Answers to Physics 176 One-Minute Questionnaires

Lecture date: February 22, 2011

For the midterm next Thursday, will it cover everything up to HW6?

It will include material covered up to and including HW6, as well as all lectures up to and including the March 1 class, and the specific reading that I have been recommending on the Announcements webpage.

I would like one example multipart problem working several topics instead of several short examples. (I'm selfish, it helps make my reviewing of notes easier.)

Do you mean like Problem 2 in Assignment 5 or Question 4 on Quiz 2? Good problems of those kinds of problems take time to invent, so what you are asking is not a casual request.

I do plan to work out a some simple multipart problems in Thursday's and next Tuesday's lectures.

I also plan to make the 2010 midterm available with answers so you can try to work through those problems and then check your understanding against the solutions. Other than that, there are numerous thermal physics books on reserve and in the library that you can look up for examples of other problems, see

`http://www.phy.duke.edu/~hsg/176/176-syllabus.html#references`

What does entropy mean in daily life? How can we apply $S = k \ln \Omega$ to non-thermal physics concepts?

Unless you are a scientist or engineer, I don't think entropy means much to anyone in daily life.

Entropy was invented in the context of practical thermodynamic problems such as designing efficient engines and refrigerators and so is important in almost any context in which one wants to convert energy (often in the form of heat transfer) to some kind of work. Since modern society depends heavily on engines to produce electricity (through spinning steam turbines) and motion (cars, airplanes), entropy is relevant on a daily level.

Entropy also plays an important role in understanding biology and in designing communication networks of any kind, so is important also to computer scientists, electrical engineers, and neurobiologists.

Not sure how to answer your question of how to apply entropy to non-thermal physics concepts, most thermal physics concepts don't have meaning outside the context in which they were invented, e.g., we have discussed several times that the concepts of entropy and temperature can't be applied to microscopic systems like a hydrogen atom or electron, you need a system consisting of many elements for thermal physics to apply, and that system further has to be in equilibrium. Most systems on Earth and in the universe are not in equilibrium and so entropy does not give a useful description of their properties.

I've heard of entropy as a measure of increasing disorder. Who motivated this description?

I don't know the history so am not able to answer this. The Wikipedia article on entropy gives a little information about your question.

"Disorder" is a difficult and subtle concept, rather like trying to come up with an unambiguous definition of "random". It is not too hard to come up with different definitions of disorder that give different conclusions for various examples. For example, look up the definition and pictures of a so-called "quasicrystal", which is disordered in that there is no regular periodicity yet not disordered in that the irregular pattern can be precisely quantified.

The concept of paramagnet temperature. Since paramagnets use spin up and spin down for Ω, is it possible to have a hot paramagnet with the same number of spin up and spin down, as a cold paramagnet?

No, this is not possible because "spin up" and "spin down" need to be defined in the context of a fixed external magnetic field $\mathbf{B}$. Once you have set up the external field, there is no ambiguity about what is up and down. A cold paramagnet will correspond to spins mainly parallel to the magnetic field and so parallel to each other. A hot paramagnet, which would include negative temperature paramagnets, would involve many spins antiparallel to the magnetic field.

Unrelated: how does one conceptualize 2^a when a is a non-integer or even an irrational, say 2^π for example? It is somewhere between 2^3 and 2^4 but how do calculations complete this?

Your insight, that $2^\pi \approx 2^{3.14\ldots}$ must lie between $8 = 2^3$ and $16 = 2^4$ is the starting point of one way to conceptualize what 2^a means. You can formalize your insight by assuming that you know the decimal expansion of the exponent a, say we write it in scientific notation

$$a = d_1.d_2d_3\ldots \times 10^m, \tag{1}$$

where the digits d_i all lie between 0 and 9 and m indicates the power of ten. Then

$$2^a = \left[2^{d_1.d_2d_3\ldots}\right]^{10^m}. \tag{2}$$

Raising a number to a power of ten is easy to understand so the tricky part is to figure out what $2^{d_1.d_2\cdots}$ means. But we can rewrite this as

$$2^{d_1.d_2d_3\ldots} = 2^{d_1} \times 2^{d_2/10} \times 2^{d_3/100} \times \ldots, \tag{3}$$

and it is not too hard to figure out what each of these factors mean since the exponents are rational numbers. For example, $2^{d_2/10}$ is a number whose 10th-power must equal the integer 2^{d_1}, so you can easily bracket this number between two successive powers of ten.

