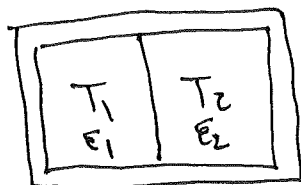


Quiz 2 Answers

Physics 176 2/15/2011

1.

1.



$E_1 + E_2 = \text{const}$ since system isolated

$$\frac{dS}{dt} = \frac{d}{dt} (S_1(E_1) + S_2(E_2)) = \frac{dS_1}{dE_1} \frac{dE_1}{dt} + \frac{dS_2}{dE_2} \frac{dE_2}{dt}$$

$$\frac{1}{T_1} = \frac{dS_1}{dE_1} \quad \frac{1}{T_2} = \frac{dS_2}{dE_2}$$

$$\frac{dE_2}{dt} = \frac{d}{dt} (E - E_1) = - \frac{dE_1}{dt}$$

$$\boxed{\frac{dS}{dt} = \frac{dE_1}{dt} \left(\frac{1}{T_1} - \frac{1}{T_2} \right) > 0}$$

key equation to start with

assume $T_1 > 0$, $T_2 < 0$. Then $\frac{1}{T_1} - \frac{1}{T_2}$ is sum of two positive terms and is positive. Since

$$\frac{dE_1}{dt} \left(\frac{1}{T_1} - \frac{1}{T_2} \right) > 0$$

this implies $\frac{dE_1}{dt} > 0$, i.e. energy of E_1 increases with time, which means heat transfers from negative temperature system to positive.

Note: what if $0 > T_1 > T_2$?

2. If $S = a + b \ln U$ then

$$\frac{1}{T} = \frac{\partial S}{\partial U} = \frac{\partial}{\partial U} (a + b \ln U) = \frac{b}{U}$$

so $U = bT$ and $\boxed{C_V = \frac{dU}{dT} = b}$

need to discover or determine T from known $S = S(U)$

3. There are $4 \times 4 = 16$ microstates since any value of die 1 can combine with any value of die 2.

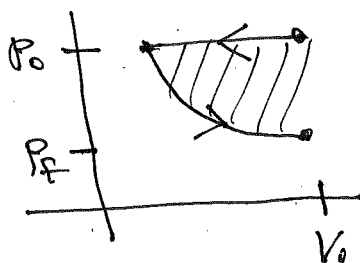
There are 7 macrostates corresponding to the sums

2, 3, 4, 5, 6, 7, 8

Their multiplicities are as follows:

<u>sum S</u>	<u>$\Omega(S)$</u>	<u>accessible microstates</u>
2	1	1+1
3	2	1+2, 2+1
4	3	1+3, 3+1, 2+2
5	4	1+4, 4+1, 2+3, 3+2
6	3	2+4, 4+2, 3+3
7	2	3+4, 4+3
8	1	4+4

(3)

Problem 4

Key step was to understand what this process looks like.

Adiabatic and isothermal expansion correspond to decreases in pressure, follows from $V^\gamma P = \text{const}$ (bigger V implies smaller P) or from $PV = \text{const}$ for isothermal.

Can now answer parts:

(a) From $U = \frac{F}{2} PV$ we conclude

$$\Delta U = \frac{F}{2} \Delta(PV) = \frac{F}{2} (P_F V_F - P_i V_i)$$

$$= \frac{F}{2} V_0 (P_F - P_0) < 0 \quad \text{so } \boxed{\Delta U < 0}$$

since $P_F < P_0$. Note how I didn't need to compute what happens for each step and I didn't need to know any details of the expansion, just that the final pressure is lower than the initial pressure

(credit)

(4)

4b: Work W is given by

$$W = - \int_{v_0}^{v_0/2} P_1(v) dv - \int_{v_0/2}^{v_0} P_2(v) dv$$

$$= - \int_{v_0}^{v_0/2} (P_1(v) - P_2(v)) dv$$

it is the area between the compression and expansion curves (shaded). The sign of W is given by which curve is higher. Here the compression curve lies above the expansion curve, more work is added than removed. Conclude

$$W > 0$$

4c Always compute heat Q after computing ΔU and W .
From conservation of energy

$$Q_{\text{tot}} = \Delta U_{\text{tot}} - W_{\text{tot}} < 0$$

since $\Delta U_{\text{tot}} < 0$ and $W_{\text{tot}} > 0 \Rightarrow -W_{\text{tot}} < 0$.

