Print your name clearly:____

Signature:____

"I agree to neither give nor receive aid during this quiz."

First Midterm Exam for Physics 176

Professor Greenside
Monday, February 9, 2009

This exam is closed book and will last the entire class period. Please note the following:

1. Only true-false and multiple choice questions should be answered on the exam itself.
2. All other questions should be answered on the extra blank pages. If you need extra pages during the exam, let me know.
3. Please be sure to write your name and the problem number at the top of each extra page.
4. Please write clearly. If I can not easily understand your answer, you will lose credit.
5. Please feel free to ask me questions during the exam if the wording of a problem is not clear.

The following data and formulas may be useful:

$$k \approx 1.4 \times 10^{-23}\,\mathrm{J/K}, \qquad \mathrm{h} \approx 6.7 \times 10^{-34}\,\mathrm{J \cdot s} \qquad 1\ \mathrm{liter} = 10^{-3}\,\mathrm{m}^{-3}, \qquad \mathrm{N_A} \approx 6.0 \times 10^{23}.$$

$$PV = NkT, \qquad \Delta U = Q + W, \qquad U = Nf\,(kT/2), \qquad v_{\rm rms} = \sqrt{3kT/m},$$

$$PV^{\gamma} = \text{const}, \qquad VT^{f/2} = \text{const}, \qquad \gamma = (f+2)/f, \qquad \int_0^{\pi/2} \sin(x)\cos(x)\,dx = 1/2.$$

$$C = Q/\Delta T, \qquad L = Q/m, \qquad C_V = (\partial U/\partial T)_V, \qquad C_P = (\partial U/\partial T)_P + P(\partial V/\partial T)_P.$$

$$\binom{n}{m} = \frac{n!}{m!(n-m)!}, \qquad n! \approx \sqrt{2\pi n}\left(\frac{n}{e}\right)^n, \qquad \Omega(n,q) = \binom{n+q-1}{q}, \qquad \ln(1+x) \approx x.$$

True or False Questions (2 points each)

For each of the following statements, please circle **T** or **F** to indicate whether a given statement is true or false respectively.

1. **T / F** For an ideal gas, the ratio of the specific heats C_P/C_V is equal to the adiabatic exponent γ.

2. **T / F** It is possible for the heat capacity C_P of a substance to be infinite at a finite temperature.

3. **T / F** When a closed path is traced out in the PV-plane of an ideal gas starting at some point A on the path and returning back to A, the total amount of heat absorbed by the gas will depend on the position of the starting point A on the path.

   Note: for this problem, assume that the total work done on the gas after following the path from A back to A is nonzero.

4. **T / F** It is possible to trace a closed path in the PV-plane of an ideal gas such that the total heat absorbed and the total work done on the gas are both zero.

   Note: In your thinking, do not include the trivial case in which the gas goes from point A to point B on some path and then returns to A by traveling backwards along the same path, i.e., assume that the closed path encloses a finite area.

5. **T / F** In an Einstein solid of N identical harmonic oscillators, increasing the number of energy units q corresponds to increasing the temperature T of the solid.

6. **T / F** A careful experimental measurement over one hour of the temperature T of some equilibrium macroscopic solid whose width is $0.1\,\mathrm{m}$ will include the effects of all the possible microstates of the solid.

7. **T / F** If an isolated Einstein solid is in thermodynamic equilibrium and has N_A oscillators in one subsystem and N_B oscillators in the other subsystem with $N_A \neq N_B$, and if the entire solid has q units of energy, then the most likely amount of energy to be observed in system A is $q\,N_A/(N_A+N_B)$. (Here the integers N_A, N_B, and q are all large numbers.)

8. **T / F** If an isolated Einstein solid has two subsystems with equal number of oscillators $N_A = N_B = N \gg 1$, and if the solid has an amount of energy units $q > N$, and if at a particular moment all of the energy q is observed to be in subsystem A, then you can conclude that the solid is not in thermodynamic equilibrium.

9. **T / F** For n a large integer and $\alpha > 1$ some real number, the sum of the n powers $1^\alpha + 2^\alpha + \ldots + n^\alpha$ is approximately equal to $(\alpha/2)n^{\alpha+1}$.

Multiple Choice Questions (4 points each)

Circle the letter that best answers each of the following questions.

1. A large solid block of metal has unequal dimensions $L_x \times L_y \times L_z$ such that $L_x > L_y > L_z$. If the thermal diffusivity of the block is κ, then the least amount of time that an experimentalist should wait for the block to be close to equilibrium is

   (a) L_x^2/κ (b) L_y^2/κ (c) L_z^2/κ (d) L_x/κ (e) L_y/κ (f) L_z/κ

2. Two identical bubbles A and B of an ideal gas form at the bottom of a lake and then rise upwards toward the lake's surface. Because the water pressure decreases from the bottom to the surface, both bubbles expand as they rise. However, bubble A rises so quickly that there is no time for heat to flow into or out of the bubble, while bubble B rises so slowly (say it is transported by a lazy water spider) that its temperature always stays the same (the surrounding water is isothermal). When both bubbles are just below the lake's surface

   (a) they will have the same diameter.
   (b) bubble A will be the larger bubble.
   (c) bubble A will be the smaller bubble.

