Answers to Physics 176 One-Minute Questionnaires

April 4 through April 21, 2011

How is that we can begin looking for energy spectra inside a black box (quite literally) and then apply that to energy emissions of objects like stars, plants, etc which aren't enclosed in a box? Or, put another way, why is the Planck spectrum applicable to many examples of radiation not inside the conditions by which it was derived?

The answer is given briefly (and somewhat incompletely) on pages 302-303 of Schroeder, which I did not have time to mention in the last lecture. It was Kirchhoff (the same Kirchhoff of the Kirchhoff laws you learned about in your intro physics course regarding laws for electrical circuits) in 1859 who argued that a black (i.e., perfectly absorbing) surface in equilibrium with a photon gas (say the wall lining a box containing the gas) must radiate away exactly as much radiation per second and per unit area, with the same Planck spectrum, as the wall receives in order for the wall and gas to be in equilibrium. But the radiation of light from the wall does not depend on the presence of the photon gas itself and so one concludes that the surface of an opaque object that is in equilibrium with temperature T must itself produce (radiate away) light that is consistent with being in equilibrium, i.e. light that satisfies the Planck spectrum and Stefan's law of radiation, Eq. (7.98) on page 303 of Schroeder. The argument depends on the existence of solid filters of different kinds that can pass light only over a small range of frequencies or that only let linearly or circularly polarized light through. Such filters are readily constructed for light in the visual range, but only exist in principle for all bands of wavelengths.

Refinements of Kirchhoff's argument also leads to the conclusion that an equilibrium photon gas must be homogeneous (have the same energy density U/V everywhere in space), isotropic (photons move in all possible directions with equal likelihood) and unpolarized. For example, if the light in an equilibrium photon gas were partially polarized, you could place a linear polarizing filter in the gas, which would not affect thermal equilibrium (if the filter has the same temperature as the gas) but would cause a temperature difference to arise by letting some light through and blocking other light, contrary to the assumption that the gas is in equilibrium and so that the temperature is everywhere the same.

The Wikipedia article "Kirchhoff's law of thermal radiation" gives more information, and the book by F. Reif on the 176 reserve in the library has a particularly clear (although detailed) discussion of the Kirchhoff radiation law.

I'm still confused about how the background radiation relates to the diagram you displayed at the end of lecture. The units were in degrees (I think). Can you elaborate on the meaning of this diagram and how it was put together with the help of the Planck spectrum?

I did go through that slide rather quickly, sorry.

The details are somewhat complicated to explain at the level of 176, and my intent in lecture was just to give you a brief sense that the extremely small spatial variations in temperature beyond the Planck spectrum provide rather extraordinary cosmological information about the geometry of the entire universe (is it curved or flat), the fraction of matter that is baryonic (neutrons and protons) and other, the fraction of matter that is dark matter, and so on.

The Wikipedia article "Wilkinson Microwave Anisotropy Probe" (abbreviated as WMAP) provides more details than what I mentioned, and also includes a version of the figure

`http://en.wikipedia.org/wiki/File:WMAP_2008_TT_and_TE_spectra.png`

that I showed in lecture.

The outline of the logic goes roughly like this. Consider a spherical coordinate system centered on Earth with the z-axis pointing from the Earth's center through the north pole, which then allows us to label any point in Earth's sky with two angles θ and ϕ. By flying a sophisticated calorimeter above the Earth's atmosphere (the atmosphere blocks many frequencies of light, especially infrared), one can measure the light intensity $I(\omega, \theta, \phi)$ as a function of frequency ω that arrives in the device from direction (θ, ϕ) in the sky. (In practice, data is collected just for a few frequencies ω, not a full spectrum.) For each one of these directions, it turns out that the light spectrum can be accurately fitted to a Planck spectrum Eq. (7.84) on page 292 of Schroeder. By subtracting this fitted curve from each spectrum recorded from a given angle, one obtains a small correction $\Delta I(\omega, \theta, \phi)$ at each point in the sky.

These are then further analyzed to extract information. For example, for a fixed frequency ω, one can fit the angular variation of the data to spherical harmonics $Y^l_m(\theta,\phi)$

$$f(\theta,\phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} f_{lm} Y_{lm}(\theta,\phi), \tag{1}$$

which generalizes the Fourier analysis of a function $f(x) = \sum_m f_m e^{imkx}$ of a single periodic function to a function $f(\theta,\phi)$ on the surface of a sphere. (See the Wikipedia article "Spherical harmonics"; this topic often first appears at the undergraduate level in an upper-level course on electrodynamcis like Physics 182, when solving a Poisson equation on the surface of a sphere.)

