Answers to Physics 176 One-Minute Questionnaires

Lecture date: January 20, 2011

What is the difference between "properties of a macroscopic system do not depend on history" versus "properties have to be independent of time"?

I gave an example of this in class: a block of wood is an example of a macroscopic system that can be time independent but its properties are dependent on its history, e.g., how it was produced by the tree. Any macroscopic solid that has some persistent non-uniform structure would be similar to wood in this regard; the rigidity of a solid prevents the atoms from mixing over human time scales so the history is frozen in.

The way to create a history-independent equilibrium solid would be to start with a liquid phase (if a liquid phase exists, this would not be the case for wood) and very slowly cool the liquid until it crystallizes into a solid. The slow cooling allows atoms to move around and eliminates the frozen structure.

What are some objects that would seem to be *not* in equilibrium that actually are (if any)?

Not sure how to answer this question since it depends on what you mean by "seems to be". A quick answer is that truly equilibrium systems are almost never found on Earth on a day-to-day basis, one has to work really hard in a lab to arrange for objects to be in equilibrium.

What are more examples that don't reach equilibrium in the same way the sandpile is not in equilibrium? Are there other way equilibrium is not reached?

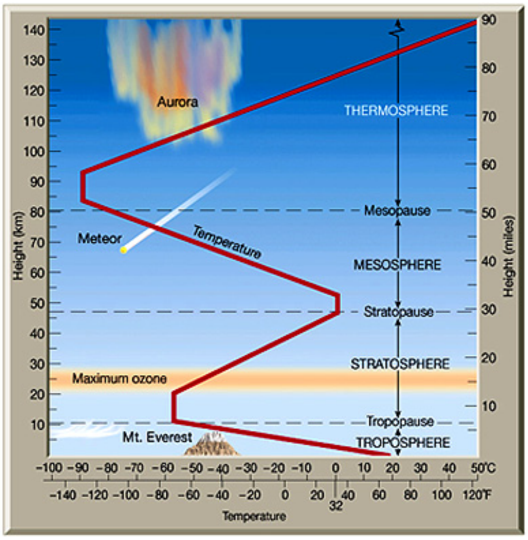
There are many ways that a system can be out of equilibrium beyond not having a constant temperature, not having a constant pressure, etc. For example, the speeds of molecules in an equilibrium gas have to obey a universal distribution called the Maxwell speed distribution (see Section 6.4 on page 242 of Schroeder). It is possible to temporarily change the speed distribution of a gas to be inconsistent with this distribution, e.g., shoot a monoenergetic beam of particles into the gas. The gas would then be out of equilibrium but in a way that is hard to detect.

Could you explain more about the condition for equilibrium that the pressure P has to be the same everywhere, except if there is an external field? What happens in an external field?

One condition for a macroscopic system to be in thermodynamic equilibrium is that all macroscopic properties are time independent. This in turn implies that the net force on any macroscopic part of the system must be zero, else that part of the system will start to move. For a body in the absence of any external fields like a gravitational field, forces inside bodies arise from pressures being applied to virtual surfaces and so the requirement for mechanical equilibrium is that the pressure is the same everywhere inside the body. In the presence of an external field, the pressure has to balance the forces from the external field.

Problem 1.16 on page 8 of Schroeder is a good example. Consider a thin horizontal block of air that has a height dz and an area A and so volume $dV = Adz$. If the mass density of air is ρ, then gravity pulls the block downward with a force $F = mg = (dV\rho)g = (Adz)\,\rho g$. In order for the block to be in mechanical equilibrium, the sum of the forces on the block have to add up to zero. If the pressure in the air varies with height z above the ground as $P(z)$, then the total force on the block would be $AP(z) - AP(z+dz) - A\rho g dz = 0$ since the pressure at the top of the block, $P(z+dz)$, pushes the block down and the pressure at the bottom of the block, $P(z)$, pushes the block up. Observing that $P(z+dz) \approx P(z) + (dP/dz)dz$ (this is the first two terms of the Taylor series expansion of $P(z+dz)$ about z), you can easily see that $dP/dz = -\rho g = -[m(N/V)]g = -(mg/(kT))P$ for mechanical equilibrium to hold. Here I used the ideal gas law to write $N/V = P/(kT)$ and where m is the mass of a gas molecule and N is the number of molecules in the block of air.

