

Answers to Physics 176 One-Minute Questionnaires

Lecture date: March 21 and March 17, 2011

Where does Earth's magnetic field come from?

It is known to arise from something called the “dynamo theory”, you can read about it in the Wikipedia article with that title. The physics involves a combination of fluid dynamics, the Maxwell equations, and nonlinear dynamics and the technical details lie beyond what most undergraduate physics majors are prepared to understand.

Briefly: the magnetic field is formed and sustained by a combination of convection of an electrically conducting fluid (for the Earth, liquid iron) and of rotation arising from the Earth spinning on its axis once every 24 hours. But the exact mechanism of how the field arises remains only roughly understood. For example, it is difficult to construct experiments involving rotating convecting liquid metal that operate in the same regime as the Earth's iron core, and the mathematics is nonlinear and difficult so that supercomputer simulations are needed, and it turns out that the parameters corresponding to the Earth's core lie barely within reach of the most powerful computers available. It also turns out that one needs some small initial field magnetic field to get things going, and the origin of the initial field is poorly understood.

It is a very good thing the Earth has such a strong magnetic field since it deflects the solar wind (consisting mainly of energetic protons, which follow the Earth's magnetic field lines to the North and South poles) from striking the Earth's surface, where it would harm life and scour the atmosphere away. Mars is believed to have had a liquid convecting iron core at some point, just like the Earth, but when this core solidified a billion years or so ago (Mars is one half the diameter of Earth so has a relaxation time that is $1/4$ Earth's relaxation time), Mars lost its magnetic field and the solar wind caused Mars to lose most of its atmosphere to space.

How good are our weather models at predicting atmospheric conditions on other planets, e.g., Saturn?

This goes beyond my knowledge. I do know that some details of planetary atmospheres, such as the formation of stripes in the gas giants (Jupiter, Saturn, Uranus, and Neptune) are well understood as a consequence of convection and rotation, and why large spots like Jupiter's Red Spot (and similar spots on Saturn) form and are stable for hundreds of years is understood in

principle. I also know that all planetary atmospheres (except Mars which has nearly no atmosphere and Venus, which has no interesting weather because the planet rotates so slowly) involve turbulence—a temporally and spatially disordered state of a fluid—which remains difficult to understand scientifically and computationally, and many details related to heat transport through a turbulent atmosphere or the formation and properties of clouds remain poorly understood.

You should appreciate that Earth’s atmosphere, and presumably that of the gas giants, has the technical property of being *chaotic*, which is a technical phrase for bounded non-periodic dynamics. Chaos implies that it is impossible to predict with any future computer and with any future mathematical algorithm precise details of Earth’s atmosphere on a time scale better than about 10 days, and presumably this holds for other atmospheres. This means that mainly certain statistical features can be predicted well using weather models, such as the average number of hurricanes per year or the presence of stripes (Hadley cells) in Earth’s atmosphere. But chaos theory implies that it will not ever be possible to use a computer code to predict an important practical detail such as whether a hurricane will pass over Duke University at some time in the future.

Can you give a more detailed explanation of the quantum mechanics relevant to the current topic?

First, take a look at Appendix A, especially Section A.4, in Schroeder where a casual summary of some key quantum mechanical results is given. This is likely too brief and qualitative to be satisfying.

Instead of guessing how much detail you are interested in, or if you have the background (Physics 211 and 212) to understand the details, let me suggest that you take a look at the graduate-level book “Quantum Mechanics: Non-Relativistic Theory, Volume 3” by Landau and Lifshitz. (I don’t know of any undergraduate books that discuss the details with any rigor.) An entire chapter, Chapter XI, is devoted to the quantum mechanics of a diatomic molecule including a discussion of the rotational and vibrational levels, while Chapter XIII discusses the vibrational levels of polyatomic molecules.

These are not topics discussed in most undergraduate physics courses, but are discussed in upper level undergraduate physical chemistry courses, although usually with just the final results given without derivation and then applied. A good book that discusses rotational motion is “Physical Chemistry” by McQuarrie and Duke’s John Simon.

Can you explain why $\sigma_E = kT\sqrt{C/k}$ approaches a constant for large T ?

The key insight is one that I made on page 5 of my March 22 lectures notes: that in the limit of high temperature, the thermal average of some quantity approaches the usual arithmetic average:

$$\lim_{T \rightarrow \infty} \langle X \rangle = \lim_{T \rightarrow \infty} \sum_s p_s X_s \quad (1)$$

$$= \sum_s \left(\lim_{T \rightarrow \infty} \frac{e^{-\beta E_s}}{Z} \right) \times X_s \quad (2)$$

$$= \sum_s \frac{1}{N} \times X_s \quad (3)$$

$$= \frac{1}{N} (X_1 + X_2 + \dots X_N), \quad (4)$$

assuming there are N distinct states. In general, the thermal average $\langle X \rangle$ of some quantity X varies with temperature because of the temperature variation of the Boltzmann factor and of the temperature variation of the partition function. But this temperature variation becomes negligible at high temperatures since each Boltzmann factor becomes approximately 1 and the partition function becomes N , the number of states.

We can apply this insight to the relation you derived in Problem 6.18 on page 231 of Schroeder, written in this way:

$$C = \frac{\sigma_E^2}{k} \frac{1}{T^2}, \quad (5)$$

where

$$\sigma_E^2 = \langle E^2 \rangle - (\langle E \rangle)^2. \quad (6)$$

If there are a finite number of energy levels, the two thermal averages in Eq. (6) both approach constants (although different constants) for large temperatures and so Eq. (5) then indeed implies that $C \propto T^{-2}$ for sufficiently large temperatures.

We can go a little further and work out the lowest-order correction to the infinite temperature limit using Taylor series and so you can see more clearly how the temperature dependence of C approaches the $1/T^2$ behavior. For large but finite T , $\beta = 1/(kT)$ is small and βE_s is also small so for each Boltzmann factor, we have

$$e^{-\beta E_s} \approx 1 - \beta E_s, \quad (7)$$

and the partition function becomes

$$Z = \sum_s e^{-\beta E_s} \quad (8)$$

$$\approx \sum_s (1 - \beta E_s) \quad (9)$$

$$\approx N - \beta \sum_s E_s \quad (10)$$

$$\approx N \left(1 - \beta \frac{1}{N} \sum_s E_s \right) \quad (11)$$

$$\approx N(1 - \beta \bar{E}), \quad (12)$$

where $\bar{E} = (1/N) \sum_s E_s$ denotes the arithmetic average of the energy levels E_s . Note that if T is sufficiently large, $\beta \bar{E}$ is also a tiny quantity since $\bar{E}$ is a finite sum of energies and so has a fixed finite magnitude.

Next, observe that the thermal average $\langle X \rangle$ for large temperatures deviates from its infinite temperature limit $\bar{X}$ (the arithmetic average) by a small quantity proportional to $1/T$:

$$\langle X \rangle = \sum_s \left(\frac{e^{-\beta E_s}}{\sum_s e^{-\beta E_s}} \right) X_s \quad (13)$$

$$\approx \sum_s \left(\frac{1 - \beta E_s}{N(1 - \beta \bar{E})} \right) X_s \quad (14)$$

$$\approx \frac{1}{N} \sum_s \left(\frac{1 - \beta E_s}{1 - \beta \bar{E}} \right) X_s \quad (15)$$

$$\approx \frac{1}{N} \sum_s (1 - \beta E_s) (1 + \beta \bar{E}) X_s \quad (16)$$

$$\approx \frac{1}{N} \sum_s [1 + \beta (\bar{E} - E_s)] X_s \quad (17)$$

$$\approx \bar{X} + \frac{1}{kT} (\bar{E} - \bar{E}) \bar{X}, \quad (18)$$

where I have dropped second or higher powers of β as being small.

Combining Eq. (5), Eq. (6), and Eq. (18), you should be able to see without further calculation that, for large but finite T , Eq. (5) picks up a $1/T$ correction:

$$C \approx \frac{c_1}{T^2} + \frac{c_2}{T^3}. \quad (19)$$

So the more precise statement is that, for large temperatures, the heat capacity can be written as a sum of powers of the small quantity $1/T$, with the first non-zero term being $1/T^2$. For large enough T , we can ignore higher powers of $1/T$ than the second.