The above plot (the one I showed in class) then shows the magnitude square of the spherical harmonic coefficients $|f_{lm}|^2$ of the light intensity, averaged over the index m. The index l can be roughly interpreted as an angle since larger l corresponds to a finer angular structure; note the angular scale at the top of the figure which starts at 90° on the left and decreases to 0.2° on the right. But the precise meaning of the curve requires an understanding of the index l for a spherical harmonic.

Can you post a link discussing the finite/infinite N particles for a boson argument?

Sections 9.3 and 9.6 of Reif's book on reserve give a detailed discussion on how to derive the Bose-Einstein distribution for a system with finitely many particles.

The argument I gave in class, and the argument given in Schroeder, is rigorous for a bosonic system in equilibrium with a reservoir whose chemical potential μ is constant. The number of particles that the system can exchange with the reservoir is then variable and in principle can vary from 0 to infinity. For a reservoir with a large but finite number of particles, the geometric series would be chopped off at some large number of order Avogadro's number, which is effectively infinity.

Reif's argument starts with the assumption that one has an isolated box of identical weakly interacting particles. The energy of the system is then given by

$$E = n_1 E_1 + n_2 E_2 + \ldots, \tag{2}$$

where n_i is the number of particles with energy E_i. The partition function Z (not the grand partition function $\mathcal{Z}$, Reif does not consider a constant chemical potential reservoir) is then given by a sum over all possible quantum states

$$Z = \sum_{n_1,n_2,\ldots} e^{-\beta(n_1E_1+n_2E_2+\ldots)}, \tag{3}$$

where the total number of particles is finite:

$$N = n_1 + n_2 + \ldots = \sum_s n_s. \tag{4}$$

Equations (3) and (4) are harder to analyze than what I discussed in class which is why I (and Schroeder) chose to avoid going down this path. For example, the occupation number for the sth state can now be calculated as

$$\overline{n_s} = \frac{\sum_{n_1,n_2,\ldots} n_s e^{-\beta(n_1E_1+n_2E_2+\ldots)}}{\sum_{n_1,n_2,\ldots} e^{-\beta(n_1E_1+n_2E_2+\ldots)}}. \tag{5}$$

With some thinking, this can be written in terms of a modified partition function $Z_s(N)$ for N particles that excludes the specific state s:

$$Z_s(N) = \sum_{n_1,n_2,\ldots}^{(s)} e^{-\beta(n_1E_1+n_2E_2+\ldots+n_{s-1}E_{s-1}+n_{s+1}E_{s+1}+\ldots)}. \tag{6}$$

One can show that Eq. (5) then be written as

$$\overline{n_s} = \frac{0 + e^{-\beta E_s}Z_s(N-1) + e^{-2\beta E_s}Z_s(N-2) + \ldots}{Z_s(N) + e^{-\beta E_s}Z_s(N-1) + e^{-2\beta E_s}Z_s(N-2) + \ldots}. \tag{7}$$

One finishes the argument by observing that, for large enough N, $Z_s(N - \Delta N) \approx Z_s(N)e^{-\mu\Delta N}$.

What physically occurs that causes He^3 to be a boson and He^4 to be a fermion?

The physical occurrence is that one consists of an odd number of fermions, the other an even number of fermions.

The He^3 nucleus (or atom) consists of an odd number of fermions, e.g., the nucleus contains two protons and one neutron. There is no way to add up the spin angular momentum of three spin-1/2 particles to get an even multiple of $h/2$ so the spin-statistics theorem then tells us that He^3 must be a fermion. He^4 consists of an even number (four) of spin-1/2 particles and it is then impossible to combine the spins of the protons and neutrons to get an odd multiple of $h/2$, implying that He^4 must be a boson.

Why is Ag used so frequently in effusion-based experiments?

I don't know. One wants a substance that can be easily vaporized to form a gas (low boiling point), a substance that has a single valence electron (the physics is simpler to understand than atoms like transition metals that have several valence electrons, which then form complicated superpositions), and one wants a substance that is easy to work with experimentally, say not too corrosive or explosive. Metals like sodium and potassium would be good choices for the first two criteria (much lower boiling point than silver, single valence electron) but fail the third criterion since the alkali metals are corrosive elements. But the answer could also be historical: someone had some silver sitting around and tried it, it worked, and this motivated other experimentalists to use silver in later experiments.

