

Print your name clearly: _____

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“I agree to neither give nor receive aid during this quiz.”

Quiz 5 for Physics 176

Professor Greenside
Thursday, April 8, 2010

This quiz is closed book and no calculators or any other electronic devices are allowed.

Please write all answers on the supplementary blank pages. Also, write your name and problem number on each extra page.

The following equations may be useful:

$$\bar{E} = -\frac{\partial \ln(Z)}{\partial \beta}, \quad Z_N = \frac{Z_1^N}{N!}, \quad Z_1 = \frac{V Z_{\text{int}}}{V_Q}, \quad V_Q = \left(\frac{h}{\sqrt{2\pi m k T}} \right)^3. \quad (1)$$

$$\bar{E} = \sum_s E(s) p_s, \quad p_s = \frac{e^{-\beta[E(s) - \mu_A N_A(s) - \mu_B N_B(s)]}}{\mathcal{Z}}, \quad \mathcal{Z} = \sum_s e^{-\beta[E(s) - \mu_A N_A(s) - \mu_B N_B(s)]}. \quad (2)$$

- (10 points)** A certain olfactory receptor in your nose has three “slots” such that one slot can bind a single molecular odorant of type A with energy $E_A < 0$, while each of the other two slots can independently bind a single molecular odorant of type B , with binding energy $E_B < 0$. (The energy of the receptor is zero if nothing binds to it, and the receptor can bind up to three molecules.) The receptor is in contact with a reservoir (the air inside your nose) whose temperature is T and whose chemical potentials for molecules A and B are μ_A and μ_B respectively, which you can assume to be constant. Write down an expression for the probability for an A odorant and a single B odorant to be bound to this receptor.
- (10 points)** Consider a *gaseous* paramagnet of N identical atoms. Each atom is a two-state magnetic dipole of strength μ and can move freely so that all the atoms together form an ideal gas in thermodynamic equilibrium with temperature T . The gas is immersed in a uniform external magnetic field of magnitude B so the allowed dipole energies are $\pm\mu B$. The density of the gas is so low that the magnetic dipoles do not influence one another.

For this gaseous paramagnet, make a qualitative sketch of how the heat capacity varies with temperature T , with the vertical axis being $C/(Nk)$ and the horizontal axis being $kT/(\mu B)$. Where possible, indicate approximate or known values of $C/(Nk)$ and $kT/(\mu B)$ associated with the significant features of your plot. Justify the details of your sketch with appropriate calculations.
- (10 points)** A two-dimensional box of area A contains an ideal two-dimensional gas of N molecules of mass M and temperature T . If a small hole of length L is opened up in the side of this box, derive an expression for the total flux Φ (particles per unit time per unit length) of particles coming out of the hole. Assume that the two-dimensional velocities of the molecules are isotropic (the vectors point

in all possible directions in the plane) and that the speeds of the molecules satisfy a two-dimensional Maxwell speed distribution $\mathcal{D}_{2D}(v)$. Your answer should be in terms of the number density $n = N/A$ and some characteristic speed such as $\bar{v}$ or v_{rms} .

Hint: The 2D analogue of the factor $\sin(\theta) d\theta d\phi/(4\pi)$ is $d\theta/(2\pi)$.

True or False Questions (2 points each)

For each of the following statements, please circle **T** or **F** to indicate respectively whether a given statement is true or false.

1. **T / F** For a small system that can exchange energy and particles with a reservoir whose temperature T and chemical potential μ are constant, the average energy $\bar{E}$ of the system is given by

$$\bar{E} = -\frac{1}{\mathcal{Z}} \frac{\partial \mathcal{Z}}{\partial \beta}, \quad (3)$$

where $\mathcal{Z}$ is the Gibb's sum (grand partition function).

2. **T / F** In setting up a computer simulation of a one-dimensional gas, a student arranges the atoms in the gas to have a constant speed distribution $\mathcal{D}(v) = \text{constant}$, which means that all speeds are equally likely between 0 and some maximum speed v_{max} . Then the most likely kinetic energy E for the atoms in this gas is $(1/4)mv_{\text{max}}^2$.
3. **T / F** The expression $\mathcal{A} = U - \mu N$ is a thermodynamic potential that would be appropriate to use for problems in which an experimentalist wants to control the variables S , P , and μ .
4. **T / F** For a small system that can exchange energy and particles with a reservoir whose temperature T_R and chemical potential μ_R are constant, the expression $\Phi_s = U_s - T_s S_s - \mu_s N_s$ will decrease monotonically until the system reaches thermodynamic equilibrium, where the subscript s denotes variables that describe the system.