I personally would conceptualize 2^x by writing this as $e^{\ln(2)x}$ which means that 2^x is simply some version of the familiar exponential growth curve e^x. The value of 2^π would just be the point on this curve corresponding to $x = \ln(2)\pi \approx 2.178$.

Making sense of 2^a requires knowing the decimal digits of a and this can be hard for many real numbers like π, e, etc. Fortunately, there are many sophisticated efficient algorithms known for generating many digits of most real numbers, especially π, so this is a solved problem.

But perhaps you are asking a different question, which is how can one compute many digits of 2^a efficiently for any real number a? For example, what is the algorithm that your calculator (or that Mathematica or Matlab) uses to rapidly compute 2^π to many digits, say by typing

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N[ 2^Pi , 10000 ]
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which will give you the first 10,000 digits of 2^π. (If you want to be creative, you can get Mathematica to "sing" the digits to you using the `Sound` command, which will help you detect any patterns in the digits.)

I don't know what algorithm Mathematica uses (Wolfram Corp most unfortunately does not publish the algorithms used in its software) but one way to evaluate 2^a would be by assuming first that you have rapid algorithms for evaluating $\ln(x)$, $\exp(x)$, and many digits of π. You could then write

$$2^\pi = \exp(\pi \ln(2)), \tag{4}$$

which requires evaluating $\ln(2)$ to many digits, multiplying this by π to many digits, and then evaluating the exponential of $\pi \times \ln(2)$. This doesn't help us conceptualize 2^π but it does give us a practical way to evaluate this number to any desired accuracy.

Most efficient algorithms used by computers do not use Taylor series (way too inefficient) but instead use some iterative method similar to Newton's method for finding the zero of a function $f(x) = 0$.

Is it true that President Kennedy was assassinated with an ice bullet like the one you mentioned?

The bullet that struck Kennedy was found, what was never found was the rifle that fired the bullet.

The idea of using some kind of ice weapon, say a dagger made of ice, is an old one. The mystery writer Agatha Christie used it in one of her novels and I believe other writers used the idea before her.

Do you believe in sentient extraterrestrial life?

I believe that chemical life similar to life on Earth is highly probable, but that sentient life is not likely.

The known physics of the universe suggests that it is extremely likely that a chemical form of life will be found elsewhere in the galaxy. For example, we know that all stars create by nucleosynthesis the elements of the periodic table in roughly the same relative abundances; in descending order, the six most abundant elements in the solar system are H, He, O, C, N, and Ne, and four of these elements—H, O, C, N—are key building blocks of chemical life on Earth. We know that a galaxy like the Milky Way, which is a fairly typical spiral galaxy, has about 10^{11} stars and there are 10^{11} galaxies in the observable universe, 10^{22} is a lot of stars! Nearly all stars that are close enough to be examined have proved to have planets (about 1,000 exoplanets have now been discovered) so planetary systems seem to be common (although, remarkably, not a single stellar system so far resembles our solar system even qualitatively, most big exoplanets are much closer to the star than our big gas giants are to our Sun). So there is nothing yet that suggests that our solar system, or Earth in particular, have unique properties that would allow life here but not elsewhere.

There is also the experimental evidence that life appeared on Earth about as soon as the Earth was cold enough to support life, which suggests that either life was easy to start spontaneously on Earth, or that life already existed somewhere else in the solar system of the galaxy and landed on Earth to seed it with life.