What is a phonon? Is it proved to exist?

A phonon is a particle associated with a quantized sound wave. It is analogous to a photon in that it is a particle that has zero mass and a finite momentum but differs from a photon by having three possible polarizations (two transverse, one longitudinal) and a more complicated dependence of the energy $E(p_x, p_y, p_z)$ on its momentum $\mathbf{p}$ arising from details of the electrical potentials that nuclei move in. (Photons have the simple energy relation $E = pc$.) Phonons also differ from photons in that they can not travel in a vacuum, they are confined to the interior of a solid medium like a crystal.

Quantum mechanics predicts that any classical continuous field such as a pressure field (which is what sound is classically), electric or magnetic field, or gravity must in fact consist of discrete quantized particles. Applying quantum mechanics to the displacement of nuclei in a solid leads to the prediction of zero-mass phonon particles with the properties mentioned above. Gravity must also consist of quantized particles called gravitons but because gravity is so weak, experiments have yet to confirm the existence and properties of gravitons.

Many experiments have confirmed in substantial detail the existence and properties of phonons, they are as real as electrons and photons in that they lead to experimental consequences whose properties are understood in quantitative detail. Besides contributing to the heat capacity of a solid, phonons play an important role in determining the resistivity of a substance (electrons and phonons scatter off one another, which reduces the flow of electrons), in the thermal expansion, and in more exotic phenomena like superconductivity. (Somewhat paradoxically, scattering of phonons off of electrons can lead to a small attractive interaction between two electrons, causing them to form a bound state that then acts like a boson called a Cooper pair such that many Cooper pairs can condense into a ground state.)

How accurate is the hemoglobin model we derived? It would seem as if oxygen binding is strongly affected by temperature and other system variables.

The simple model that I described in lecture, in which two oxygen molecules bind with greater energy than each molecule by itself, led to a pressure dependence of the oxygen occupation number $\overline{n_{O_2}}$ that was fairly close to actual experiment. (But note that Schroeder chose the values of the two binding energies ϵ_1 and ϵ_2 to give the best possible agreement with experiment.)

The important cooperative behavior of the hemoglobin, in which the binding of one oxygen molecule makes it more energetically favorable to bind a second O_2 molecule, is not explained by this model, since this was put in by hand rather crudely. Understanding this cooperative behavior is a challenging problem in the biochemistry of proteins, one has to look at the three-dimensional geometry of the folded hemoglobin protein and understand how oxygen modifies that geometry to facilitate the bonding of the next O_2 molecule. That level of detail lies beyond what can be reasonably discussed in 176.

It is the case that the ability of hemoglobin to bind and transport oxygen depends sensitively on temperature and on other details like the concentration of various ions in water. I am not knowledgeable enough to give a brief summary of these dependencies, especially on the interesting question of what evolutionary pressures led to the formation of such a complicated and capable molecule.

Could you explain this graph again (binding of O_2 to hemoglobin as a function of partial pressure P_{O_2} of oxygen) and its significance?

Not sure what to explain beyond what I mentioned in lecture and in my lecture notes. Briefly, without cooperative binding so that each O_2 molecule binds independently of the other O_2 molecules, hemoglobin holds onto its oxygen molecules too strongly (the occupation number is roughly constant with value 1 until the partial pressure of O_2 has decreased greatly) which leads to poor transport properties: the hemoglobin does not give up its oxygen readily. But with cooperative binding, the occupation number for oxygen decreases more rapidly with decreasing partial pressure of oxygen and so is able to release oxygen more readily to various parts of the body.

As usual, Wikipedia provides more information, although not always easy to understand, look up the article “Hemoglobin”.

How can we think of the chemical potential? Should it be just with the $\mu = -kT \ln(P_0/P)$ formula?

The chemical potential is must easily understood for a system in equilibrium with a reservoir with constant temperature T and constant pressure P. The system is then described by a Gibbs free energy $G(T, P, N)$ and, as explained on pages 164-165 of Schroeder, $G = N\mu$ where N is the number of particles. Thus the chemical potential $\mu = G/N$ is simply the Gibbs free energy per particle: when a particle is added to the system, its free energy changes by μ.