The equation $dP/dt = -(mg/(kT))P$ in turn predicts that the pressure P and so gas density $n = N/V = P/(kT)$ must decrease exponentially with height (for an equilibrium constant-temperature atmosphere). This prediction disagrees with observation for the simple reason that the atmosphere is not in equilibrium, e.g., its temperature varies substantially with height as shown in this image:



There is a lot of interesting physics and science related to why the temperature varies in this fashion, I'll let you explore this on your own.

If an object contains two types of differences, say temperature and concentration, how do we write the relaxation time? If $\tau_T = L^2/\kappa_1$ and $\tau_C = L^2/D$, is $\tau_{TC} < L^2/\kappa_1 + L^2/D$?

Your question is actually not meaningful since one defines a relaxation time in the context of a specific mechanism that restores a nonequilibrium system to equilibrium. Thus one talks about a thermal relaxation time or a diffusive relaxation time or a viscous relaxation time, but one does not usually talk about a general relaxation time for a macroscopic object.

The practical question you raise is: if I have a system that is out of equilibrium with respect to several different mechanisms, how long will I need to wait for the system to reach equilibrium? The answer would be some multiple of the maximum of the various relaxation times.

How much do we need to know about randomization/probability?

Surprisingly little given how much "statistical physics" involves making statistical predictions about nature.

The course is self-contained in this regard, I will explain in lecture (and Schroeder will explain in his text) what little you have to know, which is roughly a few definitions and how to use them at the high school level.

The only concept I have used so far related to randomness or probability is that the ensemble average of the dot product of two unit vectors whose directions are chosen randomly will be zero. This requires only a weak concept of randomness, that in carrying out enough experiments, there will be a roughly equal mix of positive and negative dot products whose average value will be zero.

I understand the calculations we did for the random walk but I'm still having trouble conceptually understanding how it fits in. Could you explain how it affects/fits in with what we are studying?

As we discussed in class, closed nonequilibrium systems, say one with a temperature or concentration difference, spontaneously evolve toward an equilibrium state such that the temperature is everywhere uniform, the chemical potential is everywhere uniform, and so on. (We will soon discuss why this spontaneous behavior arises when we get to Chapters 2 and 3 of Schroeder.)

In gases and liquids, collisions between molecules are the key way that a nonequilibrium system approaches equilibrium[1]. You can think of collisions as the way in which information about one part of the system ("I am hot over here") is transmitted to some other part of the system ("I am cold over here"). So the time it takes a system to approach equilibrium is determined by how rapidly you can transmit information (energy, concentration, pressure) to other parts of the system via molecular collisions.

For example, in a tube of gas that is hotter at one end than the other, the molecules are on average moving faster in the hotter region via Eq. (1.20) on page 13 of Schroeder. These faster moving molecules collide with nearby molecules from a cooler part of the gas and end up transferring some of their kinetic energy to the new molecules, which represents a transfer of energy from a hotter region to a nearby slightly colder region. Similarly, if one part of a tube of gas has a higher concentration of a certain kind of molecule (say more bromine gas Br_2, which is brown and so easily visible), collisions will slowly mix the molecules together so that, over a long period of time (many relaxation times L^2/D where D is the diffusion constant for bromine in air), the concentration will become uniform and equilibrium is attained.

The point of the random walk calculation was that collisions are a slow way to spread information about temperature or concentration through a system and so explains why the time scale to approach equilibrium (the

[1] This is true in solids also, but the objects that collide are less familiar and more complicated, such as quantized sound wave particles called phonons colliding with quasiparticle excitations related to electrons, but the idea is the same.

relaxation time) can be slow, and why relaxation times increase as the square of the system size ($\tau \propto L^2$).