To me, the big issue is whether life can evolve sufficiently to become sentient, say at the level of being able to develop technologies that would allow radio or laser communication and space travel. While on Earth, life formed early on, it stayed in a single cell form for nearly two billion years before transitioning to multicellular organisms. Even after complicated animals arrived, say the dinosaurs, no species has made the transition to a technological capability except humans. This is surprising. For example, the human species (not homo sapiens, but earlier forms) first appeared about eight million years and didn't develop technology (say fire and tools) until perhaps 20,000 years ago. In contrast, dinosaurs existed for 130,000,000 years before going extinct, about sixteen times longer than humans have so far been around, and yet to the best of our knowledge, none of the dinosaurs developed a tool-making capability, despite there existing strong evolutionary pressures to survive.

So my own guess is that life might be easy to form but sentient life, especially life capable of technology, might be unlikely. And as many scientists have pointed out, technologically capable species have a good chance of self-destruction once they master nuclear and bio-technologies.

Where does the k in $S = k \ln \Omega$ come from? Boltzmann's constant doesn't change the behavior of the entropy, it just gives it units that we can physically interpret. Why was J/K chosen for the units?

I don't know the actual history. The original discovery or invention of the entropy concept involved the ratio Q/T of heat transferred to or from a constant temperature object. This ratio automatically implies that that entropy, identified as Q/T, must have units of J/K. When people later realized that the log of the multiplicity, $\ln \Omega$, was additive over subsystems and reached a maximum in equilibrium and so basically had to be similar to and perhaps the same as the entropy identified through heat transfer, it was natural to define $S = c \ln \Omega$ where c is some constant with units of J/K, so that it could have the same units as Q/T.

I don't know the history of why the constant c was chosen by Boltzmann to be his constant k but would guess that this choice was needed to make the multiplicity $\Omega(U, V, N)$ for an ideal gas lead to the known ideal gas law $PV = NkT$. We will see this line of argument in Thursday's lecture, and it is discussed on page 110 of Schroeder.

Why did "invisible fluids" survive as such convenient, if wrong, explanations of physical phenomena? Was the ether theory of light a similar notion (transfer of energy by some invisible medium)?

I don't know the history. Schroeder mentions some historical reviews of thermodynamics at the back of his book, and Wikipedia has an article (of unknown quality) on "Caloric" which has interesting information.

It is hard mentally to go back in time and see the world from the viewpoint of 19th century scientists: they didn't know or believe that atoms and molecules existed, they didn't know or believe that atoms—if they existed—were in constant random motion, and they had imperfect understandings of energy, heat, and temperature. So I think it was not unreasonable for them to use available metaphors such as some fluid (the caloric) that would flow from one substance to another as heat was exchanged, and scientists were able to make some nontrivial deductions and predictions using the caloric theory.

It was Joule who did the definitive quantitative experiment which showed the caloric theory did not make sense; he could mechanically spin paddles in water to heat the water endlessly (there was no limit to the heat that could be transferred to other systems as long as the paddles were kept in motion) and no one was comfortable with a finite system producing an endless supply of caloric.

The ether theory is indeed analogous to the caloric theory. Maxwell used his equations to predict that there had to be electromagnetic waves that could propagate through space in the absence of charges and currents (*and* that the speed of these waves had to be the speed of light that had been measured in different circumstances). Maxwell and his contemporaries knew that waves had to have some medium to propagate through (sound waves through air, water waves through water, etc) and everyone assumed there was some unknown gas or fluid filling all of space which was the medium. But even Maxwell realized this medium had to have funny properties, e.g., visible light oscillates at high frequencies and this meant the ether had to consist of a highly stiff spring-like medium. But, if so, how come the motion of the planets through the medium didn't get damped, causing the planets to spiral into the Sun?

It took Einstein to make a crazy inspired guess that light consisted of particles (later called photons by someone else, he just called them quanta of some kind) that could then propagate through empty space and so his photon hypothesis simultaneously killed the ether theory.