

Answers to Physics 176 One-Minute Questionnaires

Lecture date: February 8, 2011

Is the experiment with the non-equilibrium system in the room dependent on anything except temperature difference (e.g., room size, room shape)?

The answer is yes, the details of the nonequilibrium states do depend on the shape and size of the room and on some properties of the walls such as whether they conduct heat well (like metal) or do not (like wood or glass).

That the patterns do depend on the room's geometry was a surprise when first discovered experimentally (in the late 1970s and early 1980s) and some details remain poorly understood. Before the experiments were done, the belief of many scientists was that, if you could make the convection cell (room with copper floor and copper ceiling held at fixed temperatures) sufficiently wide, so that the width was much greater than the depth of the air, then the patterns in the fluid would not depend on the shape or size of the room, just as the periodic array of atoms in a crystal do not depend on the shape or size of the crystal once the crystal is sufficiently big compared to the spacings of the atoms. (Near the surfaces of crystals, say about 5-50 atoms in depth, there are deviations from perfect periodic structure but the influence of the surface dies out.)

An even bigger experimental surprise was that the air could become turbulent (evolving nonperiodically in time and space) for a constant temperature difference between the floor and ceiling that theory indicated only time-independent convection rolls (stripes) could occur. Professor Robert Behringer at Duke was a coauthor of this unexpected and big discovery (he was a postdoc at Bell Labs at the time), and when I heard about this work as a graduate student at Princeton, I became fascinated with where did these patterns come from and dropped the research I was doing at the time (in condensed matter physics) to study these questions full time. Many details of the stripe, hexagon, and spiral patterns are now well understood theoretically but the question of when and how chaotic patterns (ones irregular in time and space) arise is still about as mysterious now as when they were discovered about 20 years ago.

You might think that understanding this pattern formation is an academic question—who worries about patterns in some highly artificial convection cell?—but there are many practical questions related to engineering, medicine, chemistry, biology, meteorology, and geology for which progress is slow or limited because we don't have a good understanding of nonequi-

librium pattern formation. Two medical examples would be heart attacks (physicists would describe ventricular fibrillation as a transition of an ordered periodic pattern to a disordered chaotic pattern), and epilepsy (which represents a transition from a chaotic brain state to one that more periodic in time).

With your sustained nonequilibrium example, what properties (of the gas, room, temperature) causes the different formations? I.e., what makes the distinction between the hexagons and lines?

A related question was: “What was the different initial conditions between the two air current simulations that caused two patterns that were so different? What’s special about a hexagon, in that they both shared some sort of hexagonal pattern?”

Let me answer the second question first: the formation of stripes or of hexagons is an intrinsic property of the convecting fluid and does not depend on how you started the experiment (the initial conditions of the fluid) nor on the size and shape of the room provided that the width of the room is sufficiently big compared to the depth of the air. But unlike atoms in a crystal, subtle details of the final pattern such as the final precise spacing of the stripes or hexagons *do* depend on the initial conditions (but, again, not whether you get stripes or hexagons). That there is not a unique choice of spacing was a strong experimental clue that an entropy-like quantity could not exist for nonequilibrium systems since such a quantity would force a unique spacing to be selected, as is the case for atoms in a crystal.

Whether a fluid forms stripe patterns or hexagons at the onset of convection depends on whether the kinematic viscosity ν (the dissipation coefficient that appears in the relaxation time expression L^2/ν that determines how long it takes relative motion in the fluid to damp out) is weakly dependent on temperature (giving stripes) or has a substantial dependence on temperature (giving hexagons). I don’t know of any way to predict when ν is strongly dependent on temperature, this usually has to be discovered by experiments.

Why is it that nonequilibrium systems arrange themselves in predictable patterns? What is the physical basis for this?

It is not known and remains a central question of nonequilibrium science.

In certain experimental regimes, such as a really wide room (compared to the depth of the air) and a temperature difference ΔT just big enough

to initiate convection, there is much theoretical understanding and theory is capable of correctly predicting many details about which patterns form (stripes versus hexagons versus quasicrystals versus mixtures of patterns), how localized disruptions of periodicity called defects move about, how the size and shapes of the walls affect the patterns, and when patterns become unstable to some new pattern. But once the temperature difference between the ceiling and floor becomes substantially larger than the minimum temperature difference needed to initiate convection, almost no feature of pattern formation is understood, especially the time-dependent chaotic patterns.

You mention that at a high enough magnetic field, humans would be levitating or pulled around like the frog. I am confused on how this correlates with MRI because we were told that at a high enough magnetic field, humans would feel pain inside the bone. Are the arrangements of these magnets different

The frog and other objects can only be suspended near the opening of the solenoid, where the magnetic field is not uniform and changes rapidly in strength and direction; it is the so-called gradient (spatial variation) of the magnetic field that causes a diamagnetic repulsion that pushes the frog (or any other material substance) away from the opening of the solenoid. The reason for this can be understood in terms of freshman physics (Physics 42, 52, or 62 at Duke) but for lack of time I don't give the details here.

MRI machines work by placing someone *inside* a solenoid so that a strong approximately uniform magnetic field passes through the part of the body that one is trying to image. Uniform magnetic fields do not cause a diamagnetic repulsion (there is no change in flux through small loops of a fixed orientation with respect to the magnetic field), but they can cause large forces to appear if there are any magnetic materials in the body.

I didn't know that a strong uniform magnetic field could cause pain inside bones and would be quite interested to learn more. I would not have guessed that since I thought that bones do not have magnetic atoms (like Fe, Ni) inside them, this would not happen.

Ed Purcell in his classic undergraduate book "Electricity and Magnetism" mentions briefly that, if you put your head in a strong magnetic field (say a few tesla in strength) and shake your head around, there are no ill effects but you do get a sour taste in your mouth because the shaking generates eddy currents in your tongue that activate the neurons in your taste buds. I have never tried that would be interested in doing so.

Would it be possible to levitate a human? Would it just take a very strong magnet?

I have never done the calculation and so don't know what it would take to levitate a human, which would be a lot of fun to actually do. Would some member of the class want to work out the details with me?

The hard part might be to construct a big enough superconducting solenoid that a person could fit comfortably into the opening. You would also have to be careful not to have any metal on you like fillings in your teeth, eddy currents would cause sparks and make the metal get hot.

Would levitation be a practical means of travel in the future as in the sci-fi world?

Diamagnetic repulsion, like that of the frog, is such a weak effect that I don't think that kind of levitation will ever be practical, the magnets and technology are too expensive.

Magnetic levitation of a different kind has already been studied worldwide as a way to improve the economics of trains: instead of having trains roll on wheels, you would use superconducting magnets to levitate them a small distance above other magnets and the train could move at high speed on a cushion of air. Japan and Germany built such levitating trains a decade ago to try out the technology, I assume it is still not economic to use.

In the discussion of entropy, why is it fair to assert that the entropy is monotonically increasing with thermodynamic equilibrium and that the entropy has a maximum at thermodynamic equilibrium?

At this point in the course, all I have done is show that *if* an entropy like quantity S existed and had certain properties like additivity over subsystems, then we could start to explain various properties of equilibrium systems. This Thursday, we will discuss Sections 2.3 and 2.4 of Schroeder (see especially page 59) where you will hopefully understand why isolated systems spontaneously change their internal properties over time (such as energy moving about between subsystems) so as to have this property, with S defined in terms of the multiplicity as $k \ln(\Omega)$.

The insight basically boils down to the insight that, if you have a huge number of pennies (say 10^{23} pennies), and you flip all the pennies randomly, at the same time, over and over again, it is extremely unlikely at any given time to observe anything but almost exactly 50% heads and 50% tails. (You will soon learn how to calculate this situation in full detail, so hang on.)

So if you start off with nearly all the pennies being tails and start flipping coins at random, with overwhelming probability you will soon end up with just 50% heads; even a tiny deviation like 50.00001% heads (for 10^{23} coins) will never be seen. There is no law of nature that requires this 50% heads and tails, it is a consequence of elementary laws of probability applied to a regime that people lack intuition about (except for 176 students), when there is Avogadro's number of objects that are varying randomly.

What is the relationship between the most probable macrostate and the equilibrium temperature of a system?

The relation is that the state of a macroscopic system will be the most likely macrostate only if the temperature (as defined by $1/T = \partial S/\partial U$) is the same for all macroscopic subsystems. (The uniformity of temperature is a necessary but not sufficient condition since an isolated system can be in thermal equilibrium but not yet in mechanical or diffusive equilibrium.) Note however that the actual value of the temperature T tells you nothing about the properties of the most probable macrostate since it is the uniformity of T , not its value, that determines when the most likely macrostate occurs.

Why is the harmonic oscillator so significant in physics/nature?

The importance of the harmonic oscillator is actually an artifact of how introductory physics courses are taught, not because such oscillators are so important in nature.

Newton's second law of motion, $ma = f$, is actually quite difficult to solve mathematically for many cases of interest because the equations are generally nonlinear (the force varies nonlinearly with the coordinates or velocities of the particles) and nonlinear mathematics is too hard to teach at the freshman level. (At Duke, you don't learn about nonlinear mathematics unless you take a course like Physics 213 or Math 132S.) The few cases that can be solved completely such as the two-body gravitational problem (leading to elliptical orbits and Kepler's laws) and the harmonic oscillator in various forms (damped or undamped) therefore play prominent and somewhat misleading roles in intro physics courses. If students knew more computer programming when they were freshmen, say they all knew Mathematica or Matlab, it would be possible to solve Newton's second law numerically for more realistic forces, in which case they would discover that simple harmonic behavior is actually rare.

As I mentioned briefly in lecture, most forces $\mathbf{F} = -\nabla V$ (where $V(x, y, z)$ is the potential energy of a particle) act harmonically (linear in the coordinates) only for tiny displacements about some time-independent equilibrium state. For small displacements about equilibrium, you can ignore as sufficiently small all higher-order terms (after the quadratic terms) in some Taylor series expansion of the potential energy about the equilibrium state. Small-amplitude dynamics turns out to be an accurate assumption for sound waves, for the lowest energy vibrational states of a diatomic molecule, for atoms vibrating in solids whose temperatures are well below the melting point (the case for our Einstein solid), and is an essentially exact assumption for light waves since light evolves according to the Maxwell equations which are linear. But for many other phenomena in nature and for engineered systems, motion is anharmonic and nonlinear and the evolution equations can not be solved mathematically (although are often easy to study numerically and experimentally). Thus the beating of your heart is a highly nonlinear oscillation that can not even be closely described by a harmonic oscillator, and the same holds for many dynamical states of fluids and of mechanical systems.

Can you explain again where N and q come from?

The integer N is the number of identical non-interacting quantum harmonic oscillators that was suggested by Einstein as a way to model and so understand the vibrations of atoms in a crystal. It takes $N/3$ atoms in a crystal to correspond to N oscillators since each atom can vibrate in three independent directions.

Vibrations in real crystals can be accurately described by harmonic oscillators but the oscillators have to be coupled (so the motion of one oscillator affects the motion of another oscillator) and the frequencies of the oscillators vary over a range. We will discuss later this semester the so-called Debye theory that includes these more realistic details.

Once you assume that you have N identical non-interacting quantum harmonic oscillators, you can characterize a macrostate of this system by the total energy E_{total} of the N oscillators. Since the energy of the i th harmonic oscillator has the form

$$\frac{\hbar\omega}{2} + \hbar\omega q_i, \tag{1}$$

where q_i is a non-negative integer and $1 \leq i \leq N$, the sum of the energies

of all the oscillators has the form

$$E_{\text{total}} = \left[\frac{\hbar\omega}{2} + \hbar\omega q_1 \right] + \left[\frac{\hbar\omega}{2} + \hbar\omega q_2 \right] + \dots + \left[\frac{\hbar\omega}{2} + \hbar\omega q_N \right] \quad (2)$$

$$= N \frac{\hbar\omega}{2} + \hbar\omega (q_1 + \dots + q_N) \quad (3)$$

$$= N \frac{\hbar\omega}{2} + \hbar\omega q, \quad (4)$$

where we define

$$q = q_1 + \dots + q_N, \quad (5)$$

to be the sum of the quantum states of all the harmonic oscillators.

So a macrostate of N identical noninteracting quantum harmonic oscillators can be characterized by its total energy, which in turn involves specifying a non-negative integer q via Eq. (5). A microstate of this system would be a vector of N non-negative integers $(q_1, \dots, q_N)$, which describes the state of each oscillator.

Schroeder (and myself) then simplifies the discussion by dropping the zero-point energy term $N(\hbar\omega/2)$ that is always constant and so does not show up in the thermal energy U , and by identifying q with the total energy, i.e., when he or I says “ q ”, you should think $U = \hbar\omega q$.

Is $\Omega(N, q)$ related to the degeneracy of the quantum system for the Einstein solid?

For a quantum system whose macrostate is defined by the total energy of the system, Ω is indeed the degeneracy of the total energy, i.e., the number of linearly independent physical states with the same given energy. Your insight here will be an important one when we get to Chapter 6 and discuss quantum gases of electrons and photons. But since some students in 176 have not taken Physics 143 or 211 yet, I have avoided that technical term.

But one could characterize a macrostate in ways that don't involve energy, say by the magnetization of a magnet or pressure of a gas, or the system could be classical in which cases Ω means something other than the degeneracy of the total energy.

How accurate is the Einstein model of solids? It seems pretty idealized as many solids have imperfections and places for electrons to jump around.

It is accurate and useful, see Fig. 7.29 on page 312 of Schroeder, where Einstein's theory and the more accurate (but still not perfect) Debye theory

are compared, while Fig. 1.14 on page 30 shows how Debye's theory compares with experimental data. By fitting data with just one parameter, Einstein's model gives better than 5% accuracy over the entire range of temperature. (One small detail it gets wrong is the mathematical way that $C_V(T)$ goes to zero as $T \rightarrow 0$.) The key achievement of Einstein's theory was that it showed how quantum mechanics could explain in principle the vanishing of the heat capacity of solids at low temperatures: the equipartition theorem failed specifically because the energy levels of vibrating systems were quantized rather than varying continuously like a classical harmonic oscillator.