Answers to Physics 176 One-Minute Questionnaires
Lecture date: January 25, 2011

Will all quizzes and exams be as long as today's relative to the time allotted? This seems like it would penalize students who write more slowly.

I try to design the quizzes so that 15-20 minutes should be enough time to answer all the questions provided that you have been actively and critically thinking about the course material and homework problems.

Designing a quiz or an exam is an imperfect process and I take that into account by scaling the grades. But I also purposely want the quizzes to challenge the class, at least a little bit, so as to encourage you to improve your problem solving abilities and your understanding of the material through frequent feedback.

If you find that you do write slowly, one way to help with that is to practice solving more problems before a quiz or exam. Try skimming the other thermal physics books on reserve or in the library and try to solve representative problems that other authors have identified as important, also try solving some extra problems in Schroeder.

What is the difference between the T/F question on relaxation time on the quiz (one in vacuum, on in air)?

I will be posting detailed answers to all quizzes a few days after each quiz. But briefly, the rod of length L and radius $r \ll L$ that is sitting in vacuum has a relaxation time of L^2/κ since information can only travel along the length of the rod for equilibration to occur. For the rod sitting in air of constant temperature T, it is the equilibration of the rod with the air, not with itself, that now matters since the final equilibrium temperature will be T. Since no part of the rod is further away from the air than a distance r, the relaxation time is now r^2/κ.

On the 2009 quiz 1, the solutions said that the mechanical relaxation time was $\tau_{\mathrm{mech}} = d/v_{\mathrm{sound}}$ for the balloon problem. Why does this not vary like L^2?

I was not correct in explaining that solution. In 2009, I had explained to the class that imbalances in mechanical forces typically propagate quickly

through a system compared to diffusion of heat or concentration, typically at the speed of sound in the material. Thus when you pop a balloon with a pin, the time it takes the balloon to disintegrate is roughly the circumference of the balloon divided by the speed of sound in the skin of the balloon. But this is just the time for one part of a system to know about some other part of the system having a pressure difference, it is not the time for equilibrium which indeed occurs on a slower diffusive time scale as we discussed in class.

Are there other commonly used coordinate systems besides rectangular, cylindrical, and spherical?

The three you mention are the only ones most undergraduates in math, physics, and engineering have to know. (Note: one usually says "Cartesian" instead of "rectangular" for the usual xyz coordinate system.)

There are infinitely many so-called curvilinear coordinate systems that people could consider for various problems in math, physics, or engineering and there are certain fields like general relativity and optical cloaking for which fully general coordinates are needed. More typically, people use so called "orthogonal" curvilinear coordinates which means that surfaces defined by constant values of a single coordinate (say the plane x=constant, the plane y=constant) are mutually orthogonal at any point in space. These orthogonal systems are often convenient for solving certain partial differential equations such as a Poisson equation or the Schrodinger equation. For example, with a proper choice of a coordinate system, one can use the method of "separation of variables" to reduce a partial differential equation to an infinite set of ordinary differential equations that is often easier to solve. You learn about these kinds of techniques in course like Math 108 and Physics 182.

Some names of other orthogonal coordinate systems that show up (usually at the graduate level in math and physics) are: confocal ellipsoidal, confocal paraboloidal, cyclidic, oblate spheroidal, and toroidal coordinates. The geometric symmetry of a given problem often suggests which coordinate system is appropriate to use. In my career as a theoretical physicist, I have only once used one of these less common orthogonal coordinates, the toroidal one, so my own sense is that there is not a great need to know about these other kinds.

Why was the change in momentum of a particle not simply $\Delta p = 2mv$? How come there were the extra factors of N/V and $1/6$?

The change in momentum of a single particle (for the simple anisotropic kinetic model) was indeed just $2mv$. But our goal in class was to calculate the *total* change in momentum of *many* particles striking the wall in some arbitrary short time Δt. (The change in momentum of the wall is just the negative of the change in momentum by conservation of momentum.)

This is what led us to consider a small cylinder of volume $(v\Delta t)A$ that had $[(v\Delta t A] \times (N/V)$ particles in it. But for the anisotropic model, only $1/6$ of the particles in that cylinder are actually moving in a direction that will strike the wall. So the total change in momentum in time Δt was:

$$\Delta p_{\text{total}} = (v\Delta t)A \times \frac{N}{V} \times \frac{1}{6} \times 2mv. \tag{1}$$

How would you generalize the model (deriving pressure) to accommodate different masses?