This may not be helpful since the free energy G is itself a rather abstract object and can have a complicated dependence on parameters, e.g., see Eq. (5.56) on page 182 of Schroeder where he writes down G for a van der Waals gas. But for an ideal gas, I think of the chemical potential as being related to the log of the pressure, as given by the equation you wrote down.

Could you review all C_V graphs that we should know, and how each part of the graph connects to different parts of the course?

By the end of the course, you should have memorized and understand well the heat capacity for three systems: ideal gas of diatomic molecules, an Einstein solid, and a paramagnet. You should know that the heat capacity of an electron gas increases linearly with temperature T and that the heat capacity of a photon gas is proportional to T^3 (and know how to derive that result).

Why is a correction to Sackur-Tetrode obtained when using Boltzmann? Isn't the technique still based on multiplicity counting?

In Chapter 6, when we started considering a small system in equilibrium with a reservoir, we could no longer use the argument of "all accessible microstates of an isolated system are equally likely" since the system was no longer isolated, it could exchange energy with the reservoir. So the technique of computing the entropy in terms of a multiplicity is not valid and, in particular, the entropy of an ideal gas of molecules no longer is consistent with the Sackur-Tetrode equation for an ideal gas of atoms.

Did the free energy F's relation to work or Z motivate its creation?

I unfortunately don't know the history of when and why the Helmholtz free energy was introduced, nor do I know of a book that discusses this history. I would be interested to know myself, it is unfortunate that most physics books leave out historical facts.

Is F merely a convenient definition for a decreasing thermodynamic quantity, or does it have a physical meaning?

As I discussed in lecture, F is not just a convenient definition but has to important applications: it helps to determine whether some process that changes a physical system will occur spontaneously or not (it will if the free energy decreases). And the change in F determines the maximum amount of work that can be done by the system.

Are F and G state variables? Are they more/less/equally fundamental as S?

They are indeed state variables. One way to see this is from their definitions such as $F = U - TS$ and $G = F + PV$. Since U and S are state variables, so is temperature (as the derivative of S w.r.t. U) and so $F = U - TS$ is a state variable since U, T, and S are. Similarly, P is a state variable as the derivative of F or U with respect to volume, and V is automatically a state variable (V doesn't depend on the history of how you created the volume).

The potentials F and G are nearly as fundamental as S in that they decrease monotonically over time until they reach a minimum, corresponding to thermodynamic equilibrium of the system in contact with a reservoir. But they are not as fundamental as S since only for an isolated system, can one invoke the strong assumption that all accessible microstates are equally probable, which can be used to derive the thermodynamic properties of F and of G.

Why can we write an integral as $\int e^{-mv^2/2}$? What does this mean without the dv_x or something small to integrate over?

If I wrote that during lecture, then I made a mistake. If one has an classical system whose energy $E(q)$ varies continuously with some parameter q, then the Boltzmann factor over the partition function, $e^{-\beta E(q)}/Z$ has to be interpreted as a probability density, for the system to have a value q that lies in the small range $[q, q + dq]$.

Could you direct us to some introductory chaos math with some applications or examples?

I would first encourage you to find and then read the excellent non-technical book "Chaos" by James Gleick. It was deservedly a best seller on the NY Times list of top books for many months and gives a nice overview, through the eyes and achievements of key researchers who have contributed to chaos theory and nonlinear dynamics.

A next step would be to read Robert May's nicely written Nature paper about the nonlinear behavior of the so-called logistic map $x_{n+1} = rx_n(1 - x_n)$, one of the simplest models that gives insight about nonlinear population growth. (The variable x_n is the population at integer year n normalized by the maximum possible population so is a variable between 0 and 1, the parameter r determines the growth rate of the animals like paramecia that are reproducing.) Here is a link to that paper:

`http://www.math.miami.edu/~hk/csc210/week3/May_Nature_76.pdf`

Complementary to this article, find and read the article "Chaos" in Scientific American, December 1986, pages 38-40, first author James Crutchfield.

A next step would be to sign out or purchase Stephen Strogatz's book "Nonlinear Dynamics and Chaos" which is nicely written book that is the most widely used undergraduate textbook about nonlinear dynamics. This is the book that is also used in Physics 213 at Duke (the intro nonlinear dynamics course).

What physics courses are most applicable (if I had to pick 2-3) for ECE?

This would best be discussed in person, my recommendations would change depending on your interests or goals. Please send me an email to set up a meeting.