Phy 162 - Quiz 3 (2/4/2015) - Solutions

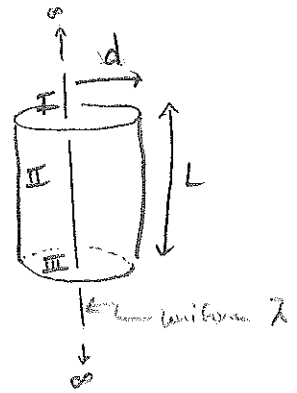
1. Cylindrical symmetry of ∞ wire implies $\vec{E}(\vec{r}) = E(r)\,\hat{r}$

a function only of radial dist from wire

pointing radially

(This is sufficient, but we could make it more explicit. Ask if you have questions.)

Choose Gaussian cylinder of radius d and arbitrary length L concentric to wire.



Use Gauss's law

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$

Evaluate LHS (Left-Hand Side) for our Gaussian surface:

$$\Phi = \oint \vec{E}\cdot d\vec{A} = \underbrace{\int_{I} \vec{E}\cdot d\vec{A}}_{0} + \int_{II} \underbrace{\vec{E}\cdot d\vec{A}}_{\text{vectors } \parallel} + \underbrace{\int_{III} \vec{E}\cdot d\vec{A}}_{0}$$

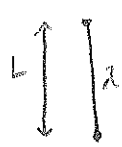
and E const at radius d

(dot product vanishes)

$$= E(d)\,A_{II} = E\cdot 2\pi d L$$

E evaluated at $r=d$

Evaluate RHS: Enclosed charge is a line segment of length L:



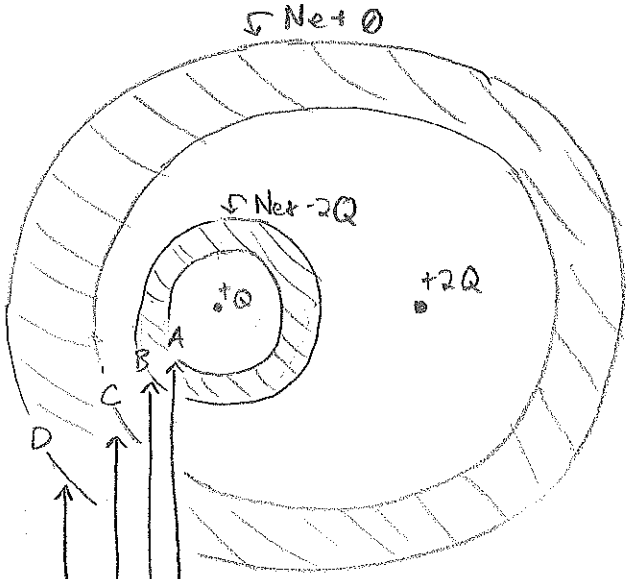
$$\Longrightarrow Q_{enc} = \lambda L$$

So Gauss's Law gives $E\,2\pi d L = \dfrac{\lambda L}{\varepsilon_0} \longrightarrow E = \dfrac{\lambda}{2\pi\varepsilon_0 d}$

Therefore the field as a function of radial coordinate r is

$$\boxed{\vec{E}(r) = \frac{\lambda}{2\pi\varepsilon_0}\frac{1}{r}\,\hat{r}}$$

2. (a)



A: $\boxed{-Q}$, because we require $Q_{enc} = 0$ for any Gaussian surface residing between A + B

$\boxed{\text{Uniform}}$, b/c point charge +Q is centered and surface A is oblivious to charges outside

B: $\boxed{-Q}$, leftover from net −2Q

$\boxed{\text{Nonuniform}}$, because the pt charge +2Q and surface C will polarize it in some nontrivial way

C: $\boxed{-Q}$, b/c require $Q_{enc} = 0$ for a G surf residing b/t C and D, and the charge within C is $(+2Q) + (+Q) + (-Q) + (-Q) = +Q$

$\boxed{\text{Nonuniform}}$ because surface B and pt charge +2Q polarize it

D: $\boxed{+Q}$, leftover from net 0 charge on outside shell

$\boxed{\text{Uniform}}$, because it is not influenced by the charges inside D, there are no charges outside D, and D is a sphere.

