

Print your name clearly:_____

Signature:_____

“I agree to neither give nor receive aid during this quiz.”

Quiz 1 for Physics 176

Professor Greenside
Friday, January 26, 2010

This quiz is closed book.

1. **Please write your name and the problem number on each page of paper** that you use for the problems that required writing. However, the true/false questions should be answered on this page.
2. For questions that are not true/false or multiple choice, **justify each answer briefly**, with appropriate calculations, sketches, phrases, or sentences.

Problems That Require Writing

All answers in this section of the quiz must be written on the supplementary blank pages. Please remember to write your name and problem number on each extra page.

1. A laser consists of a long cylindrical tube of some medium that has mirrors at both ends such that light bounces back and forth between the mirrors while its intensity is amplified. (Light not parallel to the axis of the cylinder is lost rapidly since the outer surface of the cylinder is often transparent.) Assume that the laser light is monochromatic with frequency f , which means that the laser cavity is filled with photons (quantum light particles) that each have momentum $p = (h/c)f$ where h is Planck's constant and c is the speed of light. The total energy of the light in the laser is given by $E = Nhf$ where N is the number of photons in the cavity.
 - (a) **(6 points)** Use a simple kinetic theory to derive an expression for the pressure P that the light exerts on a mirror in terms of the photon number N , the volume V of the cylinder, and the light frequency f .
 - (b) **(2 points)** Discuss briefly how your expression for P is or is not similar to the expression NkT/V for the pressure of an ideal gas at temperature T .
2. **(4 points)** A bacterium like *E. coli* is about $d = 2\mu\text{m}$ in length ($1\mu\text{m} = \text{one micron} = 10^{-6}\text{m}$) and swims with an approximately steady speed v . A bacterium will starve if it depletes resources in the surrounding water faster than diffusion can supply the resources. By comparing the time scale for a bacterium to swim one body length with the time scale for diffusion to supply nutrients over the size of its body, and given that nearly all small molecules in water at room temperature have a diffusion constant of $D \approx 10^{-9}\text{m}^2/\text{s}$, deduce an approximate minimum speed v that a bacterium must swim to “beat” diffusion and so survive. Express your answer v in units of microns per second (i.e., approximate body lengths per second).

Note: by comparing your answer with experimental measurements of v , you can learn whether diffusion plays a role in determining how fast a bacterium has to swim.

3. **(3 points)** Give three distinct criteria for a macroscopic system to be in thermodynamic equilibrium.

4. **(4 points)** Given that

$$e^x \approx 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3, \quad \text{for } |x| \ll 1, \quad (1)$$

find the Taylor series expansion of the hyperbolic tangent

$$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}},$$

up to third-order order in x , i.e., ignore all powers higher than x^3 , which provides a 4th-order accurate approximation.

True or False Questions (2 points each)

For each of the following statements, please circle **T** or **F** to indicate respectively whether a given statement is true or false. No justification is needed, but small drawings or comments in the margins can be taken into account.

1. **T / F** There can be macroscopic motion at absolute zero, $T = 0$,
2. **T / F** If you are blindfolded and then touch a flat polished piece of metal with your left hand and simultaneously touch a flat polished piece of wood with your right hand, and if both the wood and metal have the same temperature as your body (about 37°C), then the piece of metal will feel colder than the wood.
3. **T / F** If k is the Boltzmann constant and T is the temperature in kelvin, then kT has units of joules/second.
4. **T / F** As a non-equilibrium system approaches thermodynamic equilibrium, it typically reaches thermal equilibrium before it reaches diffusive equilibrium.
5. **T / F** Rooms A and B are the same size and room A is kept warmer than room B by a heating unit. Then after the rooms are connected by an open door and allowed to reach a steady state, room B will have a greater mass of air than room A.
6. **T / F** If $x = 2 + y^{1/2}$ then $x \propto y^{1/2}$.
7. **T / F**

$$\int_0^1 dx \int_0^x dy \, x^2 y = \frac{1}{6}.$$