To put this in perspective, if a molecule is moving with a speed v on average and could move in a straight line, it would travel a distance vt in time t. In air at STP (standard temperature and pressure), the average speed of molecules is about the speed of sound, say about 300 m/s, so the time to travel directly across a room that is 5 m in width would be about $5/300 \approx 20\,\text{ms}$. But because an air molecule only travels a short distance of about 300 nm before colliding with another molecule (this is the so-called mean free path), the time for an air molecule to diffuse a distance of 5 m with an experimentally measured diffusion constant of $D \approx 10^{-5}\,\text{m}^2/\text{s}$ would be of order $L^2/D = (5^2/(10^{-5})) \approx 3 \times 10^6\,\text{s} \approx 1\,\text{month}$. In most rooms, there are air currents (say from convection or people moving through the room) that would cause the molecule to move around much more quickly.

If you were forced to bet on where a random walk would end, would it be at the origin (by symmetry)? For example, although the walk may radiate outward, is still the best chance for it to land back at the origin?

There were two related questions:

1. 'Do we need to take a certain number of steps in a random walk to guarantee that the ensemble average term $\langle \sum_{ij} \delta\mathbf{x}_i \cdot \delta\mathbf{x}_j \rangle$ drops out? Are there any good resources you might recommend in random walks?"

2. "Is it possible (albeit highly improbable) that a random walk could produce a distance much greater than $\sqrt{N}d$ for N steps?"

By symmetry, the average value of the final location $\mathbf{X}$ of the random walk must indeed be $\mathbf{0} = (0, 0, 0)$. You can see this also by taking the ensemble average of the final location:

$$\langle \mathbf{X} \rangle = \left\langle \sum_{i=1}^{N} \Delta\mathbf{X}_i \right\rangle = \sum_{i=1}^{N} \langle \Delta\mathbf{X}_i \rangle = \mathbf{0}, \tag{1}$$

since each step vector $\Delta\mathbf{X}_i$ has a random direction and the same length, so the ensemble average of each step must be zero, $\langle \Delta\mathbf{X}_i \rangle = \mathbf{0}$, since you are adding up a lot of vectors that are all the same length but that point in all possible directions.

However, you hopefully appreciate that even though the average final location must be zero, that doesn't mean the standard deviation about the mean is zero, and the calculation we did in class showed that the standard deviation about the origin is $\sqrt{N}d$ if each step has length d.

You can get some intuition of the probability of a random walk ending up some distance away from the origin by running a computer simulation. If you download the Mathematica file

```
www.phy.duke.edu/~hsg/176/lectures/2d-random-walk.nb
```

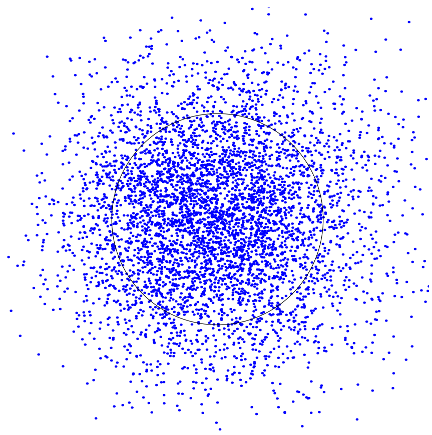
and execute the function

```
RandomWalk2DFinalPoints[ nwalks, nsteps ]
```

you will generate a plot of the final end points of `nwalks` random walks, each with `nsteps` steps of length 1. For example, the command

```
RandomWalk2DFinalPoints[ 5000 , 400 ]
```

generates 5,000 random walks, each with 400 steps. The set of the 5,000 final points looks like this for one particular run:



(and will look slightly different when you run the function because Mathematica uses a different sequence of random numbers each time) where the black circle has radius $R = \sqrt{400} = 20$ and so indicates the typical distance that a random walk of this length will lie from the origin. You can see that the plot is approximately but not exactly symmetrical around the origin (but will be in the limit of infinitely many walks), and that it is not likely for the random walk to end up a distance greater than $\sqrt{N}$ from the origin.

