

Print your name clearly:\_\_\_\_\_

Signature:\_\_\_\_\_

“I agree to neither give nor receive aid during this quiz.”

### Quiz 4 for Physics 176

Professor Greenside  
Friday, March 6, 2009

This quiz is closed book. Please:

1. **Write your answers on separate pages of blank paper** that will be handed out. Do not write your answers on this exam except for the true/false answers.
2. **Write your name and the problem number on each page of paper** that you use.
3. **Justify each answer briefly**, with appropriate calculations, sketches, phrases, or sentences.

The following data and formulas may be useful:

$$dU = TdS - PdV + \mu dN, \quad S = \frac{Q}{T}, \quad k \approx 1.4 \times 10^{-23} \text{ J/K}.$$

$$M = -\frac{U}{B} = N\mu \tanh\left(\frac{\mu B}{kT}\right).$$

### True or False Questions (2 points each)

For each of the following statements, please circle **T** or **F** to indicate respectively whether a given statement is true or false.

1. **T / F** A paramagnet in its lowest energy state (all  $N$  magnetic dipoles parallel to the external magnetic field  $B$ ) has a negative temperature  $T$ .
2. **T / F** It is possible for the entropy of a system to increase after the system undergoes an adiabatic process (no heat into or out of the system).
3. **T / F** If two macroscopic systems  $A$  and  $B$  can exchange particles of the same type and together form an isolated system, and if the chemical potentials of these systems satisfy  $\mu_A < \mu_B$ , then particles will spontaneously move from system  $A$  to system  $B$ .
4. **T / F** There are substances for which their experimentally measured entropy  $S(0)$  at  $T = 0$  is not zero.
5. **T / F** Consider an isolated system of two macroscopic subsystems  $A$  and  $B$  that each contain a mixture of two kinds of molecules, labeled type 1 and type 2, that can move between the two subsystems. If  $\mu_{1A}$  denotes the chemical potential for molecule 1 in system  $A$  with a similar notation for other combinations of molecules and subsystems, then a necessary condition for the isolated system to be in thermodynamic equilibrium is:  $\mu_{1A} = \mu_{1B} = \mu_{2A} = \mu_{2B}$ .

## Problems That Require Writing

All answers in this section of the quiz must be written on the supplementary blank pages. Please remember to write your name and problem number on each extra page.

1. **(6 points)** The Sun has a surface temperature of about 6000 K while the Earth has a surface temperature of about 300 K on a sunlit day. A square meter of Earth's surface receives about 1000 watts (J/s) of power from sunlight when the Sun is high in the sky. From this information and assuming eight hours of daylight per day, estimate to one significant digit the total change in entropy (Earth plus Sun) when a square meter of soil is exposed to sunlight for a 30-day month.

Note: You can compare your answer with the decrease in entropy associated with growing several kilograms of plants on this square meter of land over a month. In an earlier homework problem (2.36 of Schroeder), you learned to estimate the order-of-magnitude of the entropy associated with  $N$  particles, namely  $S \approx Nk$ . Several kilograms of plants corresponds to about 1,000 moles of atoms and so the entropy decrease associated with the plant growth should be of order  $\Delta S \approx -Nk \approx -[1,000 \text{ moles} \times (6 \times 10^{23} \text{ atoms/mole})] \times (1.4 \times 10^{-23} \text{ J/K}) \approx -10^4 \text{ J/K}$ .

2. **(6 points)** Consider an equilibrium paramagnet that consists of  $N$  magnetic dipoles each of magnitude  $\mu$ . The paramagnet is at temperature  $T$  in the presence of a uniform magnetic field of strength  $B$ . For two points each, draw qualitatively
- (a) the total magnetization  $M/(N\mu)$  as a function of temperature  $kT/(\mu B)$ . (Note that  $-1 \leq M/(N\mu) \leq 1$  and  $-\infty \leq kT/(\mu B) < \infty$ .)
  - (b) the temperature  $kT/(\mu B)$  as a function of energy  $U/(\mu B)$ . (Note that  $-1 \leq U/(\mu B) \leq 1$ .)
  - (c) The specific heat  $C/(Nk)$  as a function of temperature  $kT/(\mu B)$ .

For this problem only, you do *not* have to write anything to justify these figures.

3. **(4 points)** The entropy  $S = S(N, U, A)$  of an ideal two-dimensional monoatomic gas consisting of  $N$  atoms, energy  $U$ , and area  $A$  is given by the following version of the Sackur-Tetrode equation:

$$S = Nk \left[ 2 + \ln \left( \frac{2\pi m}{h^2} \frac{A}{N} \frac{U}{N} \right) \right]. \quad (1)$$

Derive an expression for the pressure  $P$  of this two-dimensional gas, simplify your expression as much as possible, and discuss briefly whether your expression makes sense physically.