:
- (a) the total change in energy ΔU_{total} .
 - (b) the total work done on the gas W_{total} .
 - (c) the total heat Q_{total} transferred to the gas.
 - (d) the total change in temperature ΔT .

Note: do not waste precious time calculating the details of ΔU , W , and Q for the two steps, think qualitatively to get the signs directly. Also, make sure to explain briefly how you got your answers.

5. **(8 points)** At atmospheric pressure, water molecules H_2O freeze at $T \approx 270 \text{ K}$, boil at $T \approx 370 \text{ K}$, and, in the liquid phase, have a heat capacity at constant pressure for one mole of $C_P \approx 75 \text{ J/K}$ that is roughly independent of temperature.

Using your knowledge of how the heat capacity C varies with temperature experimentally for solids and for gaseous H_2 , draw a plot of the ratio $C_P(T)/R$ versus T (where the gas constant $R \approx 8.3 \text{ J/(mole}\cdot\text{K)}$) that represents your prediction about how the heat capacity at constant pressure of one mole of water molecules will vary over the temperature range $0 \leq T \leq 3000 \text{ K}$.

Please be sure to indicate the numerical values on the vertical axis where various plateaus occur so that your prediction can be quantitatively compared with experimental data.

You can assume that at the highest temperature of your plot, all degrees of freedom are active and that, as the temperature decreases, the degrees start to freeze out in ways that are similar to systems that you are familiar with.

The following formulas may be useful:

$$PV = NkT = nRT, \quad \Delta U = Q + W, \quad U = Nf \frac{kT}{2} = \frac{f}{2}PV, \quad W = - \int_{V_1}^{V_2} P(V) dV.$$

$$f = 2(3K - 6) + 3 + 3 \quad \text{or} \quad f = 2(3K - 5) + 3 + 2.$$

$$C = \frac{Q}{\Delta T}, \quad C_V = \left(\frac{\partial U}{\partial T} \right)_V, \quad C_P = \left(\frac{\partial U}{\partial T} \right)_P + P \left(\frac{\partial V}{\partial T} \right)_P.$$

$$S = S_1(U_1) + S_2(U_2), \quad U_1 + U_2 = U, \quad \frac{1}{T} = \left(\frac{\partial S}{\partial U} \right)_{V,N}.$$