It is possible but unlikely for a random walk to end up far from the origin. This would be equivalent to spinning N successive arrows on a circle randomly such that the arrows all point in about the same direction.

A challenge for those who like mathematical or statistical problems: can you show that the probability density for the random walk to end up a distance r from the origin is a Gaussian of the form $A\exp(-(r/r_0)^2)$, where r is the radial distance from the origin?

If you want to learn more about the properties of random walks, a more detailed discussion is given in Chapter 1 of the book "Fundamentals of Statistical and Thermal Physics" by F. Reif, which is on the 176 reserve in Perkins.

Other than biophysics applications and what we discussed today, what other types of interesting problems in this course can be solved with random walks?

Random walks are greatly valuable in many areas of science, engineering, statistics, and economics. There are many phenomena in the world where some result is the accumulation of many successive uncorrelated steps. Try Googling "random walk".

How accurate is the "random walk" method? It seems like assuming each collision will result in an entirely random direction isn't very accurate.

The random walk model is extremely accurate (say applied to atoms in a gas or liquid) and is widely used in many areas of science, as well as in economics. It is the case that a particular collision does not completely randomize the direction of a given molecule, there is a bit of history retained from one collision to the next. But this history dies out rapidly over several successive collisions so that, after say 5-10 collisions, the particle has lost all knowledge of how it started out and the gas is randomized. Since molecules in a gas at STP collide about a billion times per second (!), it doesn't matter if the randomization occurs over ten successive collisions or just over one collision.

I would have liked to know more specifics about Einstein's paper on Brownian motion and how exactly people measure random walk without computers.

Another student asked a related question: "What is η in Einstein's relation $\langle x^2\rangle = (kT/(3\pi\eta r))t$? How precisely is it known—is it possible to know N_A more precisely than η?"

The technical details of Einstein's paper lie a bit beyond the level of this course. You can find a good discussion in Chapter 15 of the book

"Fundamentals of Statistical and Thermal Physics" by F. Reif, which is on the 176 reserve in Perkins.

The parameter η is the viscosity of the fluid in which one is observing a small macroscopic ball like a pollen grain being kicked around by collisions with the fluid molecules. The viscosity of a fluid is easily measured in a separate experiment but your intuition is right: it is not easy to measure η accurately to more than about 5 digits and this would indeed limit the accuracy of a value of N_A or k that one could deduce. (This was not an issue for the first test of Einstein's theory by Jean Perrin, since this goal was just to test the theory, not to get a super-accurate measurement of N_A.)

This paper:

`prl.aps.org.proxy.lib.duke.edu/abstract/PRL/v106/i3/e030801`

published just this last week, mentions a new way to measure Avogadro's number accurately (to about eight significant digits) by counting all the atoms in a pure sphere of silicon.

There are many ways to measure random walks experimentally without computers. One is to use the method of physicist Jean Perrin, just follow the jiggling of a pollen grain with a microscope and plot the 2D position every few moments. Another you can do yourself for a one-dimensional random walk: take a penny, flip it, and take one step forward if you get heads, one step backwards if you get tail, and see how far you get in 100 tosses.

If quantum tunneling take approximately 10^{10} years to occur, then how did scientists figure out that this was in fact appearing? What experiments did they use to prove this?

The formula for the probability of tunneling of a particle of mass m through a potential barrier of height ΔV is sensitive to the mass, and decreases rapidly with increasing m. For light particles such as electrons, hydrogen or helium, it is possible to observe and measure tunneling directly in the laboratory and such tunneling is actually important on a daily basis, e.g., numerous chemical reactions in your body involve the forming and breaking of hydrogen bonds, and quantum tunneling helps these processes.

For more massive atoms such as carbon and oxygen and especially iron, one has to extrapolate the theory beyond its range of experimental testability. But the quantum mechanics of tunneling is so well understood and so thoroughly confirmed experimentally that there is great confidence about its application to phenomena that span the age of the universe or longer.