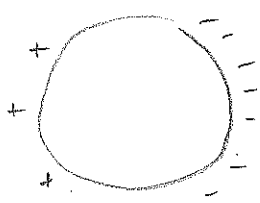
2.(b) There is not enough information to determine exactly what will happen, but we can describe the factors at play.

- $+Q$ point charge: Initially, will not move.

  It is in an unstable equilibrium at the center of the $-2Q$ shell. (Will later be attracted to nearest point of shell)

The rest of the motion depends on the interaction between the $-Q$ at surf C, the $-Q$ at surf B, and the $+2Q$ point charge. The effects are indeterminate because the distances are unknown.

- Outer shell: Surface C will polarize something like



and will experience forces

strong attraction to $+2Q$

possible attraction or repulsion to $-Q$ on B based on distance/polarization

So will probably move left

- Inner shell: Surface B will look something like

with forces

possible attr/repuls to $-Q$ on C

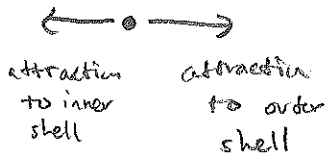
Strong attraction to $+2Q$

So it will probably move right

2. (b) cont'd

- $+2Q$ point charge:

Will experience forces

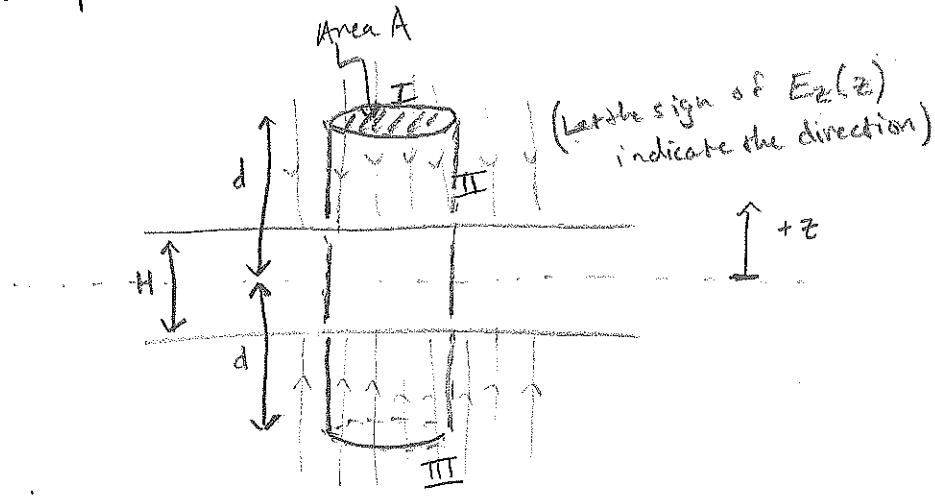


It is not clear which force will be larger — we would need explicit distances (and the problem would be a mess).

Will move left or right.

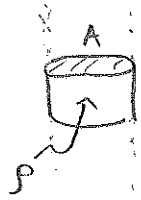
3. (a) By plane symmetry, $\vec{E} = E_z(z)\,\hat{z}$ (see coordinates below)

Choose a "Gaussian pill box" (cylinder or rect prism which straddles the slab).



Mass:

$$M_{enc} = \rho \cdot AH$$



Flux:

$$\Phi = \Phi_I + \underbrace{\cancel{\Phi_{II}}}_{0 \text{ due to dot product}} + \Phi_{III}$$

and $\Phi_I = \Phi_{III}$, so

$$\Phi = 2\Phi_I = 2\int_I \vec{E}_g \cdot d\vec{A} = 2\int_I \left(E_z(d)\,\hat{z}\right)\cdot\left(dA\,\hat{z}\right) = 2E_z(d)\,A$$

Now Gauss's law for gravitation (missing a "−" sign as given on the quiz) is

$$\Phi = \frac{-M_{enc}}{\varepsilon_G} = -4\pi G M_{enc}$$

Substituting,

$$2E_z A = -4\pi G(\rho AH)$$

So $\boxed{\vec{E} = -2\pi G H \rho\,\hat{z}}$ for points above the slab.

3. (b) $g = E = 2\pi G H \rho$

$$H = \frac{g}{2\pi G \rho}$$

$$\frac{H}{D} = \frac{g}{2\pi G \rho D}$$

$g \approx 10\ m/s^2 \longrightarrow 10$ SI units

$* \ 2\pi \approx 10 \longrightarrow 10$

$** \ G \approx 10^{-10}\ m^3 kg^{-1} s^{-1} \rightarrow 10^{-10}$ SI units

$\rho \approx 6000\ kg\ m^{-3} \rightarrow 10^4$ " "

$D \approx 13000\ km \rightarrow 10^7$ " "

$$\frac{H}{D} \approx \frac{10}{10 \cdot 10^{-10} \cdot 10^4 \cdot 10^7}$$

$$\boxed{H/D \approx 0.1}$$

* Note – you may have also taken this as $2 \rightarrow 1$ and $\pi \rightarrow 1$ separately, which would give $H/D \approx 1$

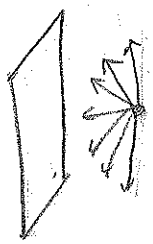
** Usually written $6.67 \times 10^{-11}\ \frac{Nm^2}{kg^2}$

T/F Questions

1. F Insufficient symmetry

2. F Not a conductor or a sphere

3. T Half of total flux goes through the plane



(Recall total flux – ie, through closed surface – is q/ε_0 by Gauss's law )

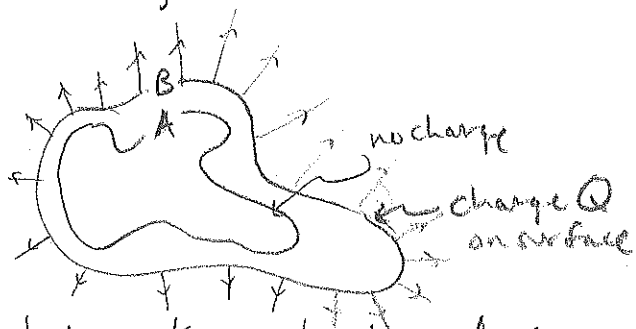
More details on the T/F Questions

1. Gauss's law is always true, but it's not always useful. We rely on the situation having a high degree of symmetry (spherical, cylindrical, or planar) in order to use Gauss's law as a calculational tool. The essential requirement is that at each piece of our Gaussian surface, we can be guaranteed that $\vec{E} \perp d\vec{A}$ or $\vec{E} \parallel d\vec{A}$ with $\vec{E}$ constant in magnitude over that piece.

   Note that the faces of the cube <u>cannot</u> be treated as ∞ planes, because we're interested in all points in space, not just close to the plane, far from the edges.

(T/F Details)

2. Consider an arbitrarily-shaped conducting shell w/ net charge and no charge in its interior. (in static equilibrium)



From what we know about conductors, $\vec{E}=0$ inside surface "B". The charge has arranged itself on the surface in just the right (non-uniform!) way to ensure this.

If we were to carefully replicate this charge distribution on the surface of an otherwise neutral insulator, we would find the same result — $\vec{E}=0$ inside. But this is the unique charge distribution that gives this result. If we move any of the charge around, we no longer have $\vec{E}=0$ inside.

The only case for which this special charge distribution is a uniform σ is (every physicist's favorite shape) the sphere.

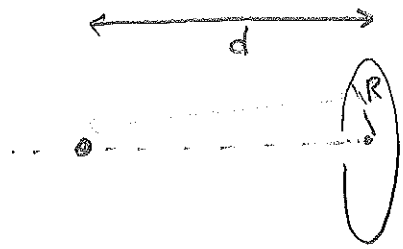
Note also that for the problem at hand, Gauss's law does not help. Okay, so you put a Gaussian surface inside the cube, and $Q_{enc}=0 \Rightarrow \Phi=0$. What does this tell you about $\vec{E}$? Not much.

$$\Phi = \oint \vec{E}\cdot d\vec{A}$$

but $\vec{E}$ is not uniform and cannot be pulled out of the integral.

(T/F Details)

3. For the skeptical, we can solve this analytically using your result from HW3, Problem 1.



$$\Phi = \frac{Q}{2\varepsilon_0}\left(1 - \frac{d}{\sqrt{d^2+R^2}}\right)$$

We want to take the infinite-plane limit:

$$\lim_{R\to\infty} \Phi = \lim_{R\to\infty} \frac{Q}{2\varepsilon_0}\left(1 - \frac{d}{R}\left(1+d^2/R^2\right)^{-1/2}\right)$$

$$= \frac{Q}{2\varepsilon_0}\left(1 - 0\right)$$

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