You would simply include new particles in the gas, that have different masses and different speeds, and carry out the exact same analysis, and you would find that the effects just add, i.e., you discover the concept of "partial pressures" in which the molecules of one kind contribute to the pressure separately, as if the other molecules weren't present. (See your intro chemistry text.)

For example, we could model air as a gas that has N_1 nitrogen molecules of mass m_1 and N_2 oxygen molecules with mass m_2. For simplicity, we can assume that all the nitrogen molecules have one speed v_1 and all the oxygen molecules have a second speed v_2, and that all velocities are parallel to one of the coordinate axes (i.e., an anisotropic model). Let's choose a short time Δt and ask: what is the total momentum transferred to an area A on the wall of the container in time Δt? Since the molecules move with different speeds, we have to use two separate cylinders, one of height $v_1\Delta t$ for the nitrogen molecules and one of height $v_2\Delta t$ for the oxygen molecules. The total momentum delivered to the area A within time Δt is then

$$
\begin{aligned}
\Delta p_{\text{total}} &= \left[(v_1\Delta t)A \times \frac{N_1}{V} \times \frac{1}{6} \times 2m_1v_1\right] \\
&\quad + \left[(v_2\Delta t)A \times \frac{N_2}{V} \times \frac{1}{6} \times 2m_2v_2\right],
\end{aligned}
$$

so the pressure is

$$P = \frac{(\Delta p / \Delta t)}{A} = \frac{\Delta p}{A \, \Delta t} = \frac{1}{3} m_1 v_1^2 \left(\frac{N_1}{V} \right) + \frac{1}{3} m_2 v_2^2 \left(\frac{N_2}{V} \right). \qquad (2)$$

But the first expression is the pressure you would have if there were only nitrogen molecules in the box, the second expression is the pressure you would get if there were only oxygen molecules in the box. The pressures then just add separately as claimed. This result is a consequence of the assumption that molecules do not interact with one another in an ideal gas.

Can you please provide more insights in the development of the solid angle calculation?

A related question was: "So is solid angle just basically a small area on the sphere we are looking at?"

The first question is unfortunately vague, I am not sure what is not clear for you. Perhaps we can meet and talk in person.

The solid angle Ω of some object is basically the area of some part of a sphere's surface compared to the square of the radius of the sphere. For example, the United States is an irregular area that spans the surface of the Earth and its solid angle Ω (with respect to the center of the Earth) would be the surface area of the US $\approx 3.7 \times 10^6 \, \text{miles}^2 \approx 9.5 \times 10^6 \, \text{km}^2$ divided by the square of the Earth's radius:

$$\Omega_{\text{UnitedStates}} = \frac{9.6 \times 10^6 \, \text{km}^2}{(6,400 \, \text{km})^2} \approx 0.2, \qquad (3)$$

which you should compare with the maximum solid angle $4\pi \approx 13$, i.e., the US occupies $0.2/(4\pi) \approx 1.6\%$ of the Earth's surface area.

What do we have v_{rms} and not v_{avg}?

When the dust cleared from our kinetic theory calculation, we had the result

$$\frac{1}{2} m v^2 = \frac{3}{2} kT, \qquad (4)$$

where v is the constant speed of each molecule. From just this derivation, there is no way to tell how to replace v with a statistical average of some kind, for example should we use the mean, root-mean-square, or some other average value.

But it is not too hard to redo the kinetic theory we discussed in lecture by allowing the molecules in the gas to have a range of speeds. We will discuss how to do this later in the course (see Section 6.4 of Schroeder) and the conclusion is that v^2 in Eq. (4) needs to be replaced by $\langle v^2 \rangle$, the average of the speed squared.

Where did that 0.9 come relating average speed to rms speed, $\overline{v} \approx 0.9 \times v_{\mathrm{rms}}$?

This question is related to the previous one. When we discuss Section 6.4 of Schroeder, you will learn how to derive from first principles the speed distribution $\mathcal{D}(v)$ of molecules in an equilibrium gas. This is the probability of observing a particular molecule in the gas to have a speed that lies in the small interval $[v, v + dv]$; alternatively, this is the fraction of molecules in the entire gas with speeds in this small range. Once one knows the precise speed distribution (this is Eq. (6.50) on page 244 of Schroeder), one can calculate the average speed and rms speed for an equilibrium gas. One finds that $v_{\mathrm{avg}}/v_{\mathrm{rms}} = \sqrt{8/(3\pi)} = 0.921 \approx 0.9$ to one digit.

What is the conceptual difference between the average speed and rms speed?

