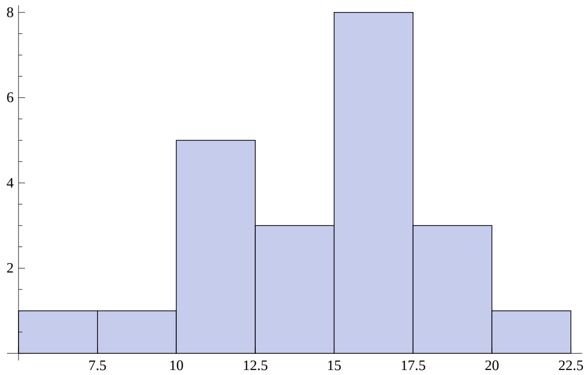
Answers to Quiz 1
Physics 176

Professor Greenside
Thursday, January 22, 2009

For future quizzes: *please* read the instructions at the beginning of your quiz (or exam). If I ask you to put all of your work on extra pages, not on the quiz itself, please do this. And especially if you are asked to justify your answers briefly, please make sure to do so. I gave zero credit to students who gave answers like $-3°$ C or $(2/3)\pi^4$ if they didn't give any information about how they obtained their answers. In this course, multiple choice and true/false questions will be the only questions for which no justification is required (and of course you can't get partial credit in those cases, so the questions that require written answers generally lead to higher scores.)

The highest and lowest quiz scores were 21 and 5 respectively, with a mean of 14.4. The histogram of quiz scores was:



1. A spherical helium-filled balloon one meter in diameter is inside an airplane, for which the pressure is 1 atm and the temperature is 27°C. The airplane is flying 6 km above the ground for which the surrounding air has a pressure and temperature of 0.5 atm and −3°C respectively. The balloon is then ejected instantly from the airplane, after which it eventually reaches thermodynamic equilibrium at the height of the airplane.

   (a) **(3 points)** Immediately after the balloon has reached its new size, will the temperature in the balloon be closer to 27°C or to −3°C?

   Answer: The temperature will be closer to 27°C right after the balloon has reached its new size.

   In justifying your answer, I was looking for phrases that mentioned that mechanical equilibrium was generally attained much more rapidly (by the speed of sound) than thermal equilibrium, which involves random collisions and slow diffusion of information.

   To fully nail this question, you should have pointed out that the time for the balloon to change its size would be of order the size of the balloon divided by the speed of sound through the He in the balloon, $1\,\mathrm{m}/(500\,\mathrm{m/s}) \approx 2\,\mathrm{ms}$. (Sound speeds in gases and solids are all of order 1000 m/s, any speed of that magnitude would have been fine to use.) In contrast, the time for the balloon to reach thermal equilibrium at the new temperature would be of order $L^2/\kappa_{\mathrm{He}} = (1\,\mathrm{m})^2/(10^{-5}\,\mathrm{m}^2/\mathrm{s}) \approx 10^5\,\mathrm{s} \approx 30\,\mathrm{hours}$ assuming that only diffusion by molecular collisions takes place (no rapid mixing of the air in the balloon by convection that would arise from uneven cooling of the balloon).

(b) **(5 points)** By what numerical factor will the volume of the balloon have changed after the balloon reaches thermodynamic equilibrium outside the airplane? (Ignore the elastic properties of the balloon.)

Answer: The volume increases by a factor of 1.8.

The fact that the balloon is sealed implies that the number N of molecules in the balloon does not change. The ideal gas law $PV = NkT$ then tells us that the product $PV/T = Nk$ is constant so $P_iV_i/T_i = P_fV_f/T_f$ where the subscript i means "initial value" and the subscript f means "final value". Solving for the desired quantity, the ratio V_f/V_i, gives:

$$\frac{V_f}{V_i} = \frac{T_f}{T_i} \times \frac{P_i}{P_f} = \frac{270}{300} \times \frac{1}{0.5} = \frac{9}{5} = 1.8 \approx 2. \tag{1}$$

So the volume will approximately double in size after being tossed out of the airplane.

Note that you could solve this problem using scientific common sense, without getting tangled in any algebra. For a fixed temperature, decreasing the pressure of a gas increases its volume, so the ratio of pressures appearing in the answer has to give a factor greater than 1, i.e., it has to be the factor $1/0.5 = 2$. For a fixed pressure, decreasing the temperature has to decrease the volume, so the ratio of temperatures has to be a factor less than 1, namely $270/300 = 9/10$. Combining the two factors gives the answer $9/5$.

Note also that nearly the entire volume change comes from the pressure change. This is because, in absolute temperatures measured in kelvin, the change from 27°C to −3°C is the change from 300 K to 270 K, not a big fractional change.

2. **(5 points)** Find the value of the following triple integral in terms of π and some ratio of integers.

$$\int_0^\pi dx \int_0^\pi dy \int_0^\pi dz\, y^2 \sin(z)\,. \tag{2}$$

Hint: If you first think about this problem conceptually, you should be able to deduce the answer quickly without evaluating any integrals.

Answer: $(2/3)\pi^4$.

A nice way to solve this problem, the way I hoped most of you would solve it, is to think first about what this integral might mean. (For scientists, it especially useful to try to think about what an integral, or different parts of an integrand, might mean scientifically.) From having worked on a spherical coordinates integral in the most recent homework assignment, you could recognize the expression $y^2 \sin(z)\, dy\, dz\, dx$ as an infinitesimal volume $dV = r^2 \sin(\theta)\, dr\, d\theta\, d\phi$ in spherical coordinates, centered on the point (r, θ, ϕ), where we think of y as r, z as θ, and x as ϕ. Thus this triple integral is asking you to add up lots of small volumes dV over some region. (The small volumes dV are all non-negative since the ranges of the integrals won't let $\sin(z)$ become negative.)

Looking at the bounds of the integrals, we see that the "radius" y goes from 0 to π, the "angle" z goes from the north pole $z = 0$ of the sphere to the south pole $z = \pi$ of the sphere, and the "angle" x goes only half-way around the equator of a sphere, from 0 to π (where once around the entire equator would go from 0 to 2π).

Thus the triple integral is one-half the volume of a sphere of radius $r = \pi$, that has been cut in half by a plane passing through the north and south pole. The integral therefore has the value

$$\frac{1}{2}\left[\frac{4}{3}\pi r^3\right] = \frac{2}{3}\pi^4. \tag{3}$$

Many students got the right answer by direct calculation. Since the integrand separates into a product of functions $f(x)g(y)h(z)$ that each depend on only one variable (with $f = 1$, $g = y^2$, and $h = \sin(z)$), the triple integral factors into a product of three separate integrals like this

$$\int_0^\pi dx \int_0^\pi dy \int_0^\pi dz\, y^2 \sin(z) = \int_0^\pi dx \times \int_0^\pi y^2\, dy \times \int_0^\pi \sin(z)\, dz, \tag{4}$$

which leads to the same answer.

3. **(5 points)** If the following Mathematica code is executed:

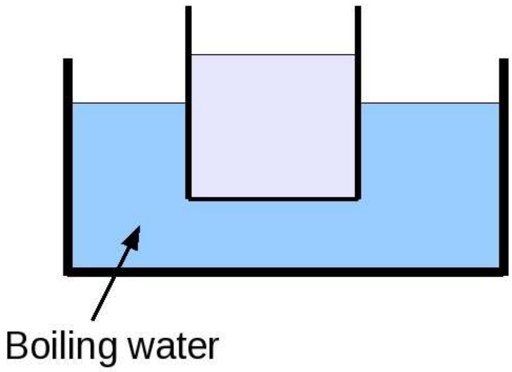
```
x = Table[ i^2 , {i,1,3} ]
y = x[[2]] x[[3]]
```

what is the value of the variable y?

Answer: 36.

Students who worked through the Mathematica tutorial and spent some time playing with Mathematica would have learned the necessary details. The `Table` command creates a list `{1,4,9}` of the squares of the first three integers. The expression `x[[2]] x[[3]]` means multiply the second element of the list x which is 4 with the third element of the list which is 9; unlike the related mathematics software Matlab and Maple, a space between two expressions in Mathematica means multiplication. The product gives 36.

4. **(5 points)** A pot of water is brought to a steady boil on a stove and then a thin plastic cup of room temperature water is suspended in the middle of the boiling water as shown in the figure below.



Explain whether or not the water in the cup will start to boil if you wait long enough.

Note: In thinking about this problem, keep in mind that it takes energy to convert water to steam.

Answer: The water in the cup will not boil.

As I discussed briefly in class, this is a problem concerning the basic fact that energy in the form of heat will only transfer between systems if there is a temperature difference between the systems. (This is, in fact, one of the most important consequences of temperature, much more important than the narrow relation defined by the equipartition theorem, that thermal energy in an ideal gas is related to the average kinetic energy of its molecules.) As we saw with the demo in class, the temperature of the water in the cup will reach within a matter of minutes the same temperature 100°C as that of the surrounding boiling water[1].

But perhaps surprisingly, the water in the cup will not start to boil. The reason is that it takes energy to convert the water in the cup to steam and there is no way to transfer the large amounts of energy needed into the cup to promote boiling once the water in the cup has the same temperature as the surrounding boiling water. Only the water in the pot, which is in direct contact with the bottom of the

[1] The water in the cup reaches equilibrium much faster than the thermal relaxation time scale L^2/κ (with $L \approx 0.1\,\mathrm{m}$ for the small cup and $\kappa \approx 10^{-6}\,\mathrm{m}^2/\mathrm{s}$ for water) since the large temperature differences will cause convection, i.e. bulk motion of the fluid that mixes the temperature much more rapidly than by molecular collisions alone.

pot which in turn has a higher temperature than boiling water, will receive a steady supply of energy in the form of heat, that will allow boiling to take place.

In their answers, several students stated incorrectly that the plastic of the cup would prevent heat from entering the water in the cup and so the cup might warm up but not reach the boiling temperature. It is impossible to prevent the flow of heat through a material substance. Different materials have different thermal diffusivities κ and so one can make the relaxation time longer or shorter by choosing a poor or good thermal conductor. But if you wait longer than the thermal relaxation time, which is always finite, a system will always attain thermal equilibrium and match the temperature of its surrounding environment.

By the way, if you have spent time in a kitchen, you might have seen or used a device called a "double boiler", which is essentially a cup of some liquid suspended in the middle of a pot of water that is brought to a boil, just like this problem. Cooks especially use double boilers to melt chocolate or heat some delicate sauce, which will otherwise decompose into some ill-tasting mess if its temperature becomes too high. The double boiler provides a clever way to raise something to but not above the boiling point of water.