4d $U = Nf \frac{kT}{2} \Rightarrow \Delta U = Nf \frac{k}{2} \Delta T$

so $\Delta U < 0 \Rightarrow \Delta T < 0$

(5)

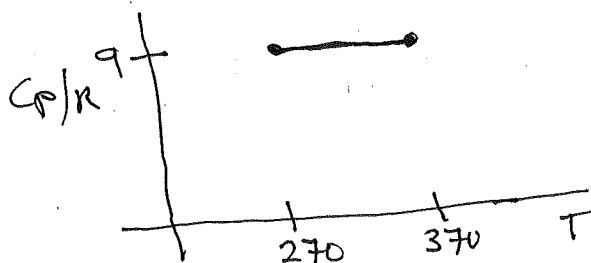
Problem 5

Let's collect information before making a plot.

We are told $C_p \approx 75 \frac{\text{J}}{\text{K}}$ in liquid phase so

$$C_p/R = \frac{75}{8.3} \approx \frac{70}{8} \approx 9 \quad 270 < T < 370$$

So part of curve will look like this:



We know from reading Schroeder page 32 bottom that, when substance melts or boils,

$$C_p = \infty \quad \text{at } T \approx 270, \approx 370$$

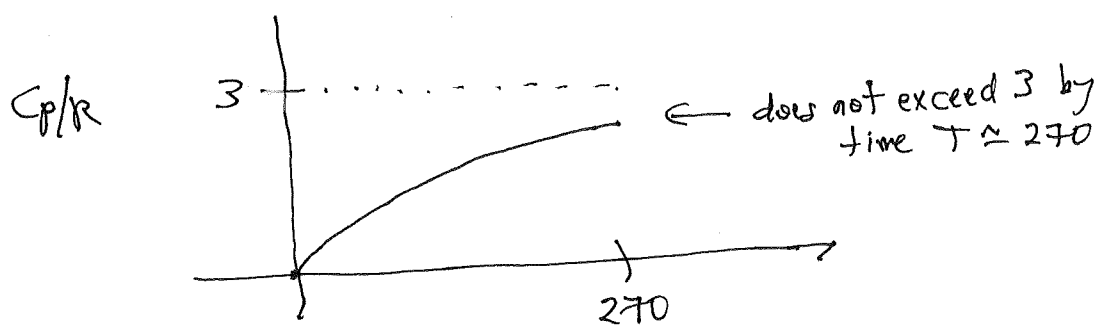
(Nearly all students missed this fact!)

For ice with $T < 270$, we have solid so expect $F=6$
and $C_p \approx C_v \approx N F \frac{k_B}{2} = 3 N k_B = 3 R$ For one mole

(next)

(6)

From Fig 1.14 on p.30 of Schroeder that you memorized for the quiz, $C_p(T)$ increases from zero at $T=0$ but reaches $3R$ by $T \approx 270$ for lead and aluminum but is below $3R$ for diamond. So all we can predict is that $C_p(T)$ will increase from zero and will not get higher than $3R$ and could fall below $3R$ when ice melts. So reasonable guess is:



Gas phase was trickiest. According to Fig 1.13¹⁵ on p. 30 that give $C_v(T)$ for H_2 , rotational degrees of freedom have already activated when water boils, $T \approx 400$ K

So for $T \approx 400$ K, there are 3 rotational and 3 translational degrees of freedom so $C_v = N \cdot \frac{7}{2} k \approx 3R$

But for ideal gas $C_p = C_v + R$

so $\boxed{C_p = 4R}$

when all rotational and translational degrees are active

(7)

Since water is non-collinear molecule, total degrees of Freedom given by formula on page 2 of quiz:

$$\begin{aligned} f &= 2(3K-6) + 6 = 6(K-1) = 6(3-1) \\ &= 12 \end{aligned}$$

$K=3$ for $\text{H}-\text{O}-\text{H}$

So at highest temperatures, when rot, trans, vibrational degrees of Freedom active expect

$$C_p = R + C_v = R + R \cdot \frac{f}{2} = 7R$$

Putting everything together, we make this prediction for $C_p(T)/R$ vs T for water:

