1. Cylindrical symmetry of an infinite wire implies $E(\hat{r}) = E(r) \hat{r}$

(This is sufficient, but we could make it more explicit.
Ask if you have questions.)

Choose a Gaussian cylinder of radius $d$ and arbitrary length $L$ concentric to wire.

Use Gauss's law

$$ \oint E \cdot d\mathbf{A} = \frac{Q_{enc}}{\varepsilon_0} $$

Evaluate LHS (Left-Hand Side) for our Gaussian surface:

$$ \Phi = \oint E \cdot d\mathbf{A} = \int_{I} E \cdot d\mathbf{A} + \int_{II} E \cdot d\mathbf{A} + \int_{III} E \cdot d\mathbf{A} $$

and $E$ is constant on radius $d$

$$ = E(d) A_{II} = E \cdot 2\pi d L $$

Evaluate RHS. Enclosed charge is a line segment of length $L$.

$$ L \int_{0}^{L} \lambda = \frac{Q_{enc}}{\varepsilon_0} = \lambda L $$

So Gauss's Law gives $E(\hat{r}) = \lambda L$.

Therefore the field as a function of radial coordinate $r$ is

$$ E(r) = \frac{\lambda}{2\pi \varepsilon_0} \frac{1}{r} \hat{r} $$
2. (a)

A: \( -Q \), because we require \( \Delta q = 0 \) for any Gaussian surface residing between A & B

- **Uniform**, b/c point charge +Q is centered and surface A is oblivious to charges outside

B: \( -Q \), leftover from net -2Q

- **Non-uniform**, because the pt charge +2Q and surface C will polarize it in some nontrivial way

C: \( -Q \), b/c require \( \Delta q = 0 \) for a G surf. residing b/t C and D, and the charge within C is \((+2Q) + (+Q) + (-Q) + (-Q) = +Q \)

- **Non-uniform**, because surface B and pt charge +2Q polarize it

D: \( +Q \), leftover from net 0 charge on outside shell

- **Uniform**, because it is not influenced by the charges inside D. there are no charges outside D, and D is a sphere.
2. (b) There is not enough information to determine exactly what will happen, but we can describe the factors at play.

- **+Q point charge**: Initially, will not move. It is in an unstable equilibrium at the center of the -2Q shell. (Will later be attracted to nearest point of shell)

The rest of the motion depends on the interaction between the -Q at surf C, the -Q at surf B, and the +2Q point charge. The effects are indeterminate because the distances are unknown.

- **Outer shell**: Surface C will polarize something like

  ![Diagram of a charged sphere with positive and negative charges]

  and will experience forces:

  Possible strong attraction or repulsion to +2Q to -Q on B based on distance/polarization

  So will probably move left.

- **Inner shell**: Surface B will look something like

  ![Diagram of a charged sphere with positive and negative charges]

  With forces:

  Possible strong attraction to -Q on C to +2Q

  So it will probably move right.
2. (b) cont'd

- +20 point charge:
  
  Will experience forces

  ![Diagram showing forces](attachment:diagram.png)

  Attractions to inner shell and to outer shell

  It is not clear which force will be larger — we would need explicit distances (and the problem would be messy).

  Will move left or right.
3. (a) By plane symmetry, \( \bar{E} = E_z(z) \hat{z} \) (see coordinates below).

Choose a "Gaussian pillbox" (cylinder or rect prism which straddles the slab).

![Diagram of a Gaussian pillbox](image)

**Mass:**

\[ M_{\text{enc}} = \int_A \rho \, dA \]

**Flux:**

\[ \Phi = \Phi_I + \Phi_{\Pi} + \Phi_{\text{III}} \]

\( \Phi_I \) due to dot product

and \( \Phi_{\Pi} = \Phi_{\text{III}} \), so

\[ \Phi = 2 \Phi_I = 2 \int_{\Pi} E_z \cdot dA = 2 \int_{\Pi} (E_z(d) \hat{z}) \cdot (dA \hat{z}) = 2E_z(d) \pi d \]

Now Gauss's law for gravitation (missing a "-" sign as given on the quiz) is

\[ \Phi = \frac{-M_{\text{enc}}}{EG} = -4\pi G \rho \text{Menc} \]

Substituting,

\[ 2E_z \pi d = -4\pi G \rho \text{AH} \]

So

\[ \bar{E} = -2\pi G \rho \hat{z} / \text{H for points above the slab} \]
3. (b) \( g = E = 2\pi G\frac{H}{\rho} \)

\[
H = \frac{g}{2\pi G\rho}
\]

\[
\frac{H}{D} = \frac{g}{2\pi G\rho D}
\]

\[
\frac{H}{D} \approx \frac{10}{10 \cdot 10^{-10} - 10^4 \cdot 10^{-7}}
\]

\[
H/D \approx 0.1
\]

\[
g \approx 10 \text{ m/s}^2 \rightarrow 10 \text{ SI units}
\]

\[
* 2\pi \approx 10 \rightarrow 10
\]

\[
* * G = 10^{-10} \text{ m}^3 \text{kg}^{-1} \text{s}^{-1} \rightarrow 10^{-10} \text{ SI units}
\]

\[
\rho \approx 6000 \text{ kg m}^{-3} \rightarrow 10^4 \text{ SI units}
\]

\[
D \approx 13000 \text{ km} \rightarrow 10^7 \text{ SI units}
\]

* Note — you may have also taken this as \(2 \rightarrow 1\) and \(n \approx 1\) separately, which would give \(H/D \approx 1\)

* * Usually written \(6.67 \times 10^{-11} \text{ Nm}^2 \text{kg}^{-2}\)
T/F Questions

1. F  Insufficient symmetry
2. F  Not a conductor or a sphere
3. T  Half of total flux goes through the plane

(Recall total flux - i.e., through closed surface - is \( \Phi \) by Gauss's law)

More details on the T/F Questions

1. Gauss's law is always true, but it's not always useful. We rely on the situation having a high degree of symmetry (spherical, cylindrical, or planar) in order to use Gauss's law as a calculational tool. The essential requirement is that at each piece of our Gaussian surface, we can be guaranteed that \( \mathbf{E} \cdot d\mathbf{A} \) or \( \mathbf{E} \cdot d\mathbf{A} \) with \( E \) constant in magnitude over that piece.

Note that the faces of the cube cannot be treated as \( \infty \) planes, because we're interested in all points in space, not just close to the plane, far from the edges.
2. Consider an arbitrarily-shaped conducting shell with net charge and no charge in its interior. (in static equilibrium)

From what we know about conductors, \( \vec{E} = 0 \) inside surface "B". The charge has arranged itself on the surface in just the right (non-uniform!) way to ensure this.

If we were to carefully replicate this charge distribution on the surface of an insulator, we would find the same result - \( \vec{E} = 0 \) inside. But this is the unique charge distribution that gives this result. If we move any of the charge around, \( \vec{E} = 0 \) inside.

The only case for which this special charge distribution is a uniform \( \sigma \) is (every physicist's favorite shape) the sphere.

Note also that for the problem at hand, Gauss's law does not help. Okay, so you put a Gaussian surface inside the cube, and \( Q_{enc} = 0 \) \( \Rightarrow \vec{I} = 0 \). What does this tell you about \( \vec{E} \)? Not much.

\[ \vec{I} = \oint \vec{E} \cdot d\vec{A} \]

but \( \vec{E} \) is not uniform and cannot be pulled out of the integral.
3. For the skeptical, we can solve this analytically using your result from HW3, Problem 1.

\[ \Phi = \frac{Q}{2\varepsilon_0} \left( 1 - \frac{d}{\sqrt{d^2 + R^2}} \right) \]

We want to take the infinite-plane limit:

\[
\lim_{R \to \infty} \Phi = \lim_{R \to \infty} \frac{Q}{2\varepsilon_0} \left( 1 - \frac{d}{R} \left( 1 + d^2/R^2 \right)^{-1/2} \right) = \frac{Q}{2\varepsilon_0} \left( 1 - 0 \right) \]

\( \checkmark \)