Answers to Physics 176 One-Minute Questionnaires

Lecture date: February 17, 2011

What is the flavor of Feynman's particle theory? As in, how did he go about showing it?

After talking with some colleagues and finding some information on the Internet, I found that what I knew about Feynman's ideas was not quite correct. First, it was evidently not Feynman but his PhD advisor, Professor John Wheeler, who suggested to Feynman that one reason why all particles could be exactly identical is that they are all the same particle, but traveling back and forth in time. Second, the formalism of quantum electrodynamics (the quantum theory of photons and electrons) formally allows particles to go back in time in just the same way that Newton's equations of motion allow particles to go backwards in time, simply by reversing the sign of the time variable t everywhere in the dynamical equations.

But in QED, an electron going back in time appears as a positron (positive antimatter particle) and vice versa (positrons going back in time behave like electrons). So one reason why the theory can't be correct is that it predicts that there should be equal numbers of positrons and electrons, while only electrons are observed except in rare collisions with enough energy to produce positrons.

Could you briefly explain why identical particles have to be exactly identical?

I don't know of an elementary argument that would be insightful for you. Quantum field theory, which is the foundation of quantum electrodynamics (the theory of electrons and photons), quantum chromodynamics (the theory of the strong interaction, in which quarks interact to form protons, neutrons, mesons, and other baryonic matter), and the Standard Model (unified theory of electromagnetic, weak, and strong interactions) shows that massive particles like an electron or proton arise from the empty vacuum by a particular mathematical mechanism that requires that all particles of a certain kind be exactly alike.

If you have taken Physics 143 or 211, you have learned that excitations of the quantum harmonic oscillator, say creating an oscillator in the nth energy state $|n\rangle$ above the ground state $|0\rangle$ can be written as multiplying the ground state by a so-called "creation operator" $a^\dagger$:

$$|n\rangle = (a^\dagger)^n|0\rangle. \quad (1)$$

Quantum field theory has analogous statements in which particles like electrons, quarks, neutrinos etc appear as excitations of a ground state, "created" by applying certain operators to the ground state. Because there is a unique way to create one or more particles in this formalism, one gets the prediction that all particles of the same type must be exactly identical. Experiments confirm this result.

Why does the $h/2 \to h$ in this equation for Ω, $\Delta x \Delta p_x \geq h/2$, right?

Not sure what question you are asking. Several different forms of the uncertainty principle appear, depending on how precisely one defines the "uncertainty" of an observable like position or momentum, and one sees an h or $\hbar/2 = h/(4\pi)$ appearing on the right side. (The precise definition is the root-mean-square of a quantum mechanical operator, which is not how Heisenberg originally wrote his uncertainty principle.)

It really doesn't matter since Schroeder is giving a heuristic explanation of a more complicated "semi-classical" limit of quantum mechanics. If you start heuristically with the uncertainty principle (which the semi-classical limit does not), you have to use an h on the right side of the uncertainty principle to match the answer you get from the semi-classical limit.

A good but advanced discussion of the semi-classical limit, and how it yields the conclusion that the classical phase space of position and momentum coordinates can be considered as small regions of size $\Delta x \Delta p_x / h$, can be found in Section 48 of Landau and Lifshitz's famous book "Quantum Mechanics, Third Edition".

Could you explain the concept of a multi-dimensional sphere. I have not dealt with the concept before.

The definition of a sphere is "the set of all points that are a given distance r from a given point C", where r is called the radius and C the center of the sphere. This definition works for any set of objects for which one can define a distance between objects. For N-dimensional vectors of the form

$$\mathbf{x} = (x_1, x_2, \ldots, x_N) \,, \tag{2}$$

one can define a distance $d(\mathbf{x}, \mathbf{y})$ between two vectors to be the Euclidean length of their difference:

$$d(\mathbf{x}, \mathbf{y}) = \left[(x_1 - y_1)^2 + \ldots (x_N - y_N)^2 \right]^{1/2} = \left(\sum_{i=1}^{N} (x_i - y_i)^2 \right)^{1/2} , \tag{3}$$

and one can show that this has the usual properties of a distance such as $d(\mathbf{0}) = 0$ (the distance is zero for the zero vector), $d \geq 0$ (distances are non-negative numbers), and the triangle inequality: $d(\mathbf{x}) + d(\mathbf{y}) \geq d(\mathbf{x} + \mathbf{y})$.

The N-dimensional hyperspheres we talked about in class had centers that were located at the origin, which is the N-dimensional zero vector $\mathbf{0} = (0, \ldots, 0)$. Setting $\mathbf{y} = 0$ in Eq. (3), the N-dimensional hypersphere centered on the origin with radius r is defined by all vectors Eq. (2) that satisfy:

$$d(\mathbf{x}) = r, \tag{4}$$

or squaring both sides,

$$x_1^2 + x_2^2 + \ldots + x_N^2 = r^2, \tag{5}$$

which is the form we used in class and that Schroeder uses in this text.

It is possible to define a distance for other mathematical objects like matrices, functions (which you can think of as an infinite vector whose components are labeled by a continuously varying index), and matrices of functions and so it is possible to talk about "N-dimensional hyperspheres of $M \times M$ matrices" if you want to meditate on that.

It is obviously difficult to think in terms of four or more spatial dimensions although some mathematicians and scientists are able to use mathematics so adeptly that they can do a pretty good job. The Wikipedia article "Fourth dimension" may be helpful, and if you hunt around the Internet (e.g., Google "visualizing four dimensions") , you can find numerous videos showing projections of four-dimensional objects into three space dimensions as the objects are rotated along various axes in 4-space, these might help you get some intuition.

You may also enjoy reading the classic book "Flatland" by Edwin Abbott that describes how two-dimensional creatures living on a plane would have a hard time understanding three-dimensional creatures like us. This would help you understand how a four-dimensional person could take you out of a room that is completely sealed off since a three-dimensional room is "open" in the fourth dimension, just as an object lying inside a square lying in a plane could easily be picked up by you and then placed outside the square on the same plane.

Is the idea of negative temperatures purely theoretical, or are there realistic experimental examples?

Paramagnets, especially based on nuclear magnetic dipoles, are a common physical example of a system that can have negative temperature state.

Section 3.3 of Schroeder has a good discussion of paramagnets and negative temperatures, and we will be discussing this in lecture this coming week.

For hypercubes, what if the length was slightly larger than the unit cube? Wouldn't the volume be infinite?

Yes, the volume $V = L^d$ would be a number greater than one raised to a large spatial dimensionality so would be a huge number. This means that the concept of volume is not so useful as a way to quantify "amount of space" in high dimensions.

However, my comment in class still holds, that high-dimensional volumes are mainly surfaces. You just have to consider the ratio of the volume of a cube of length $L(1-2\epsilon)$ to the volume of a cube of length L, the ratio $(1-2\epsilon)^d$ still becomes arbitrarily small for a large enough spatial dimensionality d.