You may enjoy reading the paper “Time without end: Physics and biology in an open universe” by Freeman Dyson, which you can download via this URL:

`http://rmp.aps.org/pdf/RMP/v51/i3/p447_1`

In this article, Dyson discusses the long-term state of the universe, including a discussion of how solid matter is actually liquid on cosmological times because of tunneling.

What would be the most efficient method to defrost a whole mammoth? Modify a microwave oven to a wavelength that can pierce through a mammoth?

You can deduce an answer from the fact that the thermal relaxation time scales as L^2/κ where κ is the thermal diffusivity. This formula suggests two strategies. One is to decrease L, which corresponds to hacking the mammoth into small pieces and cooking the pieces separately.

The second strategy is to increase κ by increasing the heat conduction. This could be achieved by driving long iron or copper nails into the mammoth, which would then conduct heat more rapidly into the interior. This idea is practical and there are numerous such “heat pipes” you can buy to speed up the cooking of a turkey.

How would organs defrost, say in relation to skin, etc?

Except for bone, all parts of your body have roughly the same amount of water and so all would defrost at about the same rate, comparable to a similar amount of ice.

Can the photon gas in intergalactic space be controlled or harnessed in any fashion?

I don’t know of any way that the photon gas (called the “cosmic background radiation” or “cosmic microwave radiation”) can be controlled or harnessed, if anything because its temperature $T \approx 3K$ is so cold. (You will learn later this semester to calculate how much thermal energy is stored per unit volume in this photon gas.)

The photon gas is mainly important for scientific and philosophical reasons since it provides some of the most convincing evidence that the universe emerged from a super-hot Big Bang about 14 billion years ago, and provides information about the universe (amount of dark matter, dark energy, geometric curvature).

The photon gas has other implications. For example, it determines the maximum possible energy of a cosmic ray that could strike the Earth's atmosphere because cosmic rays, which are ultrarelavistic protons that presumably were ejected into space by a collapsing star as it underwent a supernova explosion, collide with the photons in the photon gas on its way to Earth and so lose energy.

If the human race ever develops the capacity for interstellar flight, the photon gas will be a nuisance. For example, if you speed a rocket up to close to the speed of light, the Doppler shift will cause those harmless cold microwave photons to appear as high-energy X-rays or gamma rays that can damage the hull of the rocket and hurt the people inside. You could build some thick shielding to protect the rocket and people, but that increases the mass of the rocket a lot and so the expense of traveling to the stars.

What are some of the definitions of random processes by Knuth? What random number generator does Mathematica use?

A related question was: "What are the 16 definitions of randomness mentioned in class, along with their associated counterexamples?"

For both questions, please look at the book "The Art of Computer Programming, Volume 2: Seminumerical Algorithms (3rd edition)" by Donald Knuth, Chapter 3 with title "Random Numbers.

With a few exceptions, physicists and scientists don't worry about the subtleties of how to define a random sequence, this is rarely a key issue in trying to understand nature. For example, myself and other colleagues at Duke in our nonlinear dynamics research often want to determine whether a time series, say measured from a turbulent fluid or from an electrode in a mouse's brain, is nonperiodic or not. A practical test is to Fourier analyze the time sequence and calculate something called the power spectrum $P(\omega)$ of the time series. If this power spectrum has so-called continuous features (instead of well isolated discrete peaks), that would strongly support the hypothesis that the time series is nonperiodic, which is a weak kind of randomness known as "chaos".

I can give one example of a deeper concept of randomness which is the idea of a k-distributed sequence. The idea is that if you have an infinite sequence of numbers

$$x_1, x_2, x_3, \ldots, \tag{2}$$

you can create an infinite sequence of 2-vectors by grouping successive neighbors of numbers:

$$\mathbf{x}_1 = (x_1, x_2), \quad \mathbf{x}_2 = (x_3, x_4), \quad \mathbf{x}_3 = (x_5, x_6), \ldots, \tag{3}$$

or more generally an infinite sequence of k-vectors by taking k numbers at a time as the components of k-dimensional vectors. This is a way to "embed" a one-dimensional sequence into a high-dimensional space.

