Print your name clearly: ______________________________

Signature: ______________________________

"I agree to neither give nor receive aid during this exam."

Midterm Exam for Physics/ECE 176

Professor Greenside
Thursday, March 4, 2010

This exam is closed book and no calculators or any other electronic devices are allowed. The test will last the entire class period. Please note the following:

1. All questions except the True/False should be answered on the extra blank pages. If you need extra pages during the exam, let me know.
2. Please write your name and the problem number at the top of each extra page.
3. Please write clearly and justify your answers (unless otherwise stated). If I can not easily understand your answer, you will lose credit.

Problems That Require Writing

1. **(30 points)** The year is 2020 and you have been invited by one of your professor friends to be a guest speaker in her undergraduate thermal physics class. Using as a specific example one of the three key systems that you have studied so far this semester (Einstein solid, ideal gas, or paramagnet), discuss concisely what you would say to the class about how microstates, macrostates, multiplicity, and the second law of thermodynamics are related to one another: why does counting microstates have anything to do with thermodynamic equilibrium?

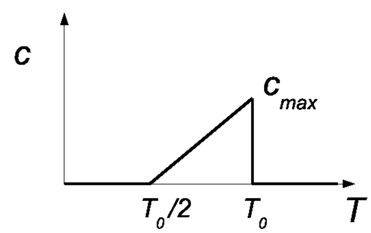
   Note: your goal is to show me that you understand the logical thread that ties various ideas together, in the context of a specific example. You don't need to write full sentences or give any mathematical derivations but you do need to state key definitions, assumptions, and mathematical expressions, and *especially* indicate through appropriate brief phrases the logic that connects the various concepts so that these future students understand what is going on.

2. A carbon nanotube is a remarkable straw-like molecule of carbon atoms that can be several millimeters long while having a tiny diameter of a few nanometers. Assume that $N \gg 1$ identical atoms with a fixed total energy U adsorb onto a nanotube of length L, such that the adsorbed atoms act as a one-dimensional ideal gas.

   (a) **(15 points)** Derive the Sackur-Tetrode equation for the entropy $S = S(U, L, N)$ of this one dimensional gas.

   (b) **(5 points)** Find the equation of state $P = P(T, L, N)$ for this one-dimensional gas.

3. **(10 points)** A ferromagnet at absolute zero has all of its $N \gg 1$ two-state (spin-1/2) magnetic moments aligned in parallel, with no external magnetic field present. The ferromagnet is then heated until its temperature exceeds its Curie temperature T_0, at which point the magnetization M of the magnet has decreased to zero. A crude model of how the heat capacity $C_V(T)$ of the ferromagnet depends on temperature is given by the following plot:



which corresponds to the mathematical expression

$$C_V(T) = \begin{cases} C_{\max}\left(2T/T_0 - 1\right), & \text{for } T_0/2 \le T \le T_0 \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

By computing the entropy change of the ferromagnet from absolute zero to the Curie temperature in two different ways, determine the maximum value $C_{\max}$ of the heat capacity in terms of N and the Boltzmann constant k.

4. **(10 points)** For an ideal gas of diatomic nitrogen N_2, draw qualitatively but carefully the heat capacity $C_V(T)/(Nk)$ as a function of temperature T in kelvin. Label the horizontal axis with numerical values that indicate at approximately what temperatures various degrees of freedom start to increase as the temperature increases, and indicate the corresponding numerical values of the heat capacity on the vertical axis. Also label what type of degree of freedom corresponds to the different regimes of your drawing.

5. **(10 points)** Calculate the leading-order low-temperature (small T) and high-temperature (large T) behaviors of the expression

$$\bar{n} = \frac{1}{e^{c/T} - 1} - \frac{N}{e^{Nc/T} - 1}, \tag{2}$$

and use your results to draw qualitatively how $\bar{n}$ varies with T from low to high temperatures. Here $N \gg 1$ is a large positive integer and $c > 0$ is a positive constant. This expression arises when trying to understand how a zipper-like DNA molecule self-assembles as a competition between energy and entropy; $\bar{n}$ is the average number of cross-links formed.

6. **(10 points)** Two vessels of volume V_1 and V_2 each contain N atoms of the same ideal gas at the same pressure P but with temperatures of T_1 and T_2 respectively. In terms of the heat capacity C_p at constant pressure (which you can assume is constant) and the temperatures T_1 and T_2, calculate the change of entropy ΔS that arises when a tube connects the two vessels and thermodynamic equilibrium is attained. Determine also whether $\Delta S_{\text{total}} \ge 0$ for your system.

7. (a) **(6 points)** Starting with a qualitative plot of how the entropy $S(U, B)$ of a two-state paramagnet in a uniform magnetic field of strength B varies with energy U, deduce and then draw schematically for two different magnetic field strengths B_{low} and $B_{\text{high}} > B_{\text{low}}$ how the entropy $S(T)$ varies with temperature T over the range $-\infty < T < \infty$. Make sure to label your two entropy curves clearly, for low B and high B.

(b) **(4 points)** For a thermally isolated two-state paramagnet in the presence of a uniform magnetic field with strength B, discuss whether increasing B will increase, decrease, or leave unchanged the temperature of the paramagnet.

True or False Questions (2 points each)

For each of the following statements, please circle **T** or **F** to indicate whether a given statement is true or false respectively.

1. **T / F** To the nearest power of ten, the entropy of a mole of Belgian chocolate molecules is approximately 10 J/K.

2. **T / F** $C_V(\text{steam}) > C_V(\text{water})$.

3. **T / F** The heat capacity $C_{\text{Einstein}}(T)$ of an Einstein solid accurately describes the shape of the heat capacity $C_V(T)$ of an ideal gas of diatomic molecules in the regime for which vibrational degrees of freedom are freezing out (decreasing) as the temperature of the gas is decreased.

4. **T / F** If an ideal gas is compressed isothermally and then expanded adiabatically back to its original volume, the final pressure will be greater than the initial pressure.

5. **T / F** For a two-state paramagnet in a uniform external magnetic field $\mathbf{B}$, all the magnetic dipoles are antiparallel to the direction of the magnetic field in the limit $T \to \infty$.

6. **T / F** The ratio W/Q of work produced to heat added is 2/7 for an ideal gas of diatomic N_2 that undergoes an isobaric expansion close to room temperature.

7. **T / F** The surface area $A_d(1)$ of a d-dimensional hypersphere with unit radius is a decreasing function of d for sufficiently large d.

8. **T / F** The expression $S = S(U, V) = S_0\, U^{3/4} V^{1/4}$ (where S_0 is a positive constant) is not a physically reasonable functional form for the entropy of some substance.