3. For x a sufficiently small number, the expression

$$\ln\left[\frac{1-\ln\bigl(1-\ln(1-x)\bigr)}{1+\ln\bigl(1+\ln(1+x)\bigr)}\right]$$

is approximately equal to

(a) 0 (b) $2x$ (c) $-2x$ (d) x^2 (e) $-x^2$

4. A long thermally isolated plastic cylinder is divided into two equal halves by a thin circular metallic impermeable partition that has an extremely small mass and that can easily slide back and forth along the axis of the cylinder with negligible friction. One half of the cylinder is filled with He gas and the other half with gaseous sulfur hexafluoride SF_6 (the ratio of the molecular masses is $(32+6\cdot 19)/4 = 146/4 \approx 37$) so that thermodynamic equilibrium is attained with constant pressures, temperatures, and volumes on both sides of the partition. An electrical current is then passed through the partition so that its temperature instantly increases by a large amount. Immediately after this temperature increase and before the temperatures of the two gases can change, the piston will

   (a) not move at all.
   (b) move a tiny bit so as to decrease the volume of He gas.
   (c) move a tiny bit so as to increase the volume of He gas.

Problems That Require Writing

Please write your answers to the following problems on extra blank sheets of paper. Also make sure to write your name and the problem number at the top of each sheet. In this part of the exam, you need to justify all of your answers to get full credit.

1. (8 points) Using Stirling's approximation, derive an approximation for the multiplicity of an Einstein solid in the low temperature limit for which the number N of oscillators and the amount q of energy satisfy $N \gg q \gg 1$. To obtain a simple final expression, ignore large numbers that multiply very large numbers.

2. (10 points) An isolated box of volume V is divided into two compartments, A and B. Compartment A contains N_A atoms of an ideal gas at temperature T_A in a volume V_A while compartment B contains N_B atoms of the same gas at a *higher* temperature $T_B > T_A$ in a volume V_B such that $V = V_A + V_B$. A thin immovable impermeable metal partition separates the two compartments.

   Using an appropriate argument supported with mathematical equations, use the second law of thermodynamics in the form $dS/dt > 0$ together with the definition of temperature $1/T = dS/dE$ to explain why heat necessarily flows from the hotter compartment to the cooler compartment. Make sure that you clearly indicate any assumptions that you have to make.

3. (6 points) Without using any mathematics, explain why the expression

$$\binom{\alpha_1+\alpha_2+\alpha_3+\beta-1}{\beta} = \sum_{\beta_1+\beta_2+\beta_3=\beta}\binom{\alpha_1+\beta_1-1}{\beta_1}\binom{\alpha_2+\beta_2-1}{\beta_2}\binom{\alpha_3+\beta_3-1}{\beta_3},$$

   is a valid mathematical identity by interpreting this expression physically. The symbols α_i and β are nonnegative integers and the sum goes over all possible nonnegative integer values of the β_i that sum to β.

4. (10 points) Consider two identical cubic dice of the sort used in board games, with each die having the numbers 1 through 6 on its six surfaces. Assume the dice are fair in that each number has the same probability of appearing when tossed.

   Consider the two dice as a macroscopic system whose macrostate is characterized by the sum of the values that appear on the top most surfaces after throwing the two dice. With an appropriate table, determine all possible macrostates of the two dice, list all the possible microstates for each macrostate, and give the probabilities for observing each macrostate.

5. Consider a box of volume V that contains an ideal gas in thermodynamic equilibrium that consists of N identical molecules, each of mass m, each moving with speed v, and such that the molecular velocities are isotropic (they point in all possible directions with equal likelihood). The gas is initially at temperature T with pressure P. A small flat metallic square of area A is attached to the inside surface of the box so that the square lies flush with the wall and such that the square is cooled to such a low temperature (by external machinery) that any gas molecules that come in contact with the square permanently stick.

   (a) (10 points) In terms of the variables m, P, T, and A (but not in terms of v, N, or V), derive a formula for the amount of the heat per unit time that must be removed from the metal square in order to keep its temperature constant and cold. (Please make sure to use some brief phrases that explain key details or assumptions in your derivation.)

   (b) (6 points) Using your formula, estimate to *one* significant digit the heat per unit time (in units of joules/second or J/s) that needs to be removed from a one-millimeter metal square if the square is attached to the inside of a one liter box containing diatomic nitrogen (molecular weight 28 g) at STP (standard temperature and pressure so $T \approx 290\,K$ and $P \approx 1.0 \times 10^5\,\mathrm{N/m^2}$). Also assume that the gas parameters have not yet had time to change significantly from their initial values.

   Note: You will save time if you simplify your expression algebraically before substituting any numbers. Also round numbers to one digit before combining them.

   (c) (6 points) Draw schematic curves of how the pressure P, temperature T, and number of particles N vary with time for this gas that is in steady contact with an small extremely cold metal square. (You need to justify, at least briefly, why you draw your particular curves.)