Then one definition of randomness for an infinite sequence is that the sequence is "∞-distributed": for each integer $k \geq 1$, the cloud of k-dimensional points you get by embedding the sequence in k dimensions should be uniformly distributed (the cloud has no particular structure in any dimensional space).

I don't know what is the random number generator used by Mathematica. The fact that the company that sells Mathematica (Wolfram Research) does not tell the public what algorithms are being used is arguably the single greatest drawback to using Mathematica. For many casual uses, like for this course, it is not a problem, but there are delicate problems in science and mathematics such as the quality of the random numbers, that you can't answer without knowing the algorithm.

How can you explain how the formula for the ensemble average is derived, the $(1/K)\sum_{k=1}^{N} f^{(}k)$?

This formula represents a definition and so can not be derived. If you repeat an experiment many times, always starting in the exact same way, then the ensemble average would be the usual arithmetic average applied to some detail that varies from experiment to experiment.

The reason why one makes a distinction between an ensemble average and the usual statistical average is that there are phenomena in nature that are not statistically stationary, i.e., the statistical properties change over time (say because a system is slowly heating up during an experiment). This means that one has to be careful when carrying out averages if those averages involve data spread out over time. An ensemble average allows one to calculate an average at a particular moment in time by averaging over many different experiments that evolve just a little into the future. Ensemble averages usually can not be evaluated experimentally, they can be computed theoretically or approximated by computer simulations.

Is there anything that is truly random? Are quantum systems truly random?

There was a related question: "Is the movement of the atoms really random, or at least nearly random?"

This question is impossible to answer by experiment because the concept of randomness is meaningful only for infinitely long sequences of numbers and it is impossible for scientists to measure and analyze an infinite amount of data. (It is not even clear that there is an infinite amount of detail in a finite universe.) Another confound is that all measurements have a limited accuracy, typically less than six significant digits.

One does not need a deep definition of randomness for a random walk theory to be highly accurate, all you need is for collisions to decorrelate over a modest number of successive steps so that a sum of dot products will cancel each other out.

Quantum theory does make a strong prediction that observations of a system should be purely random. For example, if you shoot linearly polarized photons at a linear polarizer that is rotated 45° from the axis of polarization, then quantum mechanics predicts that the probability for a linearly polarized photon to pass through the filter should be 1/2 and each actual physical event should be random (no detectable correlations whatsoever). Experiments confirm these expectations but I don't know what the state-of-the-art is. This article might help you get into the literature:

`//www.nature.com/news/2010/100414/full/news.2010.181.html`

as well as this paper

`www.opticsinfobase.org/viewmedia.cfm?uri=josab-27-8-1594&seq=0`

If you like science fiction, try reading the novel "His Master's Voice" by Stansilaw Lem, a Polish science fiction writer who was being considered for a Nobel Prize in literature (he died in 2006 before getting a Prize). The novel concerns a crackpot scientist who comes up with the idea of pointing a radio telescope to some point in the sky between the stars and galaxies and using the recorded hiss of empty space as a source of high quality random numbers. But it turns out that scientists using his table of numbers not only find correlations but discover an alien message hidden in the numbers. The novel describes the heroic but ultimately unsuccessful effort to decode the message.

Is there a relationship between the random walk simulation we were shown and Monte Carlo simulations?

Monte Carlo simulations generally mean a simulation that uses random numbers to obtain an answer so a computer random walk would be an example of a Monte Carlo simulation although there are Monte Carlo simulations that have nothing to do with random walks.

Confusion questions

There were several questions of the following sort:

1. "I'm still unclear how we arrived at L^2/κ based on today's derivation."
2. "I was a little confused about how $|X|$ represents the average distance traveled."

Without asking me a specific question, it is too hard for me guess or know what to say. Please feel free to meet with me outside of class and I will see if I can be helpful.