

Print your name clearly:_____

Signature:_____

“I agree to neither give nor receive aid during this quiz.”

Quiz 3 for Physics 176

Professor Greenside
Monday, February 23, 2009

This quiz is closed book. Please:

1. **Write your answers on separate pages of blank paper** that will be handed out. Do not write your answers on this exam except for the true/false and multiple choice answers.
2. **Write your name and the problem number on each page of paper** that you use.
3. **Justify each answer briefly**, with appropriate calculations, sketches, phrases, or sentences.

The following data and formulas may be useful:

$$C = \frac{Q}{\Delta T}, \quad C_V = \left(\frac{\partial U}{\partial T} \right)_V, \quad C_P = \left(\frac{\partial U}{\partial T} \right)_P + P \left(\frac{\partial V}{\partial T} \right)_P.$$

$$S = k \ln \Omega, \quad \frac{1}{T} = \left(\frac{\partial S}{\partial U} \right)_{V,N}, \quad S = Nk \left(\frac{5}{2} + \ln \left[\frac{V}{N} \left(\frac{4\pi m}{3h^2} \frac{U}{N} \right)^{3/2} \right] \right).$$

$$A_d(r) = \frac{2\pi^{d/2}}{\Gamma(d/2)} r^{d-1}, \quad \Gamma(x+1) = \int_0^\infty t^x e^{-t} dt \approx \sqrt{2\pi x} \left(\frac{x}{e} \right)^x \quad \text{for } x \gg 1, \quad \Gamma(x+1) = x \Gamma(x).$$

$$[f(x)]^N \approx f_{\max}^N \exp \left[-N \frac{f''(x_{\max})}{2f_{\max}} (x - x_{\max})^2 \right], \quad \ln(1+x) \approx x.$$

True or False Questions (2 points each)

For each of the following statements, please circle **T** or **F** to indicate respectively whether a given statement is true or false.

1. **T / F** If the volume V of an ideal monoatomic gas is halved during an isothermal compression that conserves the number of particles, then the entropy of the gas will decrease by $Nk \ln(2)$.
2. **T / F** The product of an extensive thermodynamic quantity with an intensive thermodynamic quantity is also an extensive quantity.
3. **T / F** Given that the function $f(x) = 1 - x^4$ has a single maximum at $x_{\max} = 0$ with value $f_{\max} = 1$, the function $[f(x)]^N$ for N a large integer can be accurately approximated by a Gaussian e^{-NAx^2} for $A > 0$ some constant.

4. **T / F** For any sequence of experimental steps that leads to a change ΔS_{gas} in the entropy of an ideal monoatomic gas (because the variables N , V , and U of the gas have changed to new values N' , V' , and U'), the value ΔS_{gas} will depend on how the variables N , V , and U were changed to their new values (e.g., quasistatic or not quasistatic, isothermal versus adiabatic, etc.)
5. **T / F** For any sequence of experimental steps that leads to a change in the entropy of an ideal monoatomic gas (because the variables N , V , and U of the gas have changed to new values N' , V' , and U'), the total change in entropy of the gas and surrounding environment will depend on how the variables N , V , and U were changed to their new values
6. **T / F** The process of slowly, quasistatically, and isothermally introducing a thin impermeable partition into a box of an ideal gas such that the partition divides the volume V of the box in two volumes of $V/3$ and $2V/3$ is an example of a reversible thermodynamic process.

Multiple Choice Questions (4 points each)

Circle the letter that best answers each of the following questions.

1. If x is a sufficiently small number, then the lowest-order approximation to the expression

$$\ln(1-x) + \ln(1+x) + \ln(1+x^2) + \ln(1+x^4),$$

is

- (a) x^2 (b) $-x^2$ (c) x^4 (d) $-x^4$ (e) x^6 (f) $-x^8$.

Problems That Require Writing

All answers in this section of the quiz must be written on the supplementary blank pages. Please remember to write your name and problem number on each extra page.

1. (5 points) The multiplicity Ω of a certain substance is related to its thermal energy U by the relation $\Omega = aU^b$, where a and b are positive constants. Find a formula for the heat capacity C_V of the substance in terms of a , b , and the Boltzmann constant k .
2. (15 points) Many scientists and engineers are greatly interested in the properties of molecules that adsorb onto the surface of a substance. (“Adsorb” means that a molecule leaves the gas phase and bonds with the surface.) For sufficiently high temperatures and for some weakly binding surfaces (e.g. graphite or frozen argon), the adsorbed molecules can move freely about on the surface without interacting with each other and so act like a two-dimensional ideal gas.

Consider N identical atoms, each of mass m , that have adsorbed onto a flat surface of area A and of temperature T . **Derive an appropriate version of the Sackur-Tetrode equation that describes the entropy $S = S(N, A, U)$ of a two-dimensional ideal gas of monoatomic atoms as a function of N , A , and the thermal energy U .**

Some comments: By analogy to how we derived the multiplicity Ω of an ideal gas in a three-dimensional volume, first derive an expression for the multiplicity Ω of the two-dimensional ideal gas. Each adsorbed molecule is free to move about in a two-dimensional area A , and each adsorbed molecule is described by a two-dimensional momentum vector $\mathbf{p} = (p_x, p_y)$ with kinetic energy $p^2/(2m) = (p_x^2 + p_y^2)/(2m)$. For this problem, assume that the N adsorbed atoms are an isolated system so that their total energy is conserved. After you have obtained an expression for the multiplicity, use the definition of entropy plus Stirling’s approximation to derive the 2D version of the Sackur-Tetrode equation. You can drop multiplicative factors of a very large number that are small or large numbers.