There is no deep conceptual difference, one is not necessarily more important or more physical than the other although these are the two most common ways to characterize various sets of numbers.

As an example, let's say you have three positive numbers x_1, x_2, x_3. Then there is an infinite number of ways to characterize the statistical properties of these three numbers. For example, if p is a nonzero real number, we could define a pth-average value by this procedure:

$$\langle x \rangle_p = \left(\frac{x_1^p + x_2^p + x_3^p}{3} \right)^{1/p}. \tag{5}$$

This reduces to the mean of the numbers for $p = 1$, the rms of the numbers for $p = 2$ but you can now see that there is a continuum of closely related averages that we could also consider. The cases $p = 1$ and $p = 2$ are widely used because they are the easiest to compute or work with mathematically.

If the numbers x_i are all greater than 1, raising them to higher powers will tend to emphasize the bigger numbers, so you might guess correctly that $\langle x \rangle_p > \langle x \rangle_q$ for $p > q$. This insight explains why, for molecules in an equilibrium gas, one would expect generally $v_{\mathrm{rms}} = \langle v \rangle_2 > v_{\mathrm{avg}} = \langle v \rangle_1$.

What is the reason that the ideal gas law must be corrected for polyatomic gases?

I hope I didn't say this during lecture, since this statement is not correct.

The ideal gas law $PV = NkT$ does not include any information about the molecules in the gas such as their mass or chemical properties, that is one reason why the ideal gas law is so general and so remarkable. As long as a gas has a sufficiently low density, so the average spacing $(V/N)^{1/3}$ of particles is large compared to the particle size, the properties of the molecules in the gas don't matter. The ideal gas law holds accurately for He atoms, for polyatomic molecules like ethane (C_2H_6), and for mixtures of molecules like air.

When the density becomes so high that the ideal gas law breaks down, it breaks down for both atoms and polyatomic molecules, although the deviations do depend on molecular details.

What extremes that violate conditions for $PV = NkT$ can be easily accounted for?

Sorry, I am not sure what you are asking here. Do you mean physical examples of cases where the ideal gas law fails?

For Schroeder 1.3, why does vibration have two degrees of freedom?

I will discuss this in class on Thursday. The answer is that a vibrational mode contributes two quadratic terms to the total energy, a kinetic energy term like $(1/2)mv^2$ and a potential energy term like $(1/2)k(x - x_0)^2$. Each quadratic term counts as a degree of freedom so each vibrational mode contributes two degrees of freedom.

An atom in the middle of a three-dimensional crystal has three vibrational modes and so contributes six degrees of freedom (six separate quadratic terms) to the energy.

Why bother with the coordinate constrained (anisotropic) cube model relating energy and the ideal gas instead of just going straight to the isotropic case that works for all directions?

The goal was to illustrate for the class the art of making scientific assumptions that help to solve a difficult problem. Calculating the pressure P for

a real gas in a real box in terms of momentum transfer of molecules is difficult: the velocities point in all different directions, the speeds are not the same, the walls might not be smooth and elastic, and so on. It turns out that a valuable and important skill in physics (also in engineering) is trying to identify simple models that get at the heart of what is going on without drowning in details.

The anisotropic model is perhaps the simplest way to set up a kinetics calculation: just one speed, all velocities are parallel to coordinate axes, walls are perfect reflectors. Our calculation then immediately gave an interesting insight, that the average molecular kinetic energy was related to temperature. A more detailed calculation might give a different numerical coefficient, but the basic insight does not change, that the temperature is proportional to average kinetic energy.

Is there any simple reason why the two kinetic approaches (anisotropic and isotropic) calculations are the same? Is it just good luck or coincidence?

For the case of calculating pressure, the two approaches give the same result and it is pretty much a coincidence. For other cases, like the homework problem of particles delivering energy by sticking to a cold plate or the problem of effusion (particles leaving the gas through a small hole in the wall), the two approaches give different answers. But it is surprising that, in nearly all cases, the answers obtained in the two cases agree to 20% or better. This says that being isotropic is not that important compared to just allowing the velocities to point in different directions.

Was Bernoulli thinking in terms of molecules?

He was, for example here is a drawing from his paper,
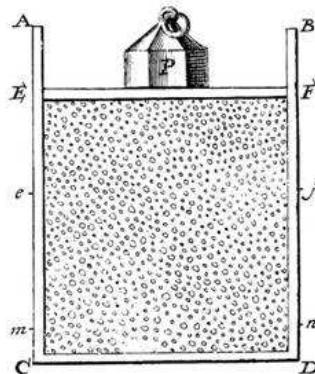
there is no mistaking his thinking about atom-like things here. But in his paper, he was defensive about having to invoke particles that no one believed to exist and over a century would pass until Maxwell was able to show that the idea really worked. My understanding is that most of Bernoulli's contemporaries rejected his insight because no one could understood how the molecules, even if they existed, would move perpetually without slowing down. People thought molecules would slow down through some kind of friction and eventually form some static lattice in space; pressure then arose from some kind of repulsion of the particles rather than by collisions with the wall.

As you perhaps know, it was the Greeks who, over 2000 years ago, first took seriously the idea that everything in the world might be composed of atoms. But it wasn't until the late 1800s that people developed the ability to actually measure the size of atoms or prove their existence, molecules were just too tiny.

By the way, one of the earliest estimates of molecular size came from a suggestion of Benjamin Franklin in the 1700s. He had observed that a teaspoon of oil (say of volume V) would spread out on a pond surface into a large contiguous region (say with area A). He then guessed that if the oil were made of identical molecules and each molecule had the same cubic volume L^3, then L would have to be the height of the oil stain and so $V = AL$ and $L = V/A$: the size of a molecule could be deduced from a teaspoon of oil and by measuring an area on a pond. Although Franklin didn't implement his own suggestion, others soon did and discovered that molecules were of order a nanometer or less, really tiny. I always that this was a neat elegant insight.

What would you have to do if you couldn't assume that the speed after hitting the wall was different from the speed before hitting the wall?

I will discuss this possibility in the next lecture.

Are there limits to the validity of the equipartition theorem? Taking into account the constraints of the derivations, this seems likely.

The equipartition is an exact result for terms in the energy of a molecule that have a quadratic form. It fails for energies that do not have a quadratic form, which occurs for liquids and solids, i.e., when atoms are close enough to influence each other.

But more importantly, as I will explain in Thursday's lecture, the equipartition theorem only describes classical (non-quantum) systems and so should not be taken too seriously. When one compares the predictions of equipartition with experiments (see the two figures on page 30 of Schroeder, which we will discuss this Thursday), it is wildly wrong and only quantum mechanics is able to explain the experiments.

When we talked about relaxation times a awhile ago, you wrote an exponential $T_0 E^{-t/\tau}$ where τ was the time scale. Does the exponential come from Newton's law of cooling?

No it does not. To give you an idea of where it comes from, I need to use some math of the sort discussed in Math 108.

Consider a one-dimensional metal rod of length L that is not in thermal equilibrium so that it's temperature $T(t, x)$ varies spatially and will vary with time as the rod approaches equilibrium. One can show that the temperature field T evolves in time according to a so-called diffusion equation:

$$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial x^2}, \tag{6}$$

where κ is the same thermal diffusivity that shows up in the relaxation time L^2/κ. By using separation of variables and Fourier analysis, one can show that the solution to Eq. (**??**) has this form:

$$T(t, x) = \sum_{n=1}^{\infty} T_n \sin\left(\frac{n\pi x}{L}\right) e^{-(n\pi)^2 \, t/(L^2/\kappa)}, \tag{7}$$

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where the coefficients T_n are determined from the temperature profile $T(x, 0)$ at time $t = 0$. Note how the solution is not a single exponential in time, it involves an infinite sum of decaying exponentials that each multiply some spatial mode. But all the exponentials in the solution involve the quantity L^2/κ which has units of time. So this example helps to illustrate why L^2/κ is a time scale associated with the approach to equilibrium of the rod, but the mathematical behavior is not a simple exponential decay.

Now that you know something about relaxation times, you should be able to understand why Newton's law of cooling can't be an accurate description in many cases. This law says that the time it takes for an object of temperature $T(t)$ to reach equilibrium with its constant-temperature surroundings (say with temperature T_0) is proportional to the temperature difference $T - T_0$:

$$\frac{dT}{dt} = -\frac{T - T_0}{t_0}, \tag{8}$$

where t_0 is some characteristic time of cooling. But under what conditions can you treat an object (say a cup of coffee) as having a single temperature $T(t)$? Only if the relaxation time $\tau = L^2/\kappa$ for the object is small compared to t_0. This will only be true for small objects (small L) or for objects that conduct heat rapidly (large κ). If $\tau \geq t_0$, the object will have a non-uniform temperature and so can not be described by a single temperature $T(t)$. One then has to solve a separate harder mathematical problem to figure out the spatial variation of temperature inside